1. $\frac{dy}{dx} = -2xy^2 \rightarrow \frac{dy}{y^2} = -2x\,dx \rightarrow \int y^{-2}\,dy = \int -2x\,dx$

$(1, \frac{1}{4})$

$\rightarrow -\frac{1}{y} = -x^2 + C$

$-\frac{1}{\frac{1}{4}} = -(1^2) + C \rightarrow -4 = -1 + C$

$C = -3$

$-\frac{1}{y} = -x^2 - 3$

$\frac{1}{y} = x^2 + 3$

$x^2 + 3 \neq 0$
$x^2 \neq -3$

$\boxed{y = \frac{1}{x^2+3} \text{ for } (-\infty, \infty)}$

2. $\frac{dy}{dx} = (\cos x)e^{y+\sin x} \rightarrow \frac{dy}{dx} = (\cos x)e^y e^{\sin x}$

$\frac{dy}{e^y} = \cos x\, e^{\sin x}\,dx$

$u = \sin x$
$du = \cos x\,dx$

$\int e^{-y}\,dy = \int e^u\,du$

$(0,0)$

$\Rightarrow -e^{-y} = e^{\sin x} + C \rightarrow -e^0 = e^{\sin 0} + C$

$-1 = 1 + C$

$C = -2$

$-e^{-y} = e^{\sin x} - 2$

$e^{-y} = 2 - e^{\sin x}$

$-y = \ln(2 - e^{\sin x})$

$2 - e^{\sin x} = 0$
if $e^{\sin x} = 2$
$\sin x = \ln 2$
$x = \sin^{-1}(\ln 2)$ (1st qad)
or $-\pi - \sin^{-1}(\ln 2)$

$\boxed{y = -\ln(2 - e^{\sin x}) \text{ for } (-\pi - \sin^{-1}(\ln 2), \sin^{-1}(\ln 2))}$

3. $\frac{dy}{dx} = (2x+1)(y+1) \quad y(-1)=1$

$\int \frac{dy}{y+1} = \int 2x+1 \, dx$

$\ln|y+1| = x^2+x+C$

$\ln 2 = (-1)^2 - 1 + C$

$\ln 2 = C$

$e^{\ln|y+1|} = e^{(x^2+x+\ln 2)}$

$|y+1| = e^{x^2+x+\ln 2}$

$y+1 = \pm 2e^{x^2+x}$

$y = \pm 2e^{x^2+x} - 1$

only + satisfies
initial condition

$\boxed{y = 2e^{x^2+x} - 1 \quad \text{for } (-\infty, \infty)}$

4. $\frac{dy}{dx} = 4xy^2 \quad y(0)=1$

$\frac{dy}{y^2} = 4x\,dx \rightarrow y^{-2}dy = 4x\,dx$

$-\frac{1}{y} = 2x^2 + C$

$-\frac{1}{1} = 0 + C$

$C = -1$

$-\frac{1}{y} = 2x^2 - 1$

$-y = \frac{1}{2x^2-1}$

$\boxed{y = \frac{1}{1-2x^2} \quad \text{for } \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)}$

$1-2x^2 \neq 0$

$2x^2 \neq 1$

$x^2 \neq \frac{1}{2}$

$x \neq \pm\frac{1}{\sqrt{2}}$

5. $\frac{dy}{dx} = \frac{y^2}{x} \rightarrow \int \frac{dy}{y^2} = \int \frac{dx}{x} \rightarrow -\frac{1}{y} = \ln|x| + C \quad y(1) = \frac{1}{3}$

$-\frac{1}{\frac{1}{3}} = \ln 1 + C$

$-3 = C$

$-\frac{1}{y} = \ln|x| - 3$

$\boxed{y = \frac{1}{3-\ln|x|} \quad \text{for } (0, e^3)}$

$x \neq 0, \quad 3 - \ln|x| = 0$

$\ln|x| \neq 3$

$|x| \neq e^3$

$x \neq \pm e^3$