1. a) $a(t) = 2$ $\quad v(t) = \int a(t)\,dt = 2t + C_1$

$v(0) = 3 = 2(0) + C_1$

$C_1 = 3$

$v(t) = 2t + 3$

$v(1) = 2(1) + 3 = \boxed{5 \text{ ft/sec}}$

b) $v(t) = 2t + 3 \rightarrow s(t) = \int 2t + 3\,dt = t^2 + 3t + C_2$

$s(0) = 5 = 0^2 + 3(0) + C_2$

$C_2 = 5$

$s(t) = t^2 + 3t + 5$

$s(2) = 2^2 + 3(2) + 5 = \boxed{15}$

2. $v(t) = \frac{1}{t} + 2t$ $\quad s(t) = \int \frac{1}{t} + 2t\,dt = \ln|t| + t^2 + C$

$s(1) = 3 = \ln 1 + 1^2 + C$

$3 = C + 1$

$C = 2$

$s(t) = \ln|t| + t^2 + 2$

$s(e) = \ln e + e^2 + 2 = \boxed{3 + e^2}$

3. $a(t) = \sec^2 t$ $\quad v(t) = \int a(t)\,dt = \int \sec^2 t\,dt = \tan t + C$

$$v\left(\frac{\pi}{4}\right) = 2 = \tan\frac{\pi}{4} + C$$

$$2 = 1 + C$$

$$C = 1$$

$$v(t) = \tan t + 1$$

$$v\left(\frac{\pi}{6}\right) = \tan\frac{\pi}{6} + 1 = \boxed{\frac{\sqrt{3}}{3} + 1}$$

4. $a(t) = \sin t + \cos t$ $\quad v(t) = \int \sin t + \cos t\,dt$

$$v(t) = -\cos t + \sin t + C_1$$

$$s(t) = \int v(t)\,dt = \int -\cos t + \sin t + C_1\,dt$$

$$s(t) = -\sin t - \cos t + tC_1 + C_2$$

$$s(0) = 2 = -\sin 0 - \cos 0 + 0(C_1) + C_2$$

$$2 = -1 + C_2$$

$$3 = C_2$$

$$s\left(\frac{\pi}{2}\right) = 3 = -\sin\frac{\pi}{2} - \cos\frac{\pi}{2} + \frac{\pi}{2}C_1 + 3$$

$$0 = -1 + \frac{\pi}{2}C_1$$

$$1 = \frac{\pi}{2}C_1$$

$$C_1 = \frac{2}{\pi} \longrightarrow s(t) = -\sin t - \cos t + \frac{2t}{\pi} + 3$$

$$s\left(\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} - \cos\frac{\pi}{6} + \frac{2}{\pi}\left(\frac{\pi}{6}\right) + 3$$

$$= -\frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{1}{3} + 3 = \boxed{\frac{17}{6} - \frac{\sqrt{3}}{2} = \frac{17 - 3\sqrt{3}}{6}}$$

5. a) $\int_0^1 4t^3-8t+1\,dt = \left[t^4-4t^2+t\right]_0^1 = 1-4+1 = \boxed{-2}$

b) $\int_1^5 4t^3-8t+1\,dt = \left[t^4-4t^2+t\right]_1^5 = 625-100+5-(1-4+1)$

$= \boxed{532}$

c) $\int_0^3 |4t^3-8t+1|\,dt \approx \boxed{53.3626080563}$

6. a) $v(t) = \int a(t)\,dt = \int 2t-1\,dt = t^2-t+C_1$

$v(1)=5 = 1^2-1+C_1 \rightarrow C_1=5$

$\boxed{v(t) = t^2-t+5}$

b) $v(2) = 2^2-2+5 = \boxed{7}$

c) $s(t) = \int t^2-t+5\,dt = \frac{t^3}{3}-\frac{t^2}{2}+5t+C_2$

$s(0)=0 = 0-0+0+C_2 \rightarrow C_2=0$

$\boxed{s(t) = \frac{t^3}{3}-\frac{t^2}{2}+5t}$

d) $\int_0^3 t^2-t+5\,dt = \left[\frac{t^3}{3}-\frac{t^2}{2}+5t\right]_0^3 = \frac{27}{3}-\frac{9}{2}+15-0 = \boxed{\frac{39}{2}}$

e) $\int_0^5 |t^2-t+5|\,dt \approx 54.1\overline{6}$

f) $s(4) = \frac{4^3}{3}-\frac{4^2}{2}+5(4) = \frac{64}{3}-8+20 = \boxed{\frac{100}{3}}$