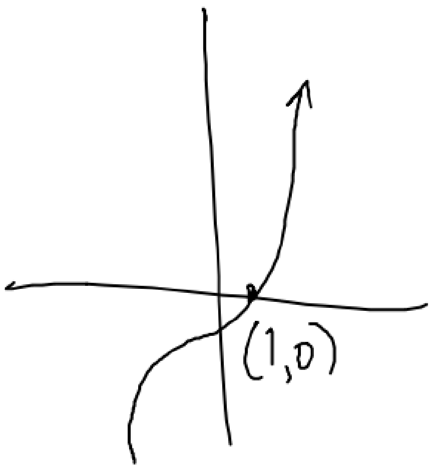
Find the area of the region between the x-axis and $y = x^3 - 1$ from $x = 1$ to $x = 2$



$$\int_1^2 (x^3 - 1)\,dx = \left[\frac{x^4}{4} - x\right]_1^2$$

$$= \left(\frac{2^4}{4} - 2\right) - \left(\frac{1^4}{4} - 1\right)$$

$$= 4 - 2 - \left(\frac{1}{4} - 1\right) = 2 + \frac{3}{4}$$

$$= \boxed{\frac{11}{4} \text{ units}^2}$$

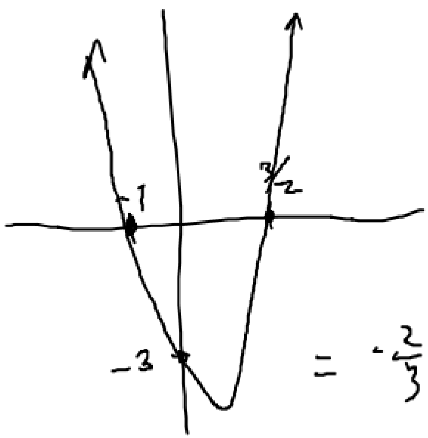
Find the area of the regions between the x-axis and $y = 2x^2 - x - 3$ from $x = -2$ to $x = 2$

$2x^2 - x - 3 = 0$

$(2x-3)(x+1) = 0$

$x = \frac{3}{2} \quad x = -1$

$$\int_{-2}^{-1} (2x^2 - x - 3)\,dx - \int_{-1}^{\frac{3}{2}} (2x^2 - x - 3)\,dx + \int_{\frac{3}{2}}^{2} (2x^2 - x - 3)\,dx$$



$$= \left[\frac{2}{3}x^3 - \frac{x^2}{2} - 3x\right]_{-2}^{-1} - \left[\frac{2}{3}x^3 - \frac{x^2}{2} - 3x\right]_{-1}^{\frac{3}{2}} + \left[\frac{2}{3}x^3 - \frac{x^2}{2} - 3x\right]_{\frac{3}{2}}^{2}$$

$$= -\frac{2}{3} - \frac{1}{2} + 3 - \left(-\frac{16}{3} - 2 + 6\right) - \left[\left(\frac{9}{4} - \frac{9}{8} - \frac{9}{2}\right) - \left(-\frac{2}{3} - \frac{1}{2} + 3\right)\right] + \left(\frac{16}{3} - 2 - 6\right) - \left(\frac{9}{4} - \frac{9}{8} - \frac{9}{2}\right)$$

$$= \frac{19}{6} - \left(-\frac{125}{24}\right) + \frac{17}{24} = \frac{109}{12} \text{ units}^2$$

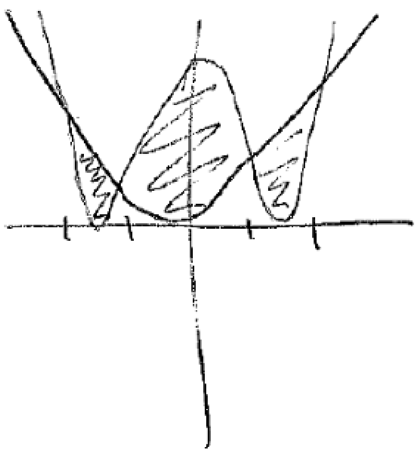
Find the area of the regions enclosed between the curves $y = x^4 - 4x^2 + 4$ and $y = x^2$

$$x^4 - 4x^2 + 4 = x^2$$

$$x^4 - 5x^2 + 4 = 0$$

$$(x^2 - 4)(x^2 - 1) = 0$$

$$x = \pm 2 \quad x = \pm 1$$



$$\int_{-2}^{-1}\left[x^2 - (x^4 - 4x^2 + 4)\right]dx + \int_{-1}^{1}(x^4 - 4x^2 + 4 - x^2)\,dx + \int_{1}^{2}\left(x^2 - (x^4 - 4x^2 + 4)\right)dx$$

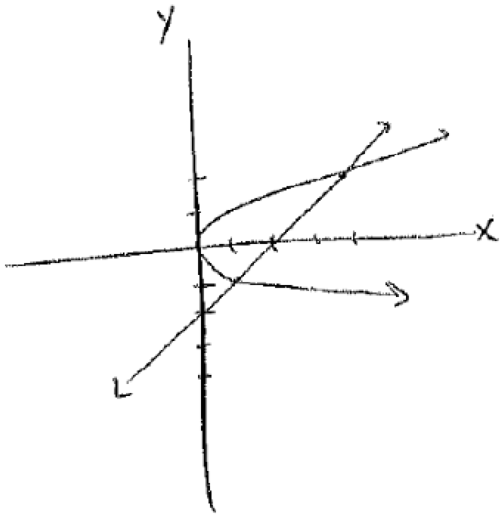
$$= \int_{-2}^{-1}(5x^2 - x^4 - 4)\,dx + \int_{-1}^{1}(x^4 - 5x^2 + 4)\,dx + \int_{1}^{2}(5x^2 - x^4 - 4)\,dx$$

$$= \left[\frac{5}{3}x^3 - \frac{x^5}{5} - 4x\right]_{-2}^{-1} + \left(\frac{x^5}{5} - \frac{5}{3}x^3 + 4x\right)_{-1}^{1} + \left[\frac{5}{3}x^3 - \frac{x^5}{5} - 4x\right]_{1}^{2}$$

$$\left(\left(-\frac{5}{3} + \frac{1}{5} + 4\right) - \left(-\frac{40}{3} + \frac{32}{5} + 8\right)\right) + \left[\left(\frac{1}{5} - \frac{5}{3} + 4\right) - \left(-\frac{1}{5} + \frac{5}{3} - 4\right)\right] + \left[\left(\frac{40}{3} - \frac{32}{5} - 8\right) - \left(\frac{5}{3} - \frac{1}{5} - 4\right)\right]$$

$$= \boxed{8 \text{ units}^2}$$

Find the area of the region enclosed by $x = y + 2$ and $x = y^2$



$$\int_{-1}^{2} \left(y + 2 - y^2\right) dy$$

$$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2}$$

$$= \left(\frac{2^2}{2} + 2(2) - \frac{2^3}{3}\right) - \left(\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3}\right)$$

$$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} + (-2) + \frac{1}{3}\right) = \boxed{\frac{9}{2} \text{ units}^2}$$

Find the area of the region enclosed by $y = 2\sin(x)$, $y = \sin(2x)$, $x = 0$, and $x = \pi$

$$\int_0^{\pi} \left[2\sin(x) - \sin(2x)\right] dx = \int_0^{\pi} 2\sin(x)\,dx - \frac{1}{2}\int_0^{\pi} 2\sin(2x)\,dx$$

$u = 2x$
$du = 2\,dx$

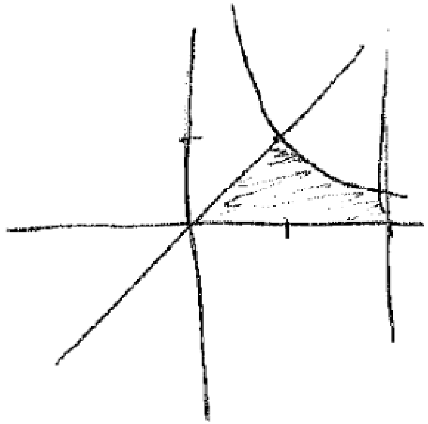
$$= \left[-2\cos(x)\right]_0^{\pi} - \frac{1}{2}\int_0^{2\pi} \sin(u)\,du$$

$$= -2\cos(\pi) - (-2\cos(0)) - \frac{1}{2}\left[-\cos(u)\right]_0^{2\pi}$$

$$= -2(-1) + 2\cos(0) - \frac{1}{2}\left[-\cos(2\pi) - (-\cos(0))\right]$$

$$= 4 - \frac{1}{2}((-1) + (1)) = \boxed{4 \text{ units}^2}$$

Find the area of the region in the first quadrant bounded by the line $y = x$, the line$x = 2$, the curve $y = \frac{1}{x^2}$, and the x-axis.



$$\int_0^1 x\,dx + \int_1^2 \frac{1}{x^2}\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 + \left[-\frac{1}{x}\right]_1^2$$

$$= \left(\frac{1^2}{2} - \frac{0^2}{2}\right) + \left(-\frac{1}{2} - (-1)\right)$$

$$= \frac{1}{2} + \left(-\frac{1}{2} + 1\right) = \frac{1}{2} + \frac{1}{2} = 1 \text{ unit}^2$$

Find the area of the region in the first quadrant bounded on the left by the y-axis and on the right by $y = \sin(x)$, $y = \cos(x)$, and their first intersection point in the first quadrant.

$$\sin(x) = \cos(x)$$

$$x = \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{4}} \left[\cos(x) - \sin(x)\right] dx = \left[\sin(x) + \cos(x)\right]_0^{\frac{\pi}{4}}$$

$$= \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) - \left(\sin(0) + \cos(0)\right)$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0+1) = \boxed{\sqrt{2} - 1 \text{ units}^2}$$

[1] Find the area of the region in the first quadrant bounded on the left by the y-axis, below by the line $y = \frac{x}{4}$, above left by the curve $y = 1 + \sqrt{x}$, and above on the right by the curve $y = \frac{2}{\sqrt{x}}$.

$$1 + \sqrt{x} = \frac{2}{\sqrt{x}}$$

$$\sqrt{x} + x = 2$$

$$x = 1$$

$$\frac{2}{\sqrt{x}} = \frac{x}{4}$$

$$x\sqrt{x} = 8$$

$$x = 4$$

$$\int_0^1 \left(1 + \sqrt{x} - \frac{x}{4}\right) dx + \int_1^4 \left(\frac{2}{\sqrt{x}} - \frac{x}{4}\right) dx$$

$$\left[x + \frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{8}\right]_0^1 + \left[4\sqrt{x} - \frac{x^2}{8}\right]_1^4$$

$$\left[\left(1 + \frac{2}{3} - \frac{1}{8}\right) - 0\right] + \left[(8 - 2) - \left(4 - \frac{1}{8}\right)\right]$$

$$= \frac{5}{3} - \frac{1}{8} + 6 - 4 + \frac{1}{8} = \frac{5}{3} + 2 = \boxed{\frac{11}{3} \text{ units}^2}$$