1. a) $v(t) = 3t^2 - 3$ $\quad x(t) = \int 3t^2 - 3 = t^3 - 3t + C$

$x(3) = 16$ $\quad 16 = 3^3 - 3(3) + C$

$16 = 27 - 9 + C$

$C = -2$

$x(t) = t^3 - 3t - 2$

$x(1) = 1^3 - 3(1) - 2 = 1 - 3 - 2 = \boxed{-4}$

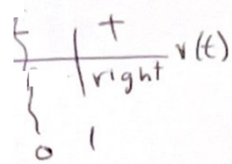
b) $v(t) = 0 = 3t^2 - 3$ $\quad t \geq 0$

$t^2 - 1 = 0$

$(t+1)(t-1) = 0$

$t = -1$ $\quad t = 1$

not in domain




The particle is moving to the left from $t=0$ to $t=1$

The particle is moving to the right from $t=1$ to $t=\infty$

c) $v(t) = 0$ at $t=1$ so particle is at rest at $t=1$

$a(t) = \frac{d}{dt} v(t) = 6t$

$a(1) = 6(1) = \boxed{6}$

d) $t^3 - 3t - 2 = 0$

| 2 | 1 | 0 | -3 | -2 |
|---|---|---|---|---|
| | | 2 | 4 | 2 |
| | 1 | 2 | 1 | 0 |

$(t-2)(t^2 + 2t + 1)$

$t = 2$ $\quad t = -1$

not in domain

$v(2) = 3(2^2) - 3 = \boxed{9}$

$a(2) = 6(2) = \boxed{12}$

e) The particle's speed is increasing at $t=2$ because velocity and acceleration have the same sign.

$u = \frac{\pi}{2}t$

$du = \frac{\pi}{2}\,dt$

2. a) $a(t) = \cos\left(\frac{\pi t}{2}\right)$ $\quad v(t) = \int \cos\left(\frac{\pi}{2}t\right) dt$

$$v(t) = \frac{2}{\pi}\int \frac{\pi}{2}\cos\frac{\pi}{2}t\ dt = \frac{2}{\pi}\int \cos u\ du$$

$$v(t) = \frac{2}{\pi}\sin\frac{\pi}{2}t + C$$

$$v(2) = 1 = \frac{2}{\pi}\sin\pi + C$$

$$C = 1$$

$$v(t) = \frac{2}{\pi}\sin\frac{\pi}{2}t + 1$$

$$v(3) = \frac{2}{\pi}\sin\frac{3\pi}{2} + 1 = \boxed{-\frac{2}{\pi} + 1}$$

b) $x(t) = \int v(t)\,dt = \frac{2}{\pi}\int \sin\frac{\pi}{2}t\,dt + \int 1\ dt$

$$x(t) = -\frac{4}{\pi^2}\cos\left(\frac{\pi}{2}t\right) + t + C_2$$

$$x(0) = 0 = -\frac{4}{\pi^2}\cos 0 + 0 + C_2$$

$$C_2 = \frac{4}{\pi^2}$$

$$x(t) = -\frac{4}{\pi^2}\cos\frac{\pi}{2}t + t + \frac{4}{\pi^2}$$

$$x(5) = -\frac{4}{\pi^2}\cos\frac{5\pi}{2} + 5 + \frac{4}{\pi^2} = \boxed{5 + \frac{4}{\pi^2}}$$

c) acceleration of particle 1 is $\cos\frac{\pi t}{2} = 0$ $\quad$ for $0 \le t \le 6$

$$\frac{\pi}{2}t = \frac{\pi}{2} + \pi n$$

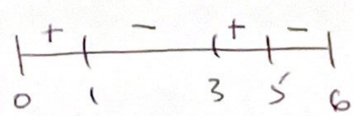
$$t = 1 + 2n$$

$t = 1,\ t = 3,\ t = 5$

acceleration of particle 2

is $v'(t) = 2 > 0$

always positive




$\boxed{\text{both positive from } t=0 \text{ to } t=1 \text{ and } t=3 \text{ to } t=5}$

d) velocity of Particle 1 $= \frac{2}{\pi}\sin\frac{\pi}{2}t + 1 = 0$

$\sin\frac{\pi}{2}t = -\frac{\pi}{2}$

Never, $-\frac{\pi}{2}$ is not on $[-1,1]$

for particle 1, $v > 0$ because $\frac{2}{\pi}\sin\frac{\pi}{2}t + 1 > 0$ for all $t$

velocity of Particle 2 $v(t) = 2t - 3 = 0$ if $t = \frac{3}{2}$

$-$ | $+$ $v(t)$

$0$ $\frac{3}{2}$ $6$

$\boxed{\text{both particles have positive velocity from } t = \frac{3}{2} \text{ to } t = 6}$

3. a) $v(t) = 9t^2 + 18t - 7$

$s(t) = \int 9t^2 + 18t - 7\, dt = 3t^3 + 9t^2 - 7t + C$

$s(0) = 3 = 0 + 0 - 0 + C \rightarrow C = 3$

$\boxed{s(t) = 3t^3 + 9t^2 - 7t + 3}$

b) $v(t) = 0 = 9t^2 + 18t - 7 \rightarrow (3t-1)(3t+7) = 0$

$t = \frac{1}{3}$ $\cancel{t = -\frac{7}{3}}$

not on $t \geq 0$

$-$ | $+$

$0$ $\frac{1}{3}$

$\boxed{\text{it changes direction at } t = \frac{1}{3}}$

c) since velocity $> 0$ on $t = 2$ to $t = 5$, speed is equal to $v(t)$

$\int_2^5 9t^2 + 18t - 7\, dt = \left[3t^3 + 9t^2 - 7t\right]_2^5 = 375 + 225 - 35 - (24 + 36 - 14)$

$= \boxed{519}$