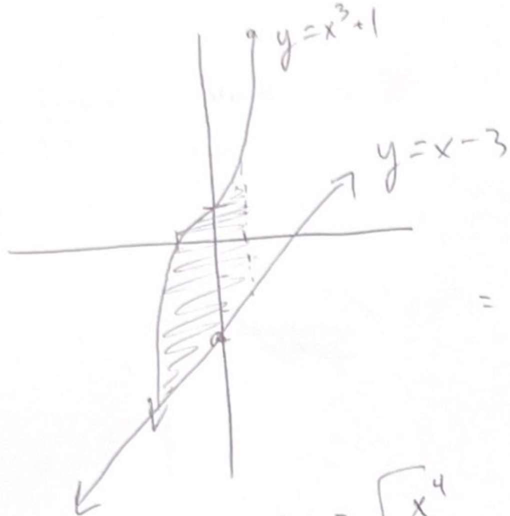
1. $y = x^3+1, \quad y = x-3, \quad x=-1, \quad x=1$



$$\int_{-1}^{1} x^3+1-(x-3)\,dx$$

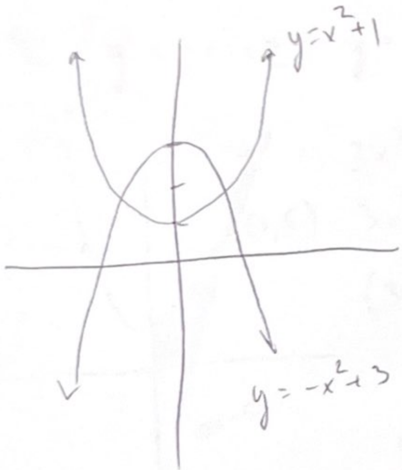
$$= \int_{-1}^{1} x^3 - x + 4 \; dx$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2} + 4x\right]_{-1}^{1} = \left(\frac{1}{4} - \frac{1}{2} + 4\right) - \left(\frac{1}{4} - \frac{1}{2} - 4\right)$$

$$= \boxed{8}$$

2. $y = x^2+1 \quad y = -x^2+3$

$$x^2+1 = -x^2+3$$
$$2x^2 = 2$$
$$x = \pm 1$$



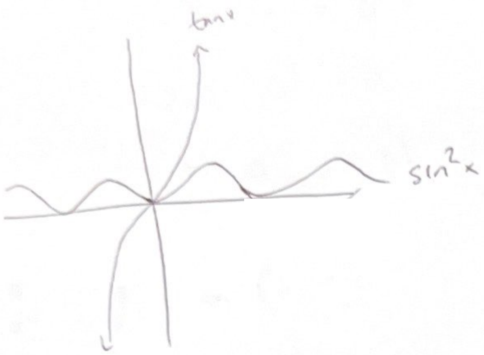
$$\int_{-1}^{1} -x^2+3-(x^2+1)\,dx$$

$$= \int_{-1}^{1} -\underline{\underline{2x^2}} + 2 \; dx$$

$$= \left[-\frac{2}{3}x^3 + 2x\right]_{-1}^{1} = \left(-\frac{2}{3}+2\right) - \left(\frac{2}{3} - 2\right) = 4 - \frac{4}{3} = \boxed{\frac{8}{3}}$$

3. $y = \sin^2 x \quad y = \tan x$ from $x = 0$ to $x = \frac{\pi}{4}$

$\tan x > \sin^2 x$ on $(0, \frac{\pi}{4}]$



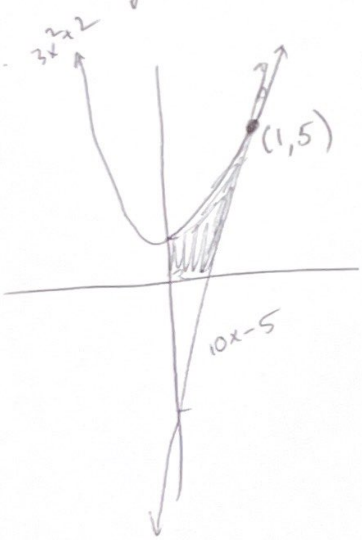
$$\int_0^{\frac{\pi}{4}} \tan x - \sin^2 x \, dx$$

$$= \Big[-\ln|\cos x|\Big]_0^{\frac{\pi}{4}} - \frac{1}{2}\int_0^{\frac{\pi}{4}} 1 - \cos 2x \, dx$$

$$= -\ln\cos\frac{\pi}{4} - (-\ln\cos 0) - \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$$

$$= -\ln\frac{\sqrt{2}}{2} - \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2} - (0 - 0)\right] = \boxed{\ln\sqrt{2} - \frac{\pi}{8} + \frac{1}{4}}$$

4. $y = 3x^2 + 2 \quad y = 10x - 5$, x-axis, y-axis



$3x^2 + 2 = 10x - 5$

$3x^2 - 10x + 7 = 0$

$(3x - 7)(x - 1) = 0$

$x = \frac{7}{3} \quad x = 1$

$y = (3(1^2) + 2) = 5$

$$\int_0^1 3x^2 + 2 - (10x - 5)\, dx - \frac{1}{2}(5)\left(\frac{1}{2}\right)$$

$$= \int_0^1 3x^2 - 10x + 7 \, dx - \frac{5}{4}$$

$$= \Big[x^3 - 5x^2 + 7x\Big]_0^1 - \frac{5}{4} = 1 - 5 + 7 - 0 - \frac{5}{4}$$

$$= \boxed{\frac{7}{4}}$$

5. $y_1 = x^3 - x + 1$ $\quad y_2 = -x^3 + x + 1$

$x^3 - x + 1 = -x^3 + x + 1$

$2x^3 - 2x = 0$

$2x(x^2 - 1) = 0$

$x = 0 \quad x = \pm 1$

$y_1 > y_2$ on $(-1, 0)$

$y_2 > y_1$ on $(0, 1)$

$$\int_{-1}^{0} x^3 - x + 1 - (-x^3 + x + 1)\,dx + \int_{0}^{1} -x^3 + x + 1 - (x^3 - x + 1)\,dx$$

$$= \int_{-1}^{0} 2x^3 - 2x\,dx + \int_{0}^{1} -2x^3 + 2x\,dx$$

$$= \left[\frac{x^4}{2} - x^2\right]_{-1}^{0} + \left[-\frac{x^4}{2} + x^2\right]_{0}^{1}$$

$$= \left[0 - \left(\frac{1}{2} - 1\right)\right] + \left[-\frac{1}{2} + 1 - 0\right]$$

$$= \frac{1}{2} + \frac{1}{2} = \boxed{1}$$