1. a) $\frac{dy}{dx} = \frac{xy}{x^2+1} \rightarrow \int \frac{dy}{y} = \int \frac{x\,dx}{x^2+1} \rightarrow \frac{1}{2}\int \frac{2x}{x^2+1}$ $u = x^2+1$, $du = 2x\,dx$

$$\ln|y| = \frac{1}{2}\int \frac{du}{u}$$

$$\ln|y| = \frac{1}{2}\ln|u| + C$$

$$\ln|y| = \frac{1}{2}\ln|x^2+1| + C$$

$$\ln|y| = \ln\sqrt{x^2+1} + C \quad \text{with } k = e^C$$

$$|y| = k\sqrt{x^2+1}$$

$$\boxed{y = \pm k\sqrt{x^2+1}}$$

b) $\frac{dy}{dx} = y^2 \sin x$

$$\int \frac{dy}{y^2} = \int \sin x\,dx$$

$$-\frac{1}{y} = -\cos x + C$$

$$\frac{1}{y} = \cos x - C$$

$$\boxed{y = \frac{1}{\cos x - C}}$$

c) $\frac{dy}{dx} = \frac{e^x}{y\sqrt{1+y^2}} \rightarrow \frac{1}{2}\int 2y\sqrt{1+y^2}\,dy = \int e^x\,dx$ $u = 1+y^2$, $du = 2y\,dy$

$$\frac{1}{2}\int u^{\frac{1}{2}}\,du = e^x + C$$

$$\frac{1}{2}\left(\frac{2}{3}(1+y^2)^{\frac{3}{2}}\right) = e^x + C$$

$$(1+y^2)^{\frac{3}{2}} = 3e^x + 3C$$

$$1+y^2 = (3e^x + 3C)^{\frac{2}{3}}$$

$$\boxed{y = \pm\sqrt{(3e^x + 3C)^{\frac{2}{3}} - 1}}$$

2. a) $\frac{dy}{dx} = 2xy \; ; \; (0,3)$

$\frac{dy}{y} = 2x\,dx$

$\ln|y| = x^2 + C$

$\ln 3 = 0 + C$

$C = \ln 3$

$\ln|y| = x^2 + \ln 3$

$|y| = e^{x^2 + \ln 3}$

$y = 3e^{x^2}$ for $(-\infty, \infty)$

b) $\frac{dy}{dx} = \cos^2 y \; ; \; (0,0)$

$\frac{dy}{\cos^2 y} = dx$

$\int \sec^2 y\,dy = \int dx$

$\tan y = x + C$

$0 = 0 + C$

$C = 0$

$\tan y = x$

$y = \tan^{-1} x$

for $(-\infty, \infty)$

c) $\frac{dy}{dx} = \frac{4\sqrt{y}\ln x}{x} \; ; \; (e,1)$

$\int y^{-\frac{1}{2}}\,dy = 4\int u\,du$ $\quad u = \ln x$, $du = \frac{1}{x}dx$

$2\sqrt{y} = \frac{4(\ln x)^2}{2} + C$

$2 = \frac{4}{2} + C$

$C = 0$

$2\sqrt{y} = \frac{4}{2}(\ln x)^2$

$\sqrt{y} = (\ln x)^2$

$y = \ln^4 x$ for $(0, \infty)$

d) $\frac{dy}{dx} = 4x^3 \tan y \; ; \; (1, \frac{\pi}{2})$

$\int \cot y\,dy = \int 4x^3\,dx$

$\ln|\sin y| = x^4 + C$

$\ln|\sin\frac{\pi}{2}| = 1 + C$

$\ln 1 = 1 + C$

$0 = 1 + C$

$C = -1$

$\ln|\sin y| = x^4 - 1$

$|\sin y| = e^{x^4 - 1}$

$-1 \le e^{x^4-1} \le 1$

$x^4 - 1 \le 0$

$x^4 \le 1$

$-1 \le x \le 1$

$y = \sin^{-1}(e^{x^4-1})$ for $[-1, 1]$

e) $\frac{dy}{dx} = \frac{2x + \sec^2 x}{2y}$; $(0, -5)$

$$2y\,dy = (2x + \sec^2 x)\,dx$$

$$y^2 = x^2 + \tan x + C$$

$$(-5)^2 = C$$

$$C = 25$$

$$y^2 = x^2 + \tan x + 25$$

$$y = -\sqrt{x^2 + \tan x + 25}$$

for $\left(-1.534255, \frac{\pi}{2}\right)$

(from calculator)