1. $\displaystyle\int_1^2 \frac{\sin x}{x}\,dx + 5 \approx \boxed{5.659329906}$

2. $\displaystyle\int_3^6 e^{-x^2}\,dx + \pi \approx \boxed{3.141612231}$

3. $\displaystyle\int_2^{\pi} \frac{2}{\ln x}\,dx + 3 \approx \boxed{5.489312637}$

4. $T_0 = 50 \quad T_s = 250 \quad T = 100 \quad t = 8$

$100 - 250 = (50 - 250)e^{8k}$

$-150 = -200e^{8k}$

$\frac{3}{4} = e^{8k}$

$\ln\frac{3}{4} = 8k$

$k = \dfrac{\ln\frac{3}{4}}{8}$

$200 - 250 = (100 - 250)e^{\frac{\ln\frac{3}{4}}{8}t}$

$-50 = -150e^{\frac{\ln\frac{3}{4}}{8}t}$

$\frac{1}{3} = e^{\frac{\ln\frac{3}{4}}{8}t}$

$-\ln 3 = \dfrac{\ln\frac{3}{4}}{8}t$

$\boxed{t \approx 30.55073343 \text{ minutes later}}$

5. $T_0 = 70 \quad T_s = 20 \quad T = 40 \quad t = 20$

$40 - 20 = (70 - 20)e^{-20k}$

$20 = 50e^{-20k}$

$\frac{2}{5} = e^{-20k}$

$\ln\frac{2}{5} = -20k$

$k = \dfrac{\ln\frac{2}{5}}{-20}$

$30 - 20 = (40 - 20)e^{\frac{\ln\frac{2}{5}}{20}t}$

$10 = 20e^{\frac{\ln\frac{2}{5}}{20}t}$

$\frac{1}{2} = e^{\frac{\ln\frac{2}{5}}{20}t}$

$-\ln 2 = \dfrac{\ln\frac{2}{5}}{20}t$

$\boxed{t \approx 15.12941595 \text{ more minutes}}$

6. $T_s = 70$ $T_0 = 200$ $T = 120$ $t = 14$

$$120 - 70 = (200-70)e^{-14k}$$

$$50 = 130e^{-14k}$$

$$\frac{5}{13} = e^{-14k}$$

$$\ln\frac{5}{13} = -14k$$

$$k = \frac{\ln\frac{5}{13}}{-14}$$

$$76 - 70 = (120-70)e^{\frac{\ln\frac{5}{13}}{14}t}$$

$$6 = 50e^{\frac{\ln\frac{5}{13}}{14}t}$$

$$\frac{3}{25} = e^{\frac{\ln\frac{5}{13}}{14}t}$$

$$\ln\frac{3}{25} = \frac{\ln\frac{5}{13}}{14}t$$

$\boxed{t \approx 31.06576029 \text{ minutes longer}}$

7. $3000\left(1 + \frac{.0236}{4}\right)^{(4)(3)} \approx \boxed{\$3219.43}$

8. $2000\left(1 + \frac{.0155}{12}\right)^{(12)(5)} \approx \boxed{\$2161.06}$

9. $400e^{(.0324)(6)} \approx \boxed{\$485.83}$

10. $8 = \frac{\ln 2}{k}$

$$k = \frac{\ln 2}{8}$$

$$3 = 40e^{-\frac{\ln 2}{8}t}$$

$$\frac{3}{40} = e^{-\frac{\ln 2}{8}t}$$

$$\ln\frac{3}{40} = -\frac{\ln 2}{8}t$$

$\boxed{t \approx 29.89572475 \text{ days}}$

11. $k = .011002$

half life $= \frac{\ln 2}{.011002}$

$\boxed{\text{half life} \approx 63.00192516 \text{ years}}$

$$33 = 125e^{-.011002t}$$

$$\frac{33}{125} = e^{-.011002t}$$

$$\ln\frac{33}{125} = -.011002t$$

$\boxed{t \approx 121.0512794 \text{ years}}$

12. $k = .2425287$

$\text{half-life} = \frac{\ln 2}{.2425287}$

$\boxed{\text{half-life} = 2.858000643 \text{ years}}$

$.22y_0 = y_0 e^{-.2425287t}$

$.22 = e^{-.2425287t}$

$\ln(.22) = -.2425287t$

$\boxed{t \approx 6.243086829 \text{ years}}$

13. a) $\int \frac{dy}{dx} = \int 3x^2 - 2x - 1$ $\quad y(2) = 3$

$y = x^3 - x^2 - x + C$

$3 = 2^3 - 2^2 - 2 + C$

$3 = 2 + C$

$C = 1$

$\boxed{y = x^3 - x^2 - x + 1 \text{ for } (-\infty, \infty)}$

b) $\int \frac{dy}{dx} = \int \cos 2x$ $\quad y = 3, x = \frac{\pi}{4}$

$y = \frac{1}{2} \sin 2x + C$

$3 = \frac{1}{2} \sin\left(2\left(\frac{\pi}{4}\right)\right) + C$

$3 = \frac{1}{2} + C$

$C = \frac{5}{2}$

$\boxed{y = \frac{1}{2} \sin 2x + \frac{5}{2} \text{ for } (-\infty, \infty)}$

c) $\int \frac{dr}{dt} = \int \frac{1}{t-2} + \frac{1}{t}$ $\quad r = 5, t = 1$

$r = \ln|t-2| + \ln|t| + C$

$5 = \ln|-1| + \ln 1 + C$ $\quad t \neq 0,\ t \neq 2$

$C = 5$

$\boxed{r = \ln|t-2| + \ln|t| + 5 \text{ for } (0, 2)}$

d) $\frac{dz}{dx} = \frac{5x+2}{\sqrt[3]{x}} \rightarrow \int \frac{dz}{dx} = \int 5x^{\frac{2}{3}} + 2x^{-\frac{1}{3}}$ $\quad z(1) = 2$

$z = 3x^{\frac{5}{3}} + 3x^{\frac{2}{3}} + C$

$2 = 3 + 3 + C$

$C = -4$

$$\boxed{z = 3x^{\frac{5}{3}} + 3x^{\frac{2}{3}} - 4 \text{ for } (-\infty, \infty)}$$

e) $\int \frac{dy}{dx} = \int \sec^2 x - x$ $\quad y(0) = 3$

$y = \tan x - \frac{x^2}{2} + C$

$3 = 0 - 0 + C$

$C = 3$

$x \neq \frac{\pi}{2} + \pi n$

$$\boxed{y = \tan x - \frac{x^2}{2} + 3 \text{ for } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}$$

f) $\int \frac{dA}{dt} = \int \frac{2}{1+t^2}$ $\quad t = 1, A = \pi$

$A = 2\tan^{-1}(t) + C$

$\pi = 2\tan^{-1}(1) + C$

$\pi = \frac{\pi}{2} + C$

$C = \frac{\pi}{2}$

$$\boxed{A = 2\tan^{-1}(t) + \frac{\pi}{2} \text{ for } (-\infty, \infty)}$$

g) $\int \frac{dy}{dx} = \int \frac{2x}{\sqrt{1-x^4}}$ $\quad u = x^2$, $du = 2x\,dx$, $x = 0, y = 3$ $\rightarrow \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C$

$y = \sin^{-1}(x^2) + C$ $\quad -1 \leq x^2 \leq 1$

$3 = \sin^{-1}(0) + C$ $\quad x^2 \leq 1$

$C = 3$ $\quad -1 \leq x \leq 1$

$$\boxed{y = \sin^{-1}(x^2) + 3 \text{ for } [-1, 1]}$$

h) $\frac{dy}{dx} = \frac{1}{xy}$ $\quad y(1) = 2$

$\int y\,dy = \int \frac{1}{x}\,dx$

$\frac{y^2}{2} = \ln|x| + C$

$2 = 0 + C$

$$\boxed{\begin{array}{c} y = \sqrt{2\ln|x| + 4} \\ \text{for } [e^{-2}, \infty) \end{array}}$$

$\frac{y^2}{2} = \ln|x| + 2$

$y^2 = 2\ln|x| + 4$

$y = \pm\sqrt{2\ln|x| + 4}$

only + satisfies initial condition

$x \neq 0$

$2\ln|x| + 4 \geq 0$

$\ln|x| \geq -2$

$|x| \geq e^{-2}$

$x \geq e^{-2}$ or $x \leq -e^{-2}$

i) $\frac{dy}{dt} = e^{y+t} \rightarrow \frac{dy}{dt} = e^{y}e^{t}$

$\frac{dy}{e^y} = e^t\,dt \rightarrow \int e^{-y}\,dy = \int e^t\,dt$

$-e^{-y} = e^t + C$

$-e^{-1} = e^0 + C$

$C = -\frac{1}{e} - 1$

$1 + \frac{1}{e} - e^t > 0$

$-e^t > -1 - \frac{1}{e}$

$e^t < 1 + \frac{1}{e}$

$t < \ln\left(1 + \frac{1}{e}\right)$

$y = 1,\ t = 0$

$-e^{-y} = e^t - \frac{1}{e} - 1$

$e^{-y} = 1 + \frac{1}{e} - e^t$

$-y = \ln\left(1 + \frac{1}{e} - e^t\right)$

$$\boxed{\begin{array}{c} y = -\ln\left(1 + \frac{1}{e} - e^t\right) \\ \text{for } \left(-\infty, \ln\left(1 + \frac{1}{e}\right)\right) \end{array}}$$

j) $\frac{dy}{dx} = xe^{x^2 - \ln(y^2)} \rightarrow \frac{dy}{dx} = \frac{xe^{x^2}}{e^{\ln(y^2)}} \rightarrow \frac{dy}{dx} = \frac{xe^{x^2}}{y^2}$

$\rightarrow \int y^2 dy = \int xe^{x^2} dx \rightarrow \frac{y^3}{3} = \frac{1}{2}e^{x^2} + C$ $\quad x=0, y=1$

$u = x^2$
$du = 2x\,dx$

$\frac{1}{3} = \frac{1}{2}e^0 + C$

$C = -\frac{1}{6}$

$\frac{y^3}{3} = \frac{1}{2}e^{x^2} - \frac{1}{6}$

$\boxed{y = \sqrt[3]{\frac{3}{2}e^{x^2} - \frac{1}{2}} \quad \text{for } (-\infty, \infty)}$

$y^3 = \frac{3}{2}e^{x^2} - \frac{1}{2}$

k) $\frac{dy}{dx} = \frac{\sin x \cos x \cos^2 y}{\sin y} \rightarrow \frac{\sin y}{\cos^2 y} dy = \sin x \cos x \, dx$

$u = \sin x$
$du = \cos x \, dx$

$\rightarrow \int \sec y \tan y \, dy = \int u \, du$

$y(\frac{\pi}{4}) = 0$

$\sec y = \frac{\sin^2 x}{2} + C$

$\sec y = \frac{\sin^2 x}{2} + \frac{3}{4}$

$\sec 0 = \frac{\sin^2(\frac{\pi}{4})}{2} + C$

$\boxed{y = \sec^{-1}\left(\frac{\sin^2 x}{2} + \frac{3}{4}\right) \quad \text{for } \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]}$

$1 = \frac{1}{4} + C$

$C = \frac{3}{4}$

$\frac{\sin^2 x}{2} + \frac{3}{4} \geq 1$ or $\frac{\sin^2 x}{2} + \frac{3}{4} \leq -1$ NEVER

$\sin^2 x \geq \frac{1}{2} \rightarrow \sin x \leq -\frac{1}{\sqrt{2}}$

$\sin x \geq \frac{1}{\sqrt{2}}$ $\quad x \leq -\frac{\pi}{4} + \pi n$

$\hookrightarrow x \geq \frac{\pi}{4} + \pi n$ $\quad$ which implies $\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$

l) $\frac{dy}{dx} = \frac{x+1}{y} \rightarrow \int y\,dy = \int x+1\,dx \rightarrow \frac{y^2}{2} = \frac{x^2}{2} + x + C$

$y(0) = -2$

$\frac{y^2}{2} = \frac{x^2}{2} + x + 2$

$2 = 0 + 0 + C$

$y^2 = x^2 + 2x + 4$

$y = \pm\sqrt{x^2+2x+4}$

$\boxed{y = -\sqrt{x^2+2x+4} \text{ for } (-\infty, \infty)}$

only $-$ satisfies initial condition

$x^2+2x+4 > 0$ so no restrictions

m) $\frac{dy}{dx} = \frac{y+3}{x-2} \rightarrow \int \frac{dy}{y+3} = \int \frac{dx}{x-2} \rightarrow \ln|y+3| = \ln|x-2| + C$

$y=1,\ x=3$

$\ln 4 = \ln 1 + C$

$C = \ln 4$

$x \neq 2$

$e^{\ln|y+3|} = e^{(\ln|x-2| + \ln 4)} = e^{\ln|x-2|}e^{\ln 4}$

$|y+3| = 4|x-2| \rightarrow y+3 = \pm 4|x-2|$

only + satisfies initial condition

$\boxed{y = -3 + 4|x-2| \text{ for } (2, \infty)}$

n) $\frac{dy}{dx} = \frac{x^2-1}{y-2} \rightarrow \int y-2\,dy = \int x^2-1\,dx \rightarrow \frac{y^2}{2} - 2y = \frac{x^3}{3} - x + C$

$y=-4,\ x=3$

$8 + 8 = 9 - 3 + C$

$C = 10$

$\frac{y^2}{2} - 2y = \frac{x^3}{3} - x + 10$

$y^2 - 4y = \frac{2}{3}x^3 - 2x + 20$

$y^2 - 4y + 4 = \frac{2}{3}x^3 - 2x + 24$

$(y-2)^2 = \frac{2}{3}x^3 - 2x + 24$

$\rightarrow y - 2 = \pm\sqrt{\frac{2}{3}x^3 - 2x + 24}$

only $-$ satisfies initial condition

$\boxed{y = 2 - \sqrt{\frac{2}{3}x^3 - 2x + 24} \text{ for } [-3.604, \infty)}$

this value came from the calculator solution of $\frac{2}{3}x^3 - 2x + 24 = 0$, this will not be required on the calculator part of the test