Use your calculator to approximate each value.

1. $f(2)$ if $f'(x) = e^{-x^2}$ and $f(3) = 1$

$f(x) = \int_3^x e^{-t^2}\,dt + C \rightarrow f(3) = \int_3^3 e^{-t^2}\,dt + C = 1$

$0 + C = 1 \rightarrow C = 1$

$f(x) = \int_3^x e^{-t^2}\,dt + 1 \qquad f(2) = \int_3^2 e^{-t^2}\,dt + 1 \approx \boxed{.996}$

2. $f(\pi)$ if $f'(x) = \frac{e^x}{x}$ and $f(1) = -2$

$f(x) = \int_1^x \frac{e^t}{t}\,dt + C \rightarrow f(1) = -2 = \int_1^1 \frac{e^t}{t}\,dt + C$

$-2 = 0 + C$

$C = -2$

$f(x) = \int_1^x \frac{e^t}{t}\,dt - 2 \rightarrow f(\pi) = \int_1^\pi \frac{e^t}{t}\,dt - 2 \approx \boxed{7.033}$

3. $g(3)$ if $g'(x) = \frac{-7}{\ln x}$ and $g(4) = 1$

$g(x) = \int_4^x \frac{-7}{\ln t}\,dt + C \rightarrow g(4) = \int_4^4 \frac{-7}{\ln t}\,dt + C = 1$

$\rightarrow 0 + C = 1 \rightarrow C = 1$

$g(x) = \int_4^x \frac{-7}{\ln t}\,dt + 1 \rightarrow g(3) = \int_4^3 \frac{-7}{\ln t}\,dt + 1 \approx \boxed{6.628}$

4. $y(4)$ if $\frac{dy}{dx} = \cos(x^2)$ and $y\left(-\frac{\pi}{2}\right) = 1$

$y = \int_{-\frac{\pi}{2}}^x \cos t^2\,dt + C \qquad y\left(-\frac{\pi}{2}\right) = 1 = \int_{-\frac{\pi}{2}}^{-\frac{\pi}{2}} \cos t^2\,dt + C$

$1 = 0 + C$

$C = 1$

$y = \int_{-\frac{\pi}{2}}^x \cos t^2\,dt + 1 \rightarrow y(4) = \int_{-\frac{\pi}{2}}^4 \cos t^2\,dt + 1 \approx \boxed{2.444}$

5. $f\left(\frac{1}{2}\right)$ if $f'(x) = \sqrt{1 - x^4}$ and $f\left(\frac{3}{4}\right) = \frac{5}{4}$

$f(x) = \int_{\frac{3}{4}}^x \sqrt{1-t^4}\,dt + C \qquad f\left(\frac{3}{4}\right) = \frac{5}{4} = \int_{\frac{3}{4}}^{\frac{3}{4}} \sqrt{1-t^4}\,dt + C$

$\frac{5}{4} = 0 + C \rightarrow C = \frac{5}{4}$

$f(x) = \int_{\frac{3}{4}}^x \sqrt{1-t^4}\,dt + \frac{5}{4} \rightarrow f\left(\frac{1}{2}\right) = \int_{\frac{3}{4}}^{\frac{1}{2}} \sqrt{1-t^4}\,dt + \frac{5}{4} \approx \boxed{1.022}$

6. $y(2)$ if $\frac{dy}{dx} = \sin(x^2)$ and $y(1) = 10$

$y = \int_1^x \sin t^2\,dt + C \qquad y(1) = 10 = \int_1^1 \sin t^2\,dt + C$

$10 = 0 + C \rightarrow C = 10$

$y = \int_1^x \sin t^2\,dt + 10 \rightarrow y(2) = \int_1^2 \sin t^2\,dt + 10 \approx \boxed{10.495}$

7. $f\left(\frac{\pi}{6}\right)$ if $f'(x) = \frac{\sin x}{x}$ and $f\left(\frac{\pi}{3}\right) = 2$

$f(x) = \int_{\frac{\pi}{3}}^x \frac{\sin t}{t}\,dt + C \qquad f\left(\frac{\pi}{3}\right) = 2 = \int_{\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\sin t}{t}\,dt + C$

$2 = 0 + C$

$C = 2$

$f(x) = \int_{\frac{\pi}{3}}^x \frac{\sin t}{t}\,dt + 2 \rightarrow f\left(\frac{\pi}{6}\right) = \int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{\sin t}{t}\,dt + 2 \approx \boxed{1.530}$