1.

$4-x^2=0$

$x^2=4$

$x=\pm 2$

$$\int_{-2}^{2} 4-x^2\,dx = \left[4x-\frac{x^3}{3}\right]_{-2}^{2}$$

$$= 8-\frac{8}{3}-\left(-8+\frac{8}{3}\right) = \boxed{\frac{32}{3}}$$

2. $\sin 2x = \cos x$

$2\sin x\cos x = \cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x\,(2\sin x - 1) = 0$

$\cos x = 0 \qquad \sin x = \frac{1}{2}$

$x = \frac{\pi}{2} \qquad x = \frac{\pi}{6}$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x - \cos x\,dx = \left[-\frac{1}{2}\cos 2x - \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= -\frac{1}{2}\cos\pi - \sin\frac{\pi}{2} - \left(-\frac{1}{2}\cos\frac{\pi}{3} - \sin\frac{\pi}{6}\right)$$

$$= \frac{1}{2} - 1 - \left(-\frac{1}{4} - \frac{1}{2}\right) = \frac{1}{4}$$

3. $R = \sin 2x$

$r = \cos x$

$$\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^2 2x - \cos^2 x\,dx = \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}(1-\cos 4x) - \frac{1}{2}(1+\cos 2x)\,dx$$

$$= \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} -\frac{1}{2}\cos 4x - \frac{1}{2}\cos 2x\,dx = \pi\left[-\frac{1}{8}\sin 4x - \frac{1}{4}\sin 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \pi\left[-\frac{1}{8}\sin 2\pi - \frac{1}{4}\sin\pi - \left(-\frac{1}{8}\sin\frac{2\pi}{3} - \frac{1}{4}\sin\frac{\pi}{3}\right)\right] = \pi\left[\frac{1}{8}\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \boxed{\frac{3\pi\sqrt{3}}{16}}$$

4. $\sin x + \cos x = 0$

$\sin x = -\cos x$

$x = \frac{3\pi}{4}, -\frac{\pi}{4}$

$$\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin x + \cos x \, dx = \Big[-\cos x + \sin x\Big]_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

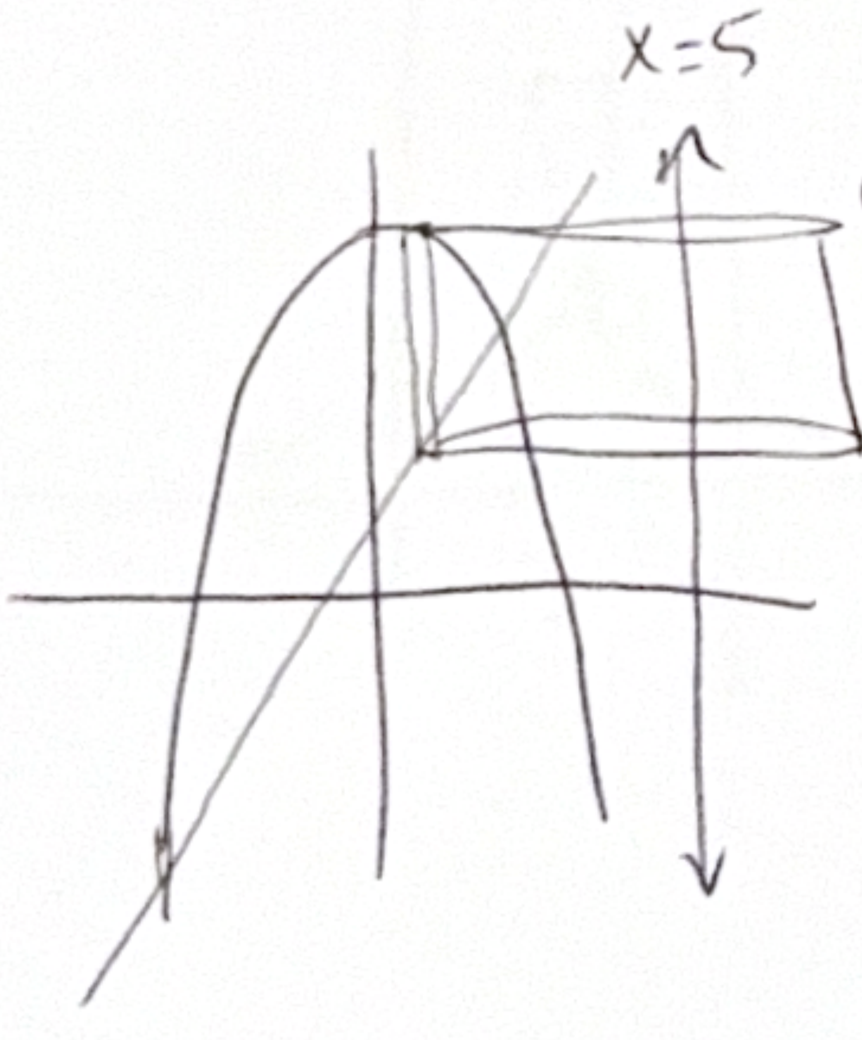
$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) = \boxed{2\sqrt{2}}$$

5. $$\pi \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sin x + \cos x)^2 dx = \pi \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2 x + \cos^2 x + 2\sin x \cos x \, dx = \pi \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} 1 + \sin 2x \, dv$$

$$= \pi \Big[x - \tfrac{1}{2}\cos 2x\Big]_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} = \pi\left[\frac{3\pi}{4} - \frac{1}{2}\cos\frac{3\pi}{2} - \left(-\frac{\pi}{4} - \frac{1}{2}\cos\left(-\frac{\pi}{2}\right)\right)\right]$$

$$= \pi(\pi - 0 + 0) = \boxed{\pi^2}$$

6.



$9 - x^2 = 2x + 1$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

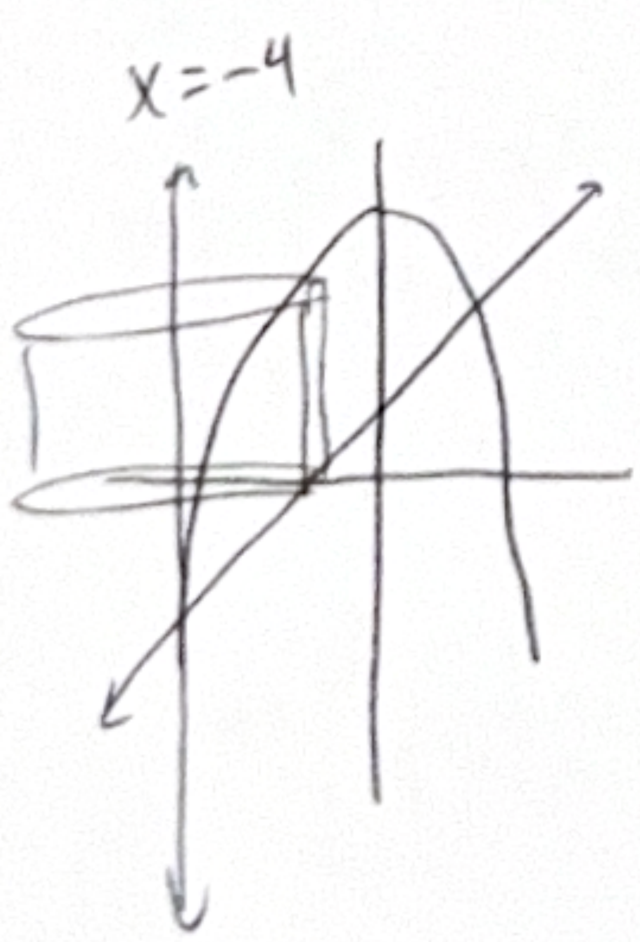
$x = -4 \quad x = 2$

$h = 9 - x^2 - (2x+1)$

$r = 5 - x$

$$2\pi \int_{-4}^{2} (5-x)(9 - x^2 - (2x+1)) \, dx \approx 1357.168$$

7.

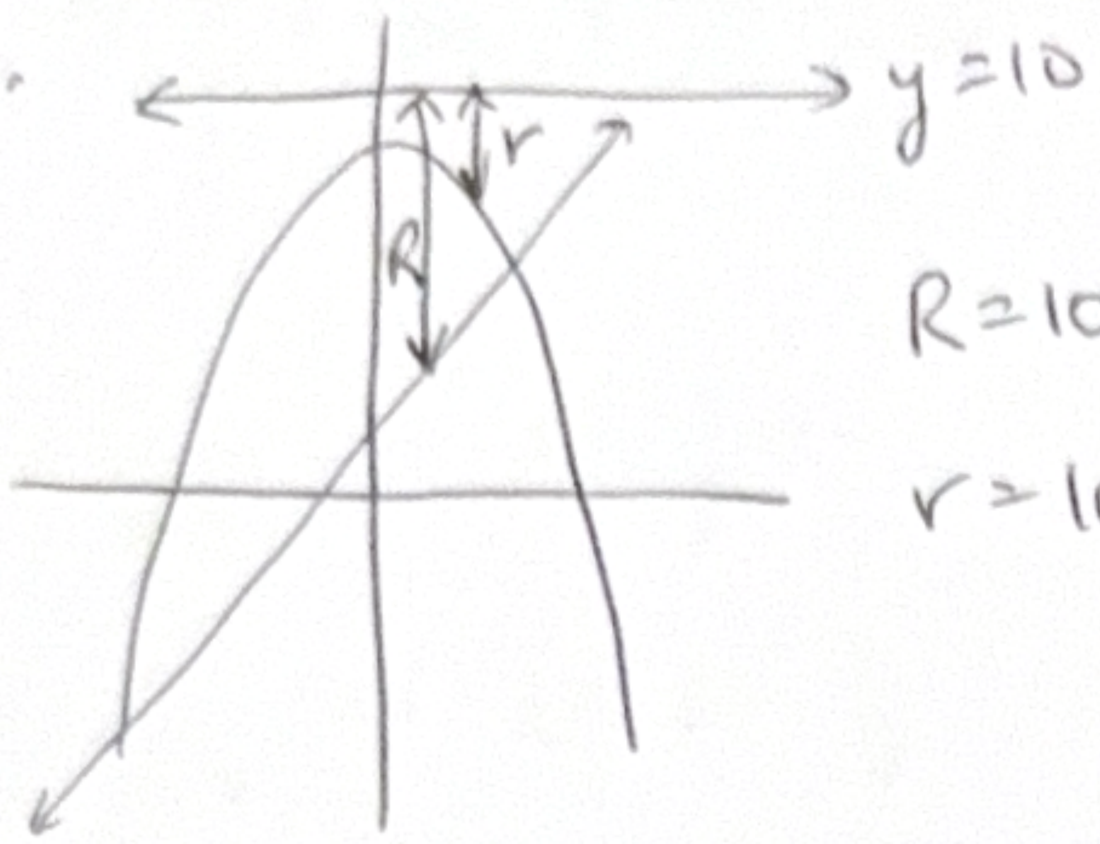


$r = 4 + x$

$h = 9 - x^2 - (2x+1)$

$$2\pi \int_{-4}^{2} (4+x)(9 - x^2 - (2x+1)) \, dx \approx 678.584$$

8.



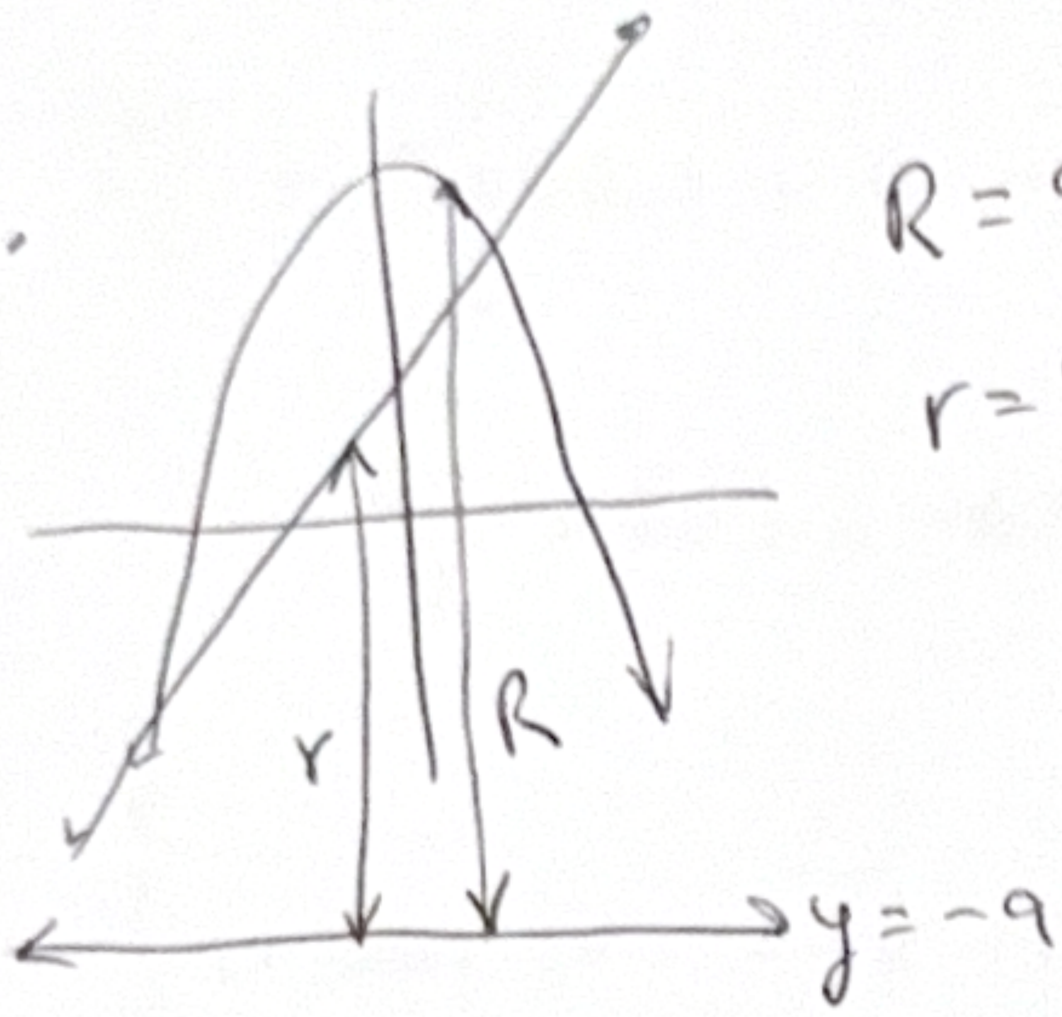
$R = 10 - (2x+1)$

$r = 10 - (9-x^2)$

$$\pi \int_{-4}^{2} [10-(2x+1)]^2 - [10-(9-x^2)]^2 \, dx$$

$$\approx 1673.841$$

9.



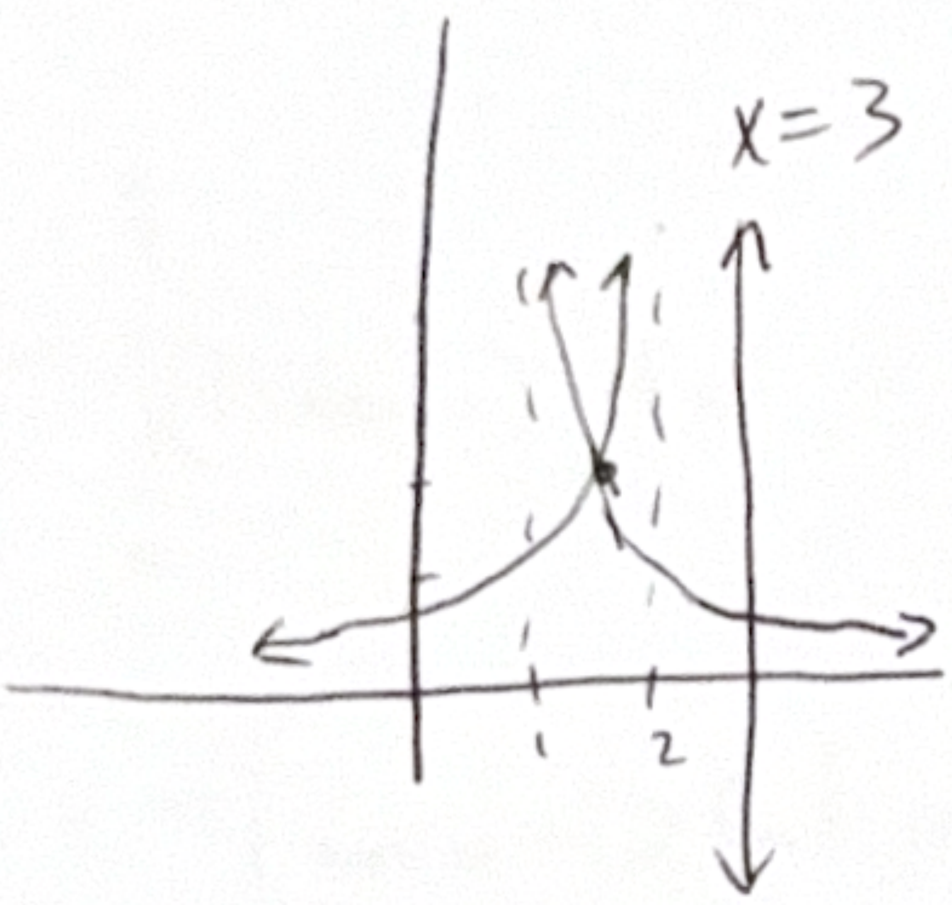
$R = 9 + (9-x^2)$

$r = 9 + (2x+1)$

$$\pi \int_{-4}^{2} (18-x^2)^2 - (2x+10)^2 \, dx$$

$$\approx 2623.858$$

10.



$\frac{1}{x-1} = \frac{1}{2-x}$

$2-x = x-1$

$2x = 3$

$x = \frac{3}{2}$

$$\int_0^{3/2} \frac{1}{2-x}\,dx + \int_{3/2}^{3} \frac{1}{x-1}\,dx$$

$$= -\ln|2-x|\Big]_0^{3/2} + \ln|x-1|\Big|_{3/2}^{3}$$

$$= -\ln\tfrac{1}{2} - (-\ln 2) + \ln 2 - \ln\tfrac{1}{2}$$

$$= 4\ln 2$$

11. $$\pi\left[\int_0^{3/2} \left(\frac{1}{2-x}\right)^2 dx + \int_{3/2}^{3} \left(\frac{1}{x-1}\right)^2 dx\right] = \pi\left[\int_0^{3/2} (2-x)^{-2}\,dx + \int_{3/2}^{3} (x-1)^{-2}\,dx\right]$$

$$= \pi\left[\left(\frac{1}{2-x}\right)_0^{3/2} + \left(-\frac{1}{x-1}\right)_{3/2}^{3}\right] = \pi\left[2 - \tfrac{1}{2} + \left(-\tfrac{1}{2} + 2\right)\right] = 3\pi$$

12. $\pi\int_0^{\frac{1}{2}} 3^2\,dy + \pi\int_{\frac{1}{2}}^{2} \left(\frac{1}{y}+1\right)^2 - \left(2-\frac{1}{y}\right)^2 dy$

$y = \frac{1}{\cancel{x}-1} \rightarrow x = \frac{1}{y}+1$

$y = \frac{1}{2-x} \rightarrow x = 2-\frac{1}{y}$

$= \pi\left[9y\right]_0^{\frac{1}{2}} + \pi\int_{\frac{1}{2}}^{2} \frac{1}{y^2} + \frac{2}{y} + 1 - \left(4 - \frac{4}{y} + \frac{1}{y^2}\right) dy$

$= \frac{9\pi}{2} + \pi\int_{\frac{1}{2}}^{2} \frac{6}{y} - 3\,dy = \frac{9\pi}{2} + \pi\left[6\ln|y| - 3y\right]_{\frac{1}{2}}^{2}$

$= \frac{9\pi}{2} + \pi\left[6\ln 2 - 6 - \left(6\ln\frac{1}{2} - \frac{3}{2}\right)\right] = \frac{9\pi}{2} + \pi\left[6\ln 2 - 6 - \left(6\ln 1 - 6\ln 2 - \frac{3}{2}\right)\right]$

$= \frac{9\pi}{2} + 6\pi\ln 2 - 6\pi + 6\pi\ln 2 + \frac{3\pi}{2} =$ $12\pi\ln 2$

13. $\int_0^{\frac{\pi}{4}} 1+\tan x\,dx = \left[x - \ln|\cos x|\right]_0^{\frac{\pi}{4}} = \frac{\pi}{4} - \ln\left|\cos\frac{\pi}{4}\right| - (0 - \ln\cos 0)$

$= \frac{\pi}{4} - \ln\frac{\sqrt{2}}{2} = \frac{\pi}{4} + \ln\sqrt{2} =$ $\frac{\pi}{4} + \frac{1}{2}\ln 2$

14. $\pi\int_0^{\frac{\pi}{4}} (1+\tan x)^2\,dx = \pi\int_0^{\frac{\pi}{4}} 1 + 2\tan x + \tan^2 x\,dx = \pi\int_0^{\frac{\pi}{4}} \sec^2 x + 2\tan x\,dx$

$= \pi\left[\tan x - 2\ln|\cos x|\right]_0^{\frac{\pi}{4}} = \pi\left[1 - 2\ln\frac{\sqrt{2}}{2} - (0 - 2\ln 1)\right]$

$= \pi(1 + 2\ln\sqrt{2}) =$ $\pi(1+\ln 2)$

15. shells: $2\pi\int_0^{\frac{\pi}{4}} x(1+\tan x)\,dx \approx$ 3.105210958

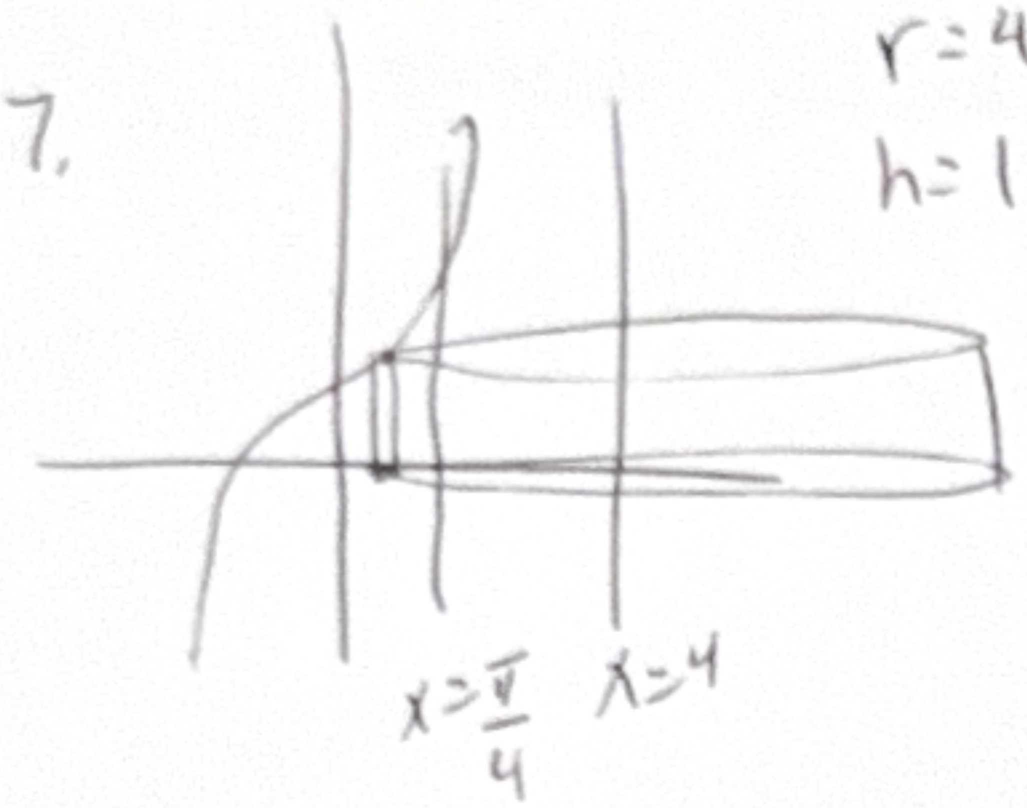
16.

$r = \frac{\pi}{4} - x$

$h = 1+\tan x$

$2\pi\int_0^{\frac{\pi}{4}} \left(\frac{\pi}{4}-x\right)(1+\tan x)\,dx =$ 2.480845743

17.

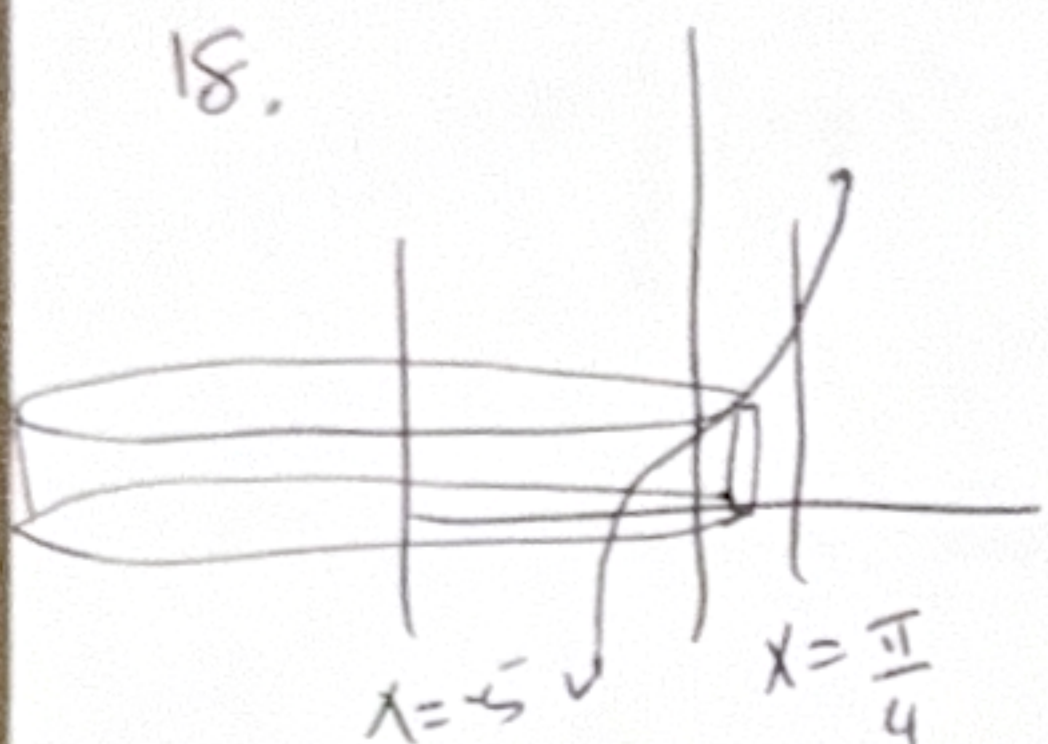


$r = 4 - x$

$h = 1 + \tan x$

$$2\pi \int_0^{\frac{\pi}{4}} (4-x)(1+\tan x)\,dx \approx 25.34434221$$

18.

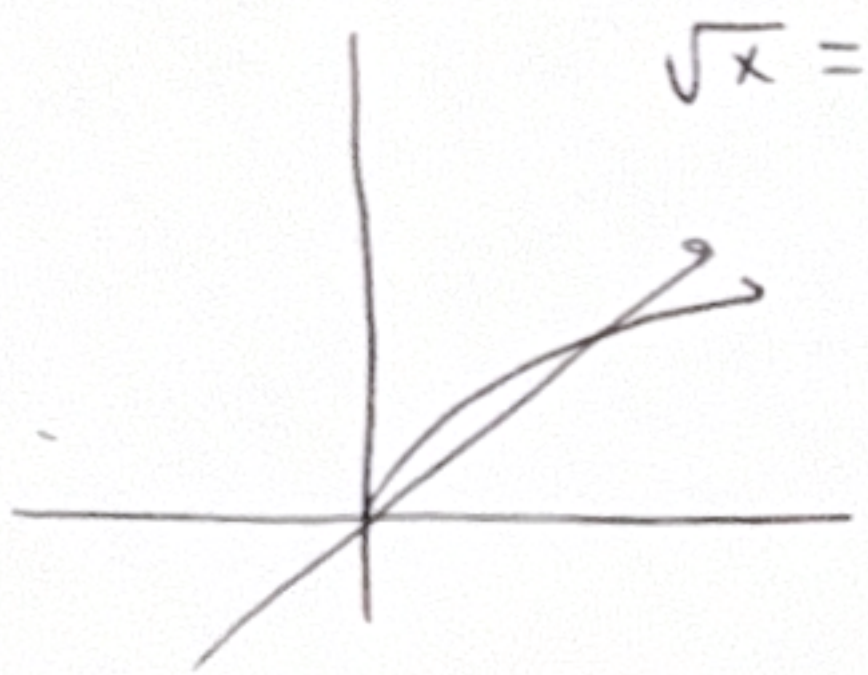


$r = 5 + x$

$h = 1 + \tan x$

$$2\pi \int_0^{\frac{\pi}{4}} (5+x)(1+\tan x)\,dx \approx 38.66715241$$

19.



$\sqrt{x} = x$

$x = 1$

$y = 1$

washers around x-axis

$$\pi \int_0^1 \sqrt{x}^2 - x^2 \,dx$$

20. $y = \sqrt{x} \rightarrow x = y^2$

$y = x \rightarrow x = y$

shells around x-axis

$$2\pi \int_0^1 y(y - y^2)\,dy$$

21. washers around y-axis

$$\pi \int_0^1 y^2 - (y^2)^2 \,dy$$

22. shells around y-axis

$$2\pi \int_0^1 x(\sqrt{x} - x)\,dx$$

23. washers around $x = 1$

$$\pi \int_0^1 (1-y^2)^2 - (1-y)^2 \,dy$$

24. shells around $x = 1$

$$2\pi \int_0^1 (1-x)(\sqrt{x} - x)\,dx$$

25. washers around $y=1$

$$\pi\int_0^1 (1-x)^2 - (1-\sqrt{x})^2\,dx$$

26. Shells around $y=1$

$$2\pi\int_0^1 (1-y)(y-y^2)\,dy$$

27. washers around $x=-1$

$$\pi\int_0^1 (1+y)^2 - (1+y^2)^2\,dy$$

28. Shells around $x=-1$

$$2\pi\int_0^1 (1+x)(\sqrt{x}-x)\,dx$$

29. washers around $y=-2$

$$\pi\int_0^1 (2+\sqrt{x})^2 - (2+x)^2\,dx$$

30. shells around $y=-2$

$$2\pi\int_0^1 (2+y)(y-y^2)\,dy$$