

Extra Integrals Test Review

Directions: Do the problems without a calculator.

1. Find the value of each integral using a limit and summation:

a. $\int_0^2 (2x - 3) dx$

b. $\int_0^3 (x^2 - x) dx$

c. $\int_1^4 (-x + 6) dx$

d. $\int_{-1}^1 (x^2 - 2) dx$

2. Suppose that $\int_0^4 f(x) dx = 3$, $\int_0^7 f(x) dx = -2$, $\int_1^4 f(x) dx = 5$, $\int_3^7 f(x) dx = -5$, $\int_0^3 g(x) dx = 2$, $\int_0^7 g(x) dx = 11$, $\int_2^3 g(x) dx = 1$, $\int_4^1 g(x) dx = 3$. Find each of the following:

a. $\int_4^7 f(x) dx$

b. $\int_1^1 f(x) dx$

c. $\int_0^7 (3f(x) + 2g(x)) dx$

d. $\int_0^3 (g(x) - f(x)) dx$

e. $\int_3^2 g(x) dx$

f. $\int_0^2 5g(x) dx$

g. $\int_7^3 g(x) dx$

3. Evaluate each of the following:

a. $\frac{d}{dx} \int_3^x \frac{1}{\ln^2 t} dt$

b. $\frac{d}{dx} \int_{-2}^{x^2} \sec(t^3) dt$

c. $\frac{d}{dx} \int_x^{\sin x} t^3 \cos(t^2) dt$

d. $\frac{d}{dx} \int_{2x^2}^{e^{2x}} \frac{1}{\sin t} dt$

4. Find the average value of each function on the given interval. If the Mean Value Theorem for Integrals applies, find all values on the given interval for which the function is equal to its average value.

a. $f(x) = x^2 - 3$ on $[-3, 0]$

b. $f(x) = \cos x$ on $\left[0, \frac{\pi}{2}\right]$

c. $f(x) = e^{2x}$ on $[1, 4]$

d. $f(x) = x\sqrt[3]{x-1}$ on $[0, 9]$

e. $f(x) = \begin{cases} 3x^2 + 1 & \text{if } x < 3 \\ 3x^2 - 1 & \text{if } x \geq 3 \end{cases}$ on $[0, 6]$

f. $f(x) = \begin{cases} e^x & \text{if } x \leq 1 \\ \frac{1}{x} & \text{if } x > 1 \end{cases}$ on $[0, 4]$

g. $f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 2 \\ x^3 - 3 & \text{if } x > 2 \end{cases}$ on $[-1, 5]$

h. $f(x) = \begin{cases} \sin x & \text{if } x \leq \frac{\pi}{4} \\ \cos x & \text{if } x > \frac{\pi}{4} \end{cases}$ on $\left[0, \frac{\pi}{3}\right]$

5. Find each definite integral using geometry:

a. $\int_1^3 (5x + 1) dx$

b. $\int_0^3 (-2x + 6) dx$

c. $\int_{-1}^2 (-x + 4) dx$

d. $\int_0^2 \sqrt{4 - x^2} dx$

e. $\int_{-3}^0 \sqrt{9 - x^2} dx$

f. $\int_{-1}^1 \sqrt{1 - x^2} dx$

g. $\int_{-2}^2 (2 + \sqrt{4 - x^2}) dx$

h. $\int_0^2 (2 - \sqrt{4 - x^2}) dx$

i. $\int_{-8}^0 \sqrt{16 - (x + 4)^2} dx$

6. Find each definite integral:

a. $\int_0^2 (x - 3) dx$

b. $\int_{-\pi}^{\pi} \sin x \cos x dx$

c. $\int_0^3 e^{2x} dx$

d. $\int_{-1}^2 \sqrt{x + 2} dx$

e. $\int_1^6 \frac{1}{x+3} dx$

f. $\int_{-2}^3 (e^{2x} - e^{3x}) dx$

g. $\int_0^5 x\sqrt{x+4} dx$

h. $\int_1^3 \frac{2}{x+1} dx$

i. $\int_{-4}^{-1} \frac{2}{x-2} dx$

j. $\int_1^8 x^{\frac{2}{3}} dx$

k. $\int_0^1 x\sqrt{3x^2 + 1} dx$

l. $\int_{-3}^0 (2x - 1)^2 dx$

m. $\int_0^{\frac{\pi}{12}} \sec^2(3x) dx$

n. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{3}} \sin x \cos^3 x dx$

o. $\int_0^4 f(x) dx$ for $f(x) = \begin{cases} 3x^2 - 3 & \text{if } x \leq 3 \\ \frac{1}{x-2} & \text{if } x > 3 \end{cases}$

7. Find each indefinite integral:

a. $\int (x^3 - x^2) dx$

b. $\int \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

c. $\int (e^{3x} - \cos 2x) dx$

d. $\int x \sin\left(\frac{1}{2}x^2\right) dx$

e. $\int (2x^3)\sqrt[3]{x^2 - 1} dx$

f. $\int \sec \frac{1}{3}x \tan \frac{1}{3}x dx$

g. $\int \left(\frac{2}{3x-1} + \frac{4}{5x+1}\right) dx$

h. $\int \frac{x^2-8x-1}{x+1} dx$

i. $\int \frac{x^3+x^2+x+1}{x-2} dx$

j. $\int \frac{x^4-x-6}{x+5} dx$

k. $\int \frac{2}{9+x^2} dx$

l. $\int \frac{2x}{9+x^2} dx$

m. $\int \frac{3}{\sqrt{25-x^2}} dx$

n. $\int \frac{3x}{\sqrt{25-x^2}} dx$

o. $\int \frac{x^3}{1+x^2} dx$

p. $\int \frac{5}{4+x^2} dx$

q. $\int \frac{6}{\sqrt{100-x^2}} dx$

r. $\int \frac{x-2}{(x^2-4x-1)^3} dx$

s. $\int \frac{x-x^3+1}{\sqrt{x}} dx$

t. $\int \frac{\cos x}{\sin^4 x} dx$

u. $\int \frac{\sin 2x}{\cos^3 x} dx$

v. $\int \frac{\sin 2x}{\cos^4 x} dx$