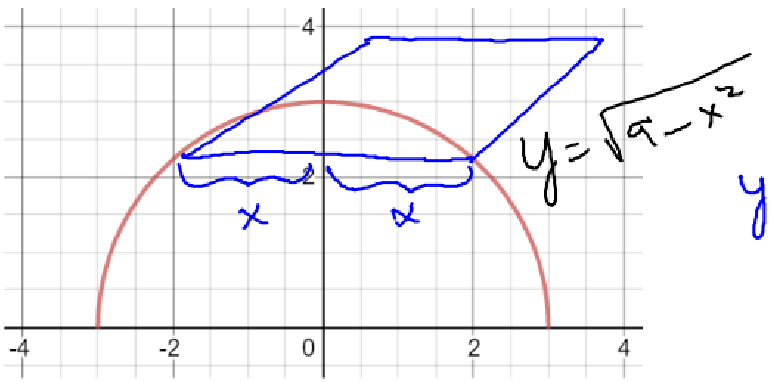
1. The base of a solid is the region enclosed by a semicircle of radius 3, lying flat on the x-axis. Cross sections perpendiculars to the y-axis are squares. Find the volume of the solid.



$x^2 = 9 - y^2$

Thickness : dy

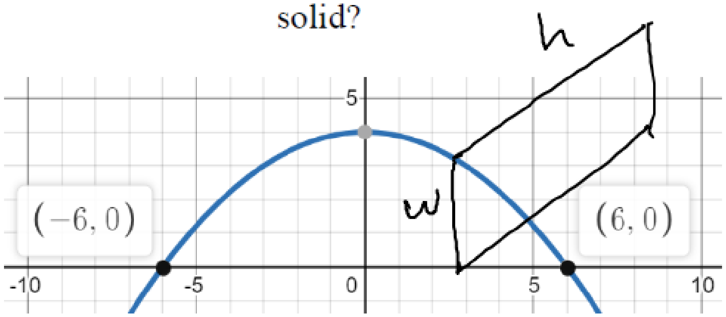
Area $= (2x)^2 = 4x^2$

$$\int_{y=0}^{y=3} 4x^2 \, dy = 4\int_0^3 9 - y^2 \, dy$$

$$= 4\left[9y - \frac{y^3}{3}\right]_0^3 = 4\left[27 - 9 - (0 - 0)\right]$$

$$= 4(18) = \boxed{72 \text{ units}^3}$$

2. The base of a solid is the region enclosed by $y = -\frac{x^2}{9} + 4$ and $y = 0$. Cross sections perpendicular to the x-axis are rectangles with heights twice that of the side in the xy-plane. What is the volume of the solid?



$w = y$ Thickness: dx

$h = 2y$

$\text{Area} = 2y^2 = 2\left(-\frac{x^2}{9}+4\right)^2$

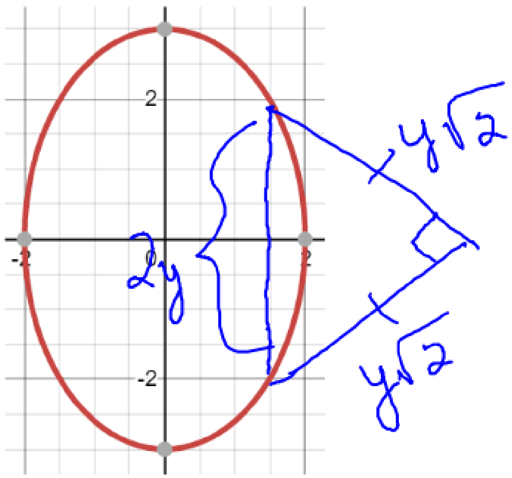
$$2\int_{-6}^{6}\left(-\frac{x^2}{9}+4\right)^2 dx$$

$$= 2\int_{-6}^{6} \frac{x^4}{81} - \frac{8x^2}{9} + 16 \, dx$$

$$= 2\left[\frac{x^5}{405} - \frac{8x^3}{27} + 16x\right]_{-6}^{6}$$

$$= 2\left[\frac{6^5}{405} - \frac{8(6^3)}{27} + 96 - \left(\frac{(-6)^5}{405} + \frac{8(-6)^3}{27} - 96\right)\right] = \boxed{\frac{1024}{5} \text{ units}^3}$$

3. The base of the solid is an elliptical region given by $9x^2 + 4y^2 = 36$. Cross sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.



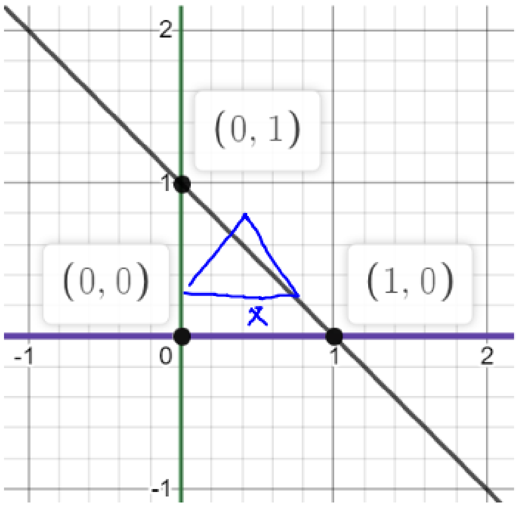
thickness: dx

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(y\sqrt{2})^2 = y^2 = \frac{36-9x^2}{4}$$

$$\frac{1}{4}\int_{-2}^{2} 36 - 9x^2 \, dx = \frac{1}{4}\left[36x - 3x^3\right]_{-2}^{2}$$

$$= \frac{1}{4}\left[72 - 24 - (-72 + 24)\right] = \boxed{24 \text{ units}^3}$$

4. The base of the solid is the triangular region with vertices (0,0), (0,1), and (1,0). Cross sections perpendicular to the y-axis are equilateral triangles.



thickness : dy

$$\text{Area} = \frac{x^2\sqrt{3}}{4}$$

$$= \frac{(1-y)^2\sqrt{3}}{4}$$

$y = -x+1$

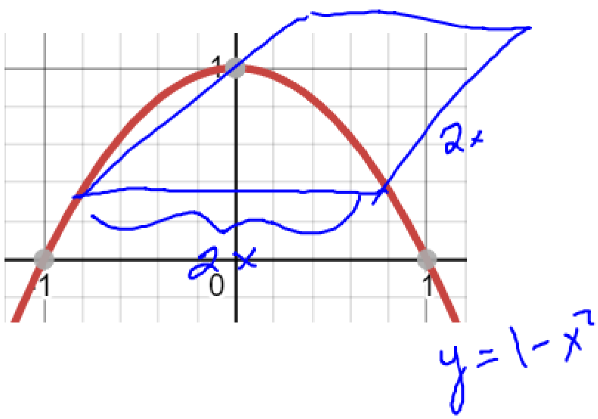
$x = 1-y$

$$\frac{\sqrt{3}}{4}\int_0^1 1 - 2y + y^2 \, dy$$

$$= \frac{\sqrt{3}}{4}\left[y - y^2 + \frac{y^3}{3}\right]_0^1$$

$$= \frac{\sqrt{3}}{4}\left[1 - 1 + \frac{1}{3} - (0 - 0 + 0)\right] = \frac{\sqrt{3}}{4}\left(\frac{1}{3}\right) = \boxed{\frac{\sqrt{3}}{12} \text{ units}^3}$$

5. The base of the solid is the region enclosed by the parabola $y = 1 - x^2$ and the x-axis. Cross sections perpendicular to the y-axis are squares.



Thickness: dy

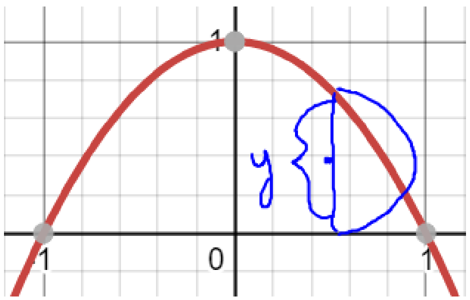
Area: $4x^2 = 4(1-y)$

$$4\int_0^1 (1-y)\,dy = 4\left[y - \frac{y^2}{2}\right]_0^1$$

$$= 4\left(1 - \frac{1}{2} - (0-0)\right)$$

$$= \boxed{2 \text{ units}^3}$$

6. The base of the solid is the region enclosed by the parabola $y = 1 - x^2$ and the x-axis. Cross sections perpendicular to the x-axis are semicircles.



$y = 1 - x^2$

Thickness : dx

$$\text{Area} = \frac{1}{2}\pi r^2 \qquad r = \frac{1}{2}y$$

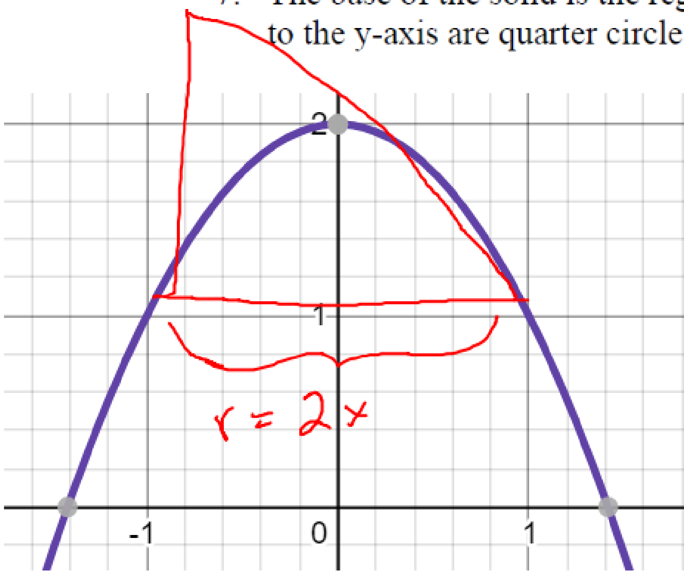
$$= \frac{1}{8}\pi y^2$$

$$= \frac{1}{8}\pi (1 - x^2)^2$$

$$\frac{\pi}{8}\int_{-1}^{1} (1 - 2x^2 + x^4)\,dx = \frac{\pi}{8}\left[x - \frac{2}{3}x^3 + \frac{x^5}{5}\right]_{-1}^{1}$$

$$= \frac{\pi}{8}\left[1 - \frac{2}{3} + \frac{1}{5} - \left(-1 + \frac{2}{3} - \frac{1}{5}\right)\right] = \frac{\pi}{8}\left(\frac{16}{15}\right) = \boxed{\frac{2\pi}{15} \text{ units}^3}$$

7. The base of the solid is the region enclosed by $y = 2 - x^2$ and the x-axis. Cross sections perpendicular to the y-axis are quarter circles.



thickness: dy

Area: $\frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2x)^2 \cdot \pi x^2$

$= \pi(2-y)$

$$\pi\int_0^2 (2-y)\,dy = \pi\left[2y - \frac{y^2}{2}\right]_0^2$$

$$= \pi\left[4 - 2 - (0-0)\right] = \boxed{2\pi \text{ units}^3}$$