Solve each differential equation given the initial condition and state the interval for which it is valid.

1. $\frac{dy}{dx} = 3\sin x$ and $y = 2$ when $x = 0$

$y = -3\cos x + C \rightarrow 2 = -3\cos 0 + C$

$2 = -3 + C$

$C = 5$

$\boxed{y = -3\cos x + 5}$ $\boxed{\text{for } (-\infty, \infty)}$

2. $\frac{dy}{dx} = -\frac{1}{x^2} - \frac{3}{x^4} + 12$ and $y = 3$ when $x = 1$

$y = \frac{1}{x} + \frac{1}{x^3} + 12x + C \rightarrow 3 = 1 + 1 + 12 + C$

$C = -11$

$\boxed{y = \frac{1}{x} + \frac{1}{x^3} + 12x - 11}$ $\boxed{\text{for } (0, \infty)}$

3. $\frac{dy}{dt} = 2e^t - \cos t$ and $y = 3$ when $t = 0$

$y = 2e^t - \sin t + C \rightarrow 3 = 2e^0 - \sin 0 + C$

$1 = C$

$\boxed{y = 2e^t - \sin t + 1}$ $\boxed{\text{for } (-\infty, \infty)}$

4. $\frac{dv}{dt} = 4\sec t\tan t + e^t + 6t$ and $v = 5$ when $t = 0$

$V = 4\sec t + e^t + 3t^2 + C$

$\rightarrow 5 = 4\sec 0 + e^0 + 3(0^2) + C$

$5 = 4 + 1 + C$

$C = 0$

$\boxed{V = 4\sec t + e^t + 3t^2}$ $\boxed{\text{for } (-\frac{\pi}{2}, \frac{\pi}{2})}$

5. $\frac{du}{dx} = 5\sec^2 x - \frac{3\sqrt{x}}{2}$ and $u = 7$ when $x = 0$

$u = 5\tan x - x^{\frac{3}{2}} + C \rightarrow 7 = 5\tan 0 - 0 + C$

$C = 7$

$\boxed{u = 5\tan x - x^{\frac{3}{2}} + 7}$ $\boxed{\text{for } [0, \frac{\pi}{2})}$

6. $\frac{dy}{dt} = (\cos t)e^{\sin t}$ and $y = 2$ when $t = 0$

$u = \sin t$

$du = \cos t \, dt$ $\rightarrow \int e^u du$ $\rightarrow$ $y = e^{\sin t} + C \rightarrow 2 = e^{\sin 0} + C$

$2 = 1 + C$

$C = 1$

$\boxed{y = e^{\sin t} + 1}$ $\boxed{\text{for } (-\infty, \infty)}$

7. $\frac{dy}{dx} = \frac{1}{x-2} + e^{x-2}$ and $y = 1$ when $x = 3$

$y = \ln|x-2| + e^{x-2} + C \rightarrow \ln 1 + e^1 + C = 1$

$C = 1 - e$

$\boxed{y = \ln|x-2| + e^{x-2} + 1 - e \quad \text{for } (2, \infty)}$

8. $\frac{dy}{dx} = 3x^2 - \frac{2x}{\sqrt{1-x^4}}$ and $y = \frac{1}{\sqrt[4]{8}}$ when $x = \frac{1}{\sqrt[4]{2}}$

$y = x^3 - \sin^{-1}(x^2) + C \rightarrow \frac{1}{\sqrt[4]{8}} = \frac{1}{\sqrt[4]{8}} - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + C$

$u = x^2$

$du = 2x$ $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u$

$-1 \le x^2 \le 1$

$-1 \le x \le 1$

$\rightarrow \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = C$

$C = \frac{\pi}{4}$

$\boxed{y = x^3 - \sin^{-1}(x^2) + \frac{\pi}{4} \quad \text{for } [-1, 1]}$