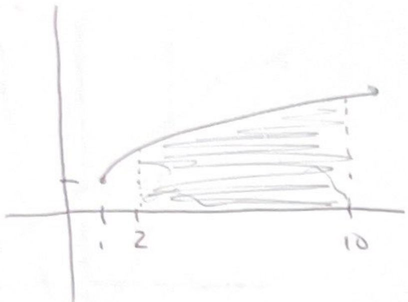
1. x-axis, $y = 1 + \sqrt{x-1}$ from $x=2$ to $x=10$



$$\int_2^{10} 1 + \sqrt{x-1}\, dx$$

$$= \left[x + \frac{2}{3}(x-1)^{\frac{3}{2}} \right]_2^{10}$$

$$= 10 + \frac{2}{3}(27) - \left(2 + \frac{2}{3}\right) = 8 + \frac{52}{3} = \boxed{\frac{76}{3}}$$

2. $f(x) = x^3 + x^2 - 2x$ and x-axis from $x=-2$ to $x=1$

$x^3 + x^2 - 2x = 0$

$x(x^2 + x - 2) = 0$

$x(x+2)(x-1) = 0$

$x=0 \quad x=-2 \quad x=1$

$$\int_{-2}^{0} x^3 + x^2 - 2x\, dx + -\int_0^1 x^3 + x^2 - 2x\, dx$$

+ − $f(x)$

-2 0 1

$$= \left[\frac{x^4}{4} + \frac{x^3}{3} - x^2 \right]_{-2}^{0} - \left[\frac{x^4}{4} + \frac{x^3}{3} - x^2 \right]_0^1$$

$f(x) > 0$ on $(-2, 0)$

$f(x) < 0$ on $(0, 1)$

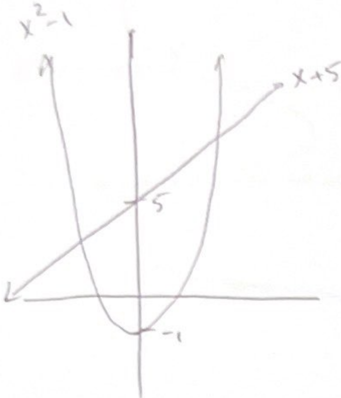
$$= 0 - \left(4 - \frac{8}{3} - 4\right) - \left[\frac{1}{4} + \frac{1}{3} - 1 - 0\right]$$

$$= \frac{8}{3} - \frac{1}{4} - \frac{1}{3} + 1$$

$$= \frac{10}{3} - \frac{1}{4} = \boxed{\frac{37}{12}}$$

3. $y = x^2 - 1 \quad y = x + 5$

$$x^2 - 1 = x + 5$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3 \quad x = -2$$

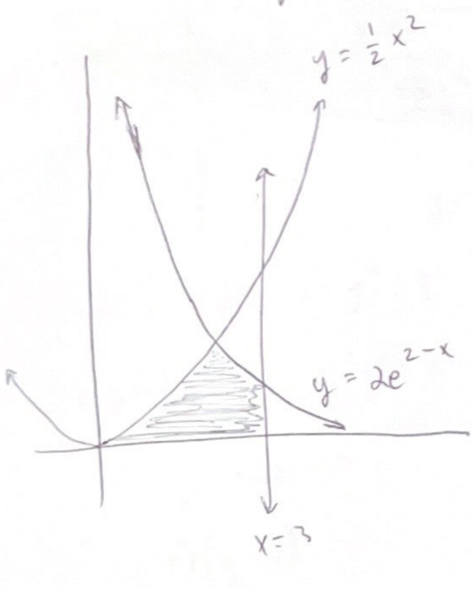


$$\int_{-2}^{3} x+5-(x^2-1)\,dx = \int_{-2}^{3} -x^2+x+6\,dx$$

$$= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 6x\right]_{-2}^{3} = -9 + \frac{9}{2} + 18 - \left(\frac{8}{3} + 2 - 12\right)$$

$$= 19 - \frac{8}{3} + \frac{9}{2} = \boxed{\frac{125}{6}}$$

4. x-axis, $y = \frac{1}{2}x^2 \quad x = 3 \quad y = 2e^{2-x}$ $\qquad \frac{1}{2}x^2 = 2e^{2-x}$ if $x = 2$



$$\int_0^2 \frac{1}{2}x^2\,dx + -\int_2^3 -2e^{2-x}\,dx$$

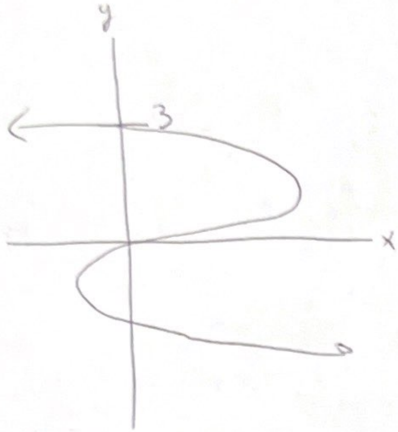
$u = 2-x$
$du = -dx$
$x=2,\ u=0$
$x=3,\ u=-1$

$$\left[\frac{x^3}{6}\right]_0^2 - \int_0^{-1} 2e^u\,du = \frac{4}{3} - 0 - \left[2e^u\right]_0^{-1}$$

$$= \frac{4}{3} - \left[2e^{-1} - 2e^0\right]$$

$$= \frac{4}{3} - \frac{2}{e} + 2 = \boxed{\frac{10}{3} - \frac{2}{e}}$$

5. y-axis and $x = y^2 - y^3 + 6y$ from $y=0$ to $y=3$

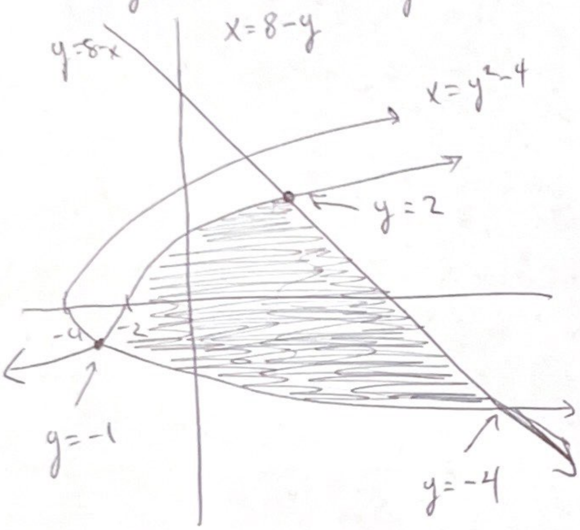


$$\int_0^3 y^2 - y^3 + 6y \, dy$$

$$= \left[\frac{y^3}{3} - \frac{y^4}{4} + 3y^2\right]_0^3 = 9 - \frac{81}{4} + 27 - 0$$

$$= \boxed{\frac{63}{4}}$$

6. $y = 8 - x \quad x = y^3 - 2 \quad x = y^2 - 4$



$y^3 - 2 = y^2 - 4$

$y^3 - y^2 + 2 = 0$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 0 & 2 \\ & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$(y+1)(y^2 - 2y + 2) = 0$

$y = -1$ $\quad \to$ no real sol.

$y = 8 - x$ and $x = y^3 - 2$

$y = 8 - (y^3 - 2) \to y = 8 - y^3 + 2$

$y^3 + y - 10 = 0$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 1 & -10 \\ & & 2 & 4 & 10 \\ \hline & 1 & 2 & 5 & 0 \end{array}$$

$(y-2)(y^2 + 2y + 5) = 0$

$y = 2$ $\quad \downarrow$ no real sol

$y = 8 - x$ and $x = y^2 - 4$

$y = 8 - (y^2 - 4) \to y = 8 - y^2 + 4$

$y^2 + y - 12 = 0$

$(y-3)(y+4) = 0$

$y = 3 \quad y = -4$

$$\int_{-4}^{-1} 8 - y - (y^2 - 4)\, dy + \int_{-1}^{2} 8 - y - (y^3 - 2)\, dy$$

$$= \int_{-4}^{-1} 12 - y - y^2 \, dy + \int_{-1}^{2} 10 - y - y^3 \, dy$$

$$= \left[12y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-4}^{-1} + \left[10y - \frac{y^2}{2} - \frac{y^4}{4}\right]_{-1}^{2} = \left[-12 - \frac{1}{2} + \frac{1}{3} - \left(-48 - 8 + \frac{64}{3}\right)\right] + \left[20 - 2 - 4 - \left(-10 - \frac{1}{2} - \frac{1}{4}\right)\right]$$

$$= \boxed{\frac{189}{4}}$$