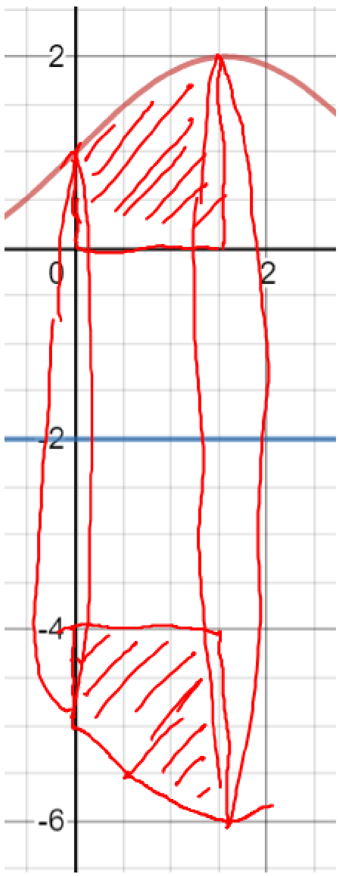
1. The region between $f(x) = 1 + \sin x$ and the x-axis from $x = 0$ to $x = \frac{\pi}{2}$ is revolved around the line $y = -2$.



$R = 2 + 1 + \sin x$

$r = 2$

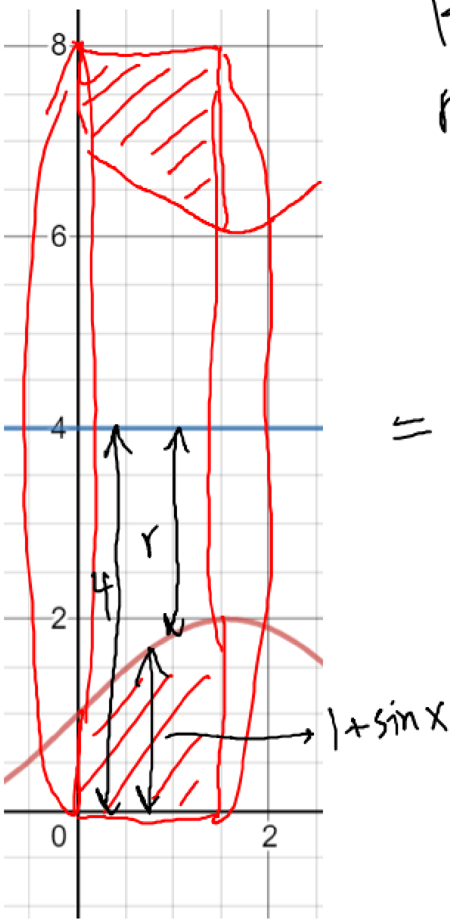
$$\pi \int_0^{\frac{\pi}{2}} (3 + \sin x)^2 - 2^2 \, dx = \pi \int_0^{\frac{\pi}{2}} 9 + 6\sin x + \sin^2 x - 4 \, dx$$

$$= \pi \int_0^{\frac{\pi}{2}} 5 + 6\sin x + \frac{1}{2}(1 - \cos 2x) \, dx = \pi \int_0^{\frac{\pi}{2}} \frac{11}{2} + 6\sin x - \frac{1}{2}\cos 2x \, dx$$

$$= \pi \left[\frac{11x}{2} - 6\cos x - \frac{1}{4}\sin 2x \right]_0^{\frac{\pi}{2}} = \pi \left[\frac{11\pi}{4} - 0 - 0 - (0 - 6 - 0) \right]$$

$$= \boxed{\frac{11\pi^2}{4} + 6\pi \text{ units}^3}$$

2. The region between $f(x) = 1 + \sin x$ and the x-axis from $x = 0$ to $x = \frac{\pi}{2}$ is revolved around the line $y = 4$.



$R = 4$

$r = 4 - (1 + \sin x) = 3 - \sin x$

$$\pi \int_0^{\frac{\pi}{2}} 16 - (3 - \sin x)^2 \, dx = \pi \int_0^{\frac{\pi}{2}} 7 + 6\sin x - \sin^2 x \, dx$$

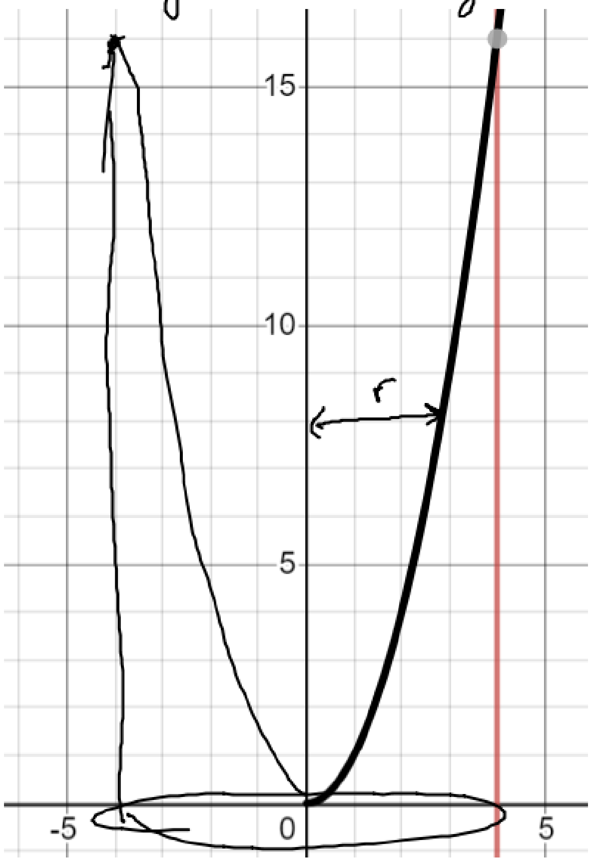
$$= \pi \int_0^{\frac{\pi}{2}} 7 + 6\sin x - \frac{1}{2}(1 - \cos 2x)\, dx = \pi \int_0^{\frac{\pi}{2}} \frac{13}{2} + 6\sin x + \frac{1}{2}\cos 2x \, dx$$

$$= \pi \left[\frac{13x}{2} - 6\cos x + \frac{1}{4}\sin 2x \right]_0^{\frac{\pi}{2}} = \pi \left[\frac{13\pi}{4} - 0 + 0 - (0 - 6 + 0) \right]$$

$$= \boxed{\frac{13\pi^2}{4} + 6\pi \text{ units}^3}$$

3. The region between the x-axis and $f(x) = x^2$ from $x = 0$ to $x = 4$ is revolved around the y-axis.

$y = x^2 \rightarrow x = \sqrt{y}$



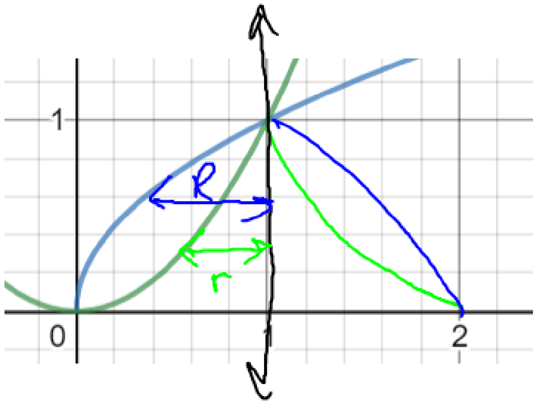
$R = 4 \quad r = x = \sqrt{y}$

$$\pi \int_0^{16} 16 - \sqrt{y}^{\,2}\, dy = \pi \int_0^{16} 16 - y \, dy$$

$$= \pi \left[16y - \frac{y^2}{2} \right]_0^{16} = \pi \left[256 - 128 \right]$$

$$= \boxed{128\pi \text{ units}^3}$$

4. The region enclosed between $f(x) = \sqrt{x}$ and $g(x) = x^2$ is revolved around the line $x = 1$.



$y = \sqrt{x}$ $\quad$ $y = x^2$

$x = y^2$ $\quad$ $x = \sqrt{y}$

$R = 1 - x$ $\quad$ $r = 1 - x$

$= 1 - y^2$ $\quad$ $= 1 - \sqrt{y}$

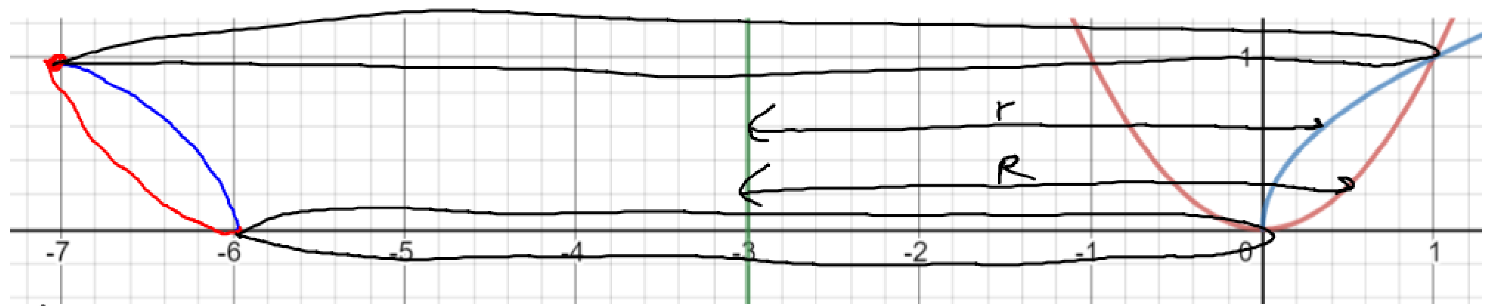
$$\pi \int_0^1 (1-y^2)^2 - (1-\sqrt{y})^2 \, dy = \pi \int_0^1 \left(1 - 2y^2 + y^4 - 1 + 2\sqrt{y} - y\right) dy$$

$$= \pi \left[-\frac{2}{3}y^3 + \frac{y^5}{5} + \frac{4}{3}y^{\frac{3}{2}} - \frac{y^2}{2} \right]_0^1 = \pi \left[-\frac{2}{3} + \frac{1}{5} + \frac{4}{3} - \frac{1}{2} - 0 \right]$$

$$= \boxed{\frac{11\pi}{30} \text{ units}^3}$$

5. The region enclosed between $f(x) = \sqrt{x}$ and $g(x) = x^2$ is revolved around the line $x = -3$.

$x = y^2$ $\qquad$ $x = \sqrt{y}$



$R = 3 + \sqrt{y}$

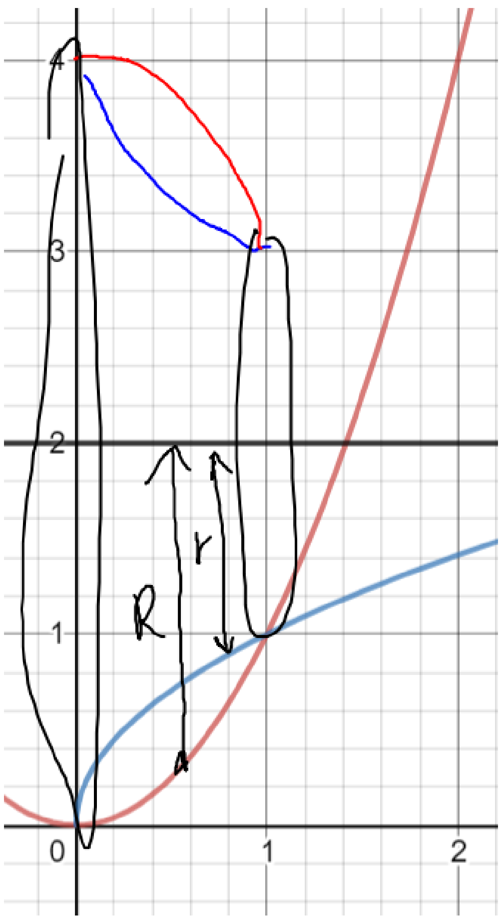
$r = 3 + y^2$

$$\pi \int_0^1 (3+\sqrt{y})^2 - (3+y^2)^2 \, dy = \pi \int_0^1 9 + 6\sqrt{y} + y - 9 - 6y^2 - y^4 \, dy$$

$$= \pi \int_0^1 6\sqrt{y} + y - 6y^2 - y^4 \, dy = \pi \left[4y^{\frac{3}{2}} + \frac{y^2}{2} - 2y^3 - \frac{y^5}{5} \right]_0^1$$

$$= \pi \left[4 + \frac{1}{2} - 2 - \frac{1}{5} - 0 \right] = \boxed{\frac{23\pi}{10} \text{ units}^3}$$

6. The region enclosed between $f(x) = \sqrt{x}$ and $g(x) = x^2$ is revolved around the line $y = 2$.



$R = 2 - x^2 \qquad r = 2 - \sqrt{x}$

$$\pi \int_0^1 (2 - x^2)^2 - (2 - \sqrt{x})^2 \, dx$$

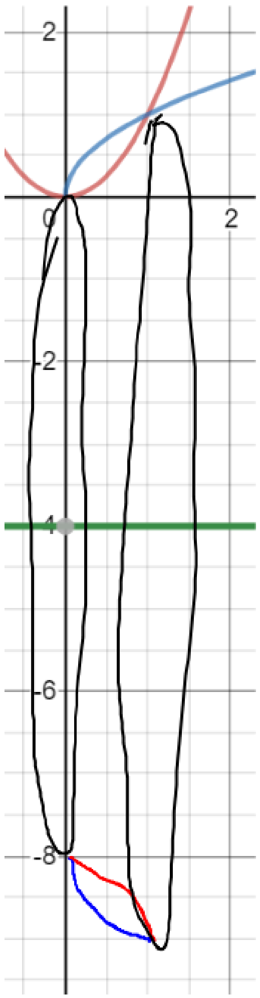
$$= \pi \int_0^1 4 - 4x^2 + x^4 - 4 + 4\sqrt{x} - x \, dx$$

$$= \pi \int_0^1 -4x^2 + x^4 + 4\sqrt{x} - x \, dx$$

$$= \pi \left[-\frac{4}{3}x^3 + \frac{x^5}{5} + \frac{8}{3}x^{\frac{3}{2}} - \frac{x^2}{2} \right]_0^1$$

$$= \pi \left[-\frac{4}{3} + \frac{1}{5} + \frac{8}{3} - \frac{1}{2} - 0 \right] = \boxed{\frac{31\pi}{30} \text{ units}^3}$$

7. The region enclosed between $f(x) = \sqrt{x}$ and $g(x) = x^2$ is revolved around the line $y = -4$.



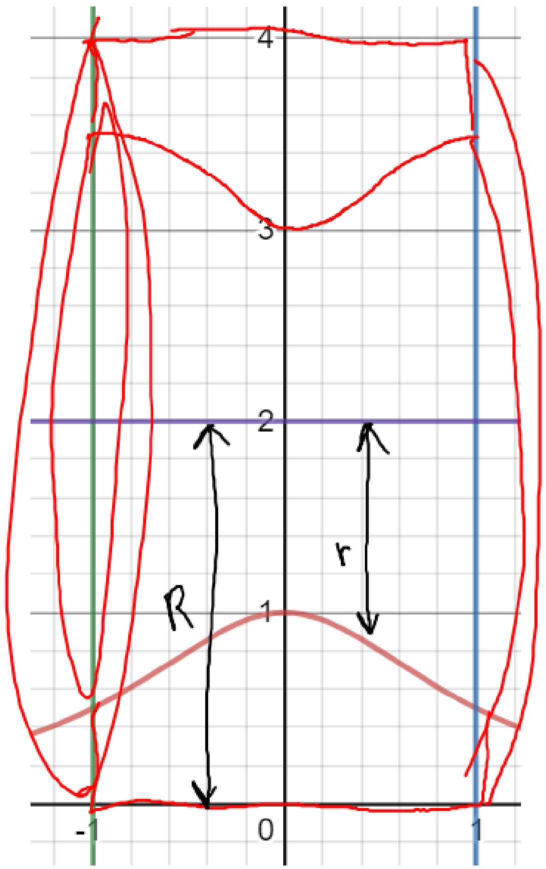
$R = 4 + \sqrt{x} \qquad r = 4 + x^2$

$$\pi\int_0^1 (4+\sqrt{x})^2 - (4+x^2)^2\,dx = \pi\int_0^1 16 + 8\sqrt{x} + x - 16 - 8x^2 - x^4\,dx$$

$$= \pi\int_0^1 8\sqrt{x} + x - 8x^2 - x^4\,dx = \pi\left[\frac{16}{3}x^{\frac{3}{2}} + \frac{x^2}{2} - \frac{8}{3}x^3 - \frac{x^5}{5}\right]_0^1$$

$$= \pi\left[\frac{16}{3} + \frac{1}{2} - \frac{8}{3} - \frac{1}{5} - 0\right] = \boxed{\frac{89\pi}{30} \text{ units}^3}$$

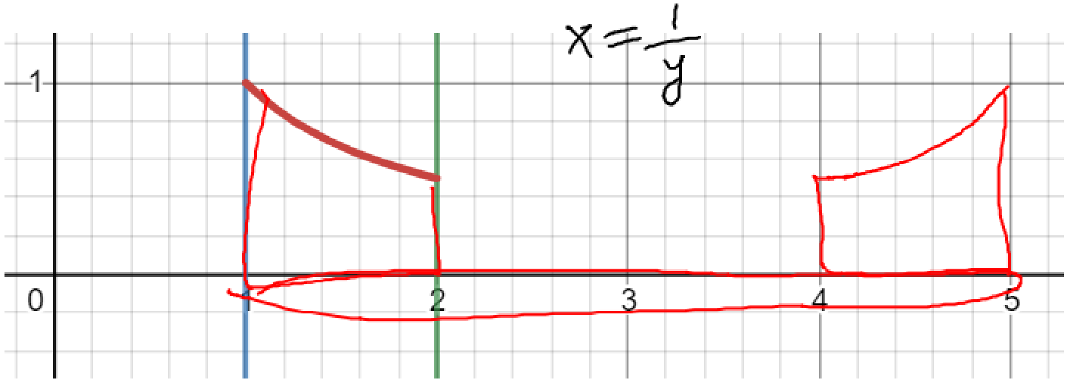
8. The region between $f(x) = \frac{1}{1+x^2}$ from $x = -1$ to $x = 1$ is revolved around the line $y = 2$.



$R = 2 \qquad r = 2 - \frac{1}{1+x^2}$

$\pi \int_{-1}^{1} 4 - \left(2 - \frac{1}{1+x^2}\right)^2 dx \approx \boxed{15.701 \text{ units}^3}$

9. The region between $f(x) = \frac{1}{x}$, $x = 1$, $x = 2$, and the x-axis is revolved around the line $x = 3$.



from $y = 0$ to $y = \frac{1}{2}$

$R = 2 \quad r = 1$

from $y = \frac{1}{2}$ to $y = 1$

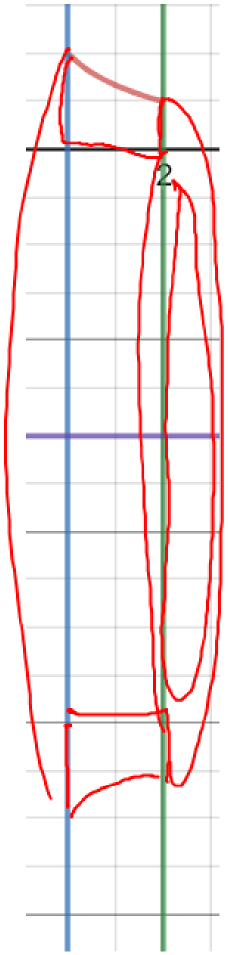
$R = 2 \quad r = 3 - x = 3 - \frac{1}{y}$

$$\pi \int_0^{\frac{1}{2}} 4 - 1 \, dy + \pi \int_{\frac{1}{2}}^{1} 4 - \left(3 - \frac{1}{y}\right)^2 dy = \pi \int_0^{\frac{1}{2}} 3 \, dy + \pi \int_{\frac{1}{2}}^{1} -5 + \frac{6}{y} - \frac{1}{y^2} \, dy$$

$$= \pi \Big[3y\Big]_0^{\frac{1}{2}} + \pi \left[-5y + 6\ln|y| + \frac{1}{y}\right]_{\frac{1}{2}}^{1} = \pi \left[\frac{3}{2} - 0 - 5 + 0 + 1 - \left(-\frac{5}{2} + 6\ln\frac{1}{2} + 2\right)\right]$$

$$= \pi\left(-2 - 6\ln\frac{1}{2}\right) = \boxed{\pi(6\ln 2 - 2) \text{ units}^3}$$

10. The region between $f(x) = \frac{1}{x}$, $x = 1$, $x = 2$, and the x-axis is revolved around the line $y = -3$.



$$R = 3 + \frac{1}{x} \qquad r = 3$$

$$\pi \int_1^2 \left(3 + \frac{1}{x}\right)^2 - 9 \, dx = \pi \int_1^2 \frac{6}{x} + \frac{1}{x^2} \, dx$$

$$= \pi \left[6 \ln|x| - \frac{1}{x} \right]_1^2 = \pi \left[6\ln 2 - \frac{1}{2} - 6\ln 1 + 1 \right]$$

$$= \boxed{\pi \left(6\ln 2 + \frac{1}{2} \right) \text{ units}^3}$$