

$$1. f'(x) = \cos 2x \rightarrow f(x) = \int \cos 2x \, dx = \frac{1}{2} \sin 2x + C$$

$$f\left(\frac{\pi}{2}\right) = \frac{1}{2} \sin\left(2\left(\frac{\pi}{2}\right)\right) + C = 1$$

$$C = 1$$

$$\boxed{f(x) = \frac{1}{2} \sin 2x + 1 \text{ for } (-\infty, \infty)}$$

$$2. \frac{dy}{dx} = \sin x - \sec^2 x + \sqrt{x}$$

$$y(0) = 1$$

$$y = -\cos x - \tan x + \frac{2}{3} x^{\frac{3}{2}} + C$$

$$1 = -\cos 0 - \tan 0 + 0 + C$$

$$1 = -1 + C$$

$$C = 2$$

$$\boxed{y = -\cos x - \tan x + \frac{2}{3} x^{\frac{3}{2}} + 2}$$

$$\text{for } \left[0, \frac{\pi}{2}\right)$$

$$3. f'(x) = x^2 + \sec x \tan x \text{ and } f(0) = 1$$

$$f(x) = \frac{x^3}{3} + \sec x + C$$

$$f(x) = \frac{x^3}{3} + \sec x$$

$$1 = 0 + \sec 0 + C$$

$$1 = 1 + C$$

$$C = 0$$

$$f(\pi) = \frac{\pi^3}{3} + \sec \pi = \boxed{\frac{\pi^3}{3} - 1}$$

$$4. f'(x) = 6x^2 - 1 \quad f(2) = 3$$

$$f(x) = 2x^3 - x + C$$

$$f(2) = 2(8) - 2 + C = 3$$

$$C = -11$$

$$f(x) = 2x^3 - x - 11$$

$$f(3) = 2(27) - 3 - 11 = \boxed{40}$$

$$5. f(x) = \int_1^x \sqrt{100 - t^4} \, dt + 3$$

$$\text{so } f(2) = \int_1^2 \sqrt{100 - t^4} \, dt + 3 \approx \boxed{12.682}$$

$$6. f(x) = \int_2^x \cos(t^2) \, dt + 7$$

$$\text{so } f(3) = \int_2^3 \cos(t^2) \, dt + 7 \approx \boxed{7.241}$$