

1. The region in the first quadrant of $x = 2y^2 - y^3$ is revolved around the x-axis.



$$h = x = 2y^2 - y^3$$

$$r = y$$

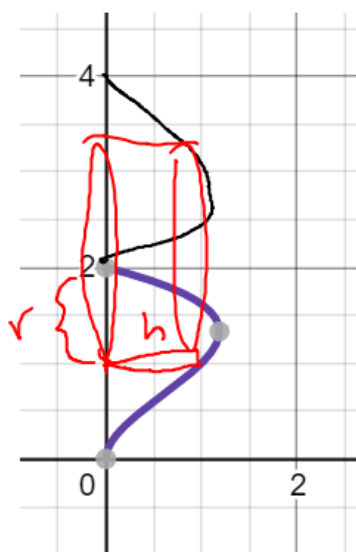
$$V = 2\pi \int_0^2 y(2y^2 - y^3) dy$$

$$= 2\pi \int_0^2 2y^3 - y^4 dy = 2\pi \left[\frac{1}{2} y^4 - \frac{y^5}{5} \right]_0^2$$

$$= 2\pi \left[8 - \frac{32}{5} - 0 \right] = 2\pi \left(\frac{8}{5} \right)$$

$$= \boxed{\frac{16\pi}{5} \text{ units}^3}$$

2. The region in the first quadrant of $x = 2y^2 - y^3$ is revolved around the line $y = 2$.



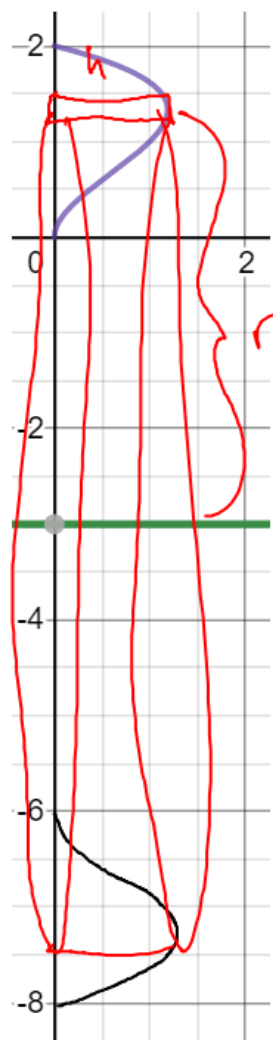
$$h = x = 2y^2 - y^3$$

$$r = 2 - y$$

$$V = 2\pi \int_0^2 (2-y)(2y^2 - y^3) dy$$

$$= 2\pi \int_0^2 4y^2 - 4y^3 + y^4 dy = 2\pi \left[\frac{4}{3}y^3 - y^4 + \frac{y^5}{5} \right]_0^2$$

$$= 2\pi \left[\frac{32}{3} - 16 + \frac{32}{5} - 0 \right] = \boxed{\frac{32\pi}{15} \text{ units}^3}$$



3. The region in the first quadrant of $x = 2y^2 - y^3$ is revolved around the line $y = -3$.

$$h = x = 2y^2 - y^3$$

$$r = 3 + y$$

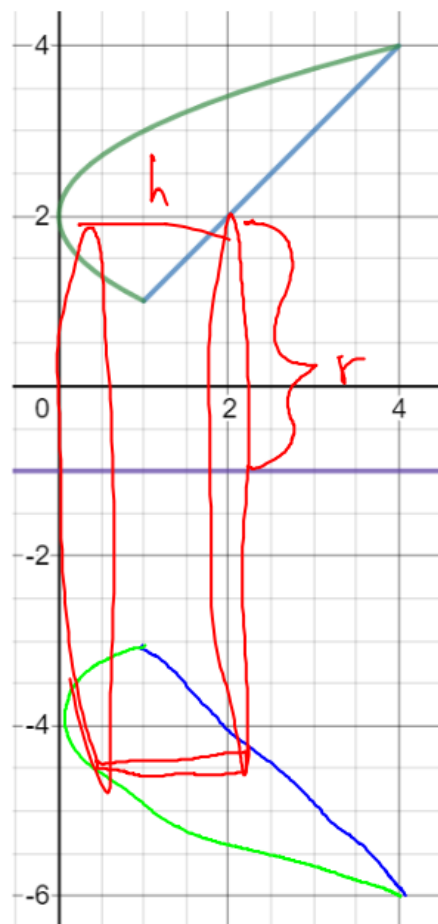
$$V = 2\pi \int_0^2 (3+y)(2y^2 - y^3) dy$$

$$= 2\pi \int_0^2 (6y^2 - y^3 - y^4) dy = 2\pi \left[2y^3 - \frac{y^4}{4} - \frac{y^5}{5} \right]_0^2$$

$$= 2\pi \left[16 - 4 - \frac{32}{5} - 0 \right] = 2\pi \left(\frac{28}{5} \right)$$

$$= \boxed{\frac{56\pi}{5} \text{ units}^3}$$

4. The region between $y = x$ and $x = y^2 - 4y + 4$ is revolved around the line $y = -1$



$$h = x_2 - x_1 = y - (y^2 - 4y + 4)$$

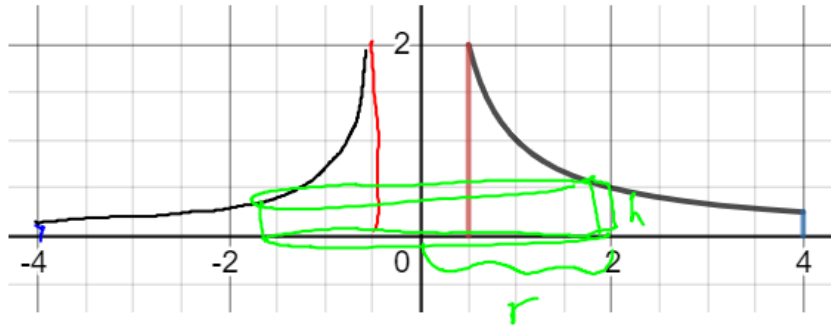
$$r = 1 + y$$

$$V = 2\pi \int_1^4 (1+y)(y - y^2 + 4y - 4) dy = 2\pi \int_1^4 (-4 + y + 4y^2 - y^3) dy$$

$$= 2\pi \left[-4y + \frac{y^2}{2} + \frac{4}{3}y^3 - \frac{y^4}{4} \right]_1^4$$

$$= 2\pi \left[-16 + 8 + \frac{256}{3} - 64 - \left(-4 + \frac{1}{2} + \frac{4}{3} - \frac{1}{4} \right) \right] = \boxed{\frac{63\pi}{2} \text{ units}^3}$$

5. The region bounded by $y = \frac{1}{x}$, $x = \frac{1}{2}$, $x = 4$, and the x-axis is revolved around the y-axis.



$$r = x \quad h = y = \frac{1}{x}$$
$$2\pi \int_{\frac{1}{2}}^4 x \left(\frac{1}{x} \right) dx = 2\pi \int_{\frac{1}{2}}^4 dx$$

$$= 2\pi \left[x \right]_{\frac{1}{2}}^4 = 2\pi \left[4 - \frac{1}{2} \right] = \boxed{7\pi \text{ units}^3}$$