Review

Directions: Do the problems. Don't use a calculator.

1. Use a right, left, and midpoint rectangular approximation as well as a trapezoidal approximation with the given number of subintervals to approximate the value of each integral. For the right, left, and trapezoidal approximations, indicate whether it is an underestimate or an overestimate and give a reason why.

a. $\int_1^4 2x^2 - 1\,dx;\ n = 3$

b. $\int_0^4 -\frac{1}{2}x^2 + x + 4\,dx;\ n = 4$

c. $\int_{-4}^0 (x^3 + x + 8)\,dx;\ n = 4$

d. $\int_1^3 \frac{1}{x^2}\,dx;\ n = 4$

2. Use Part 1 of the Fundamental Theorem of Calculus to evaluate:

a. $\frac{d}{dx}\int_{-3}^x t^2(1 + e^t)\,dt$

b. $\frac{d}{dx}\int_5^x \cos(t^3)\,dt$

c. $\frac{d}{dx}\int_x^7 e^{-t^2}\,dt$

d. $\frac{d}{dx}\int_5^{3x^2} \frac{t-3}{t-6}\,dt$

e. $\frac{d}{dx}\int_4^{\sin x} \frac{t-4}{\sin t}\,dt$

f. $\frac{d}{dx}\int_{3x}^{x^2-1} (t + \tan t)\,dt$

g. $\frac{d}{dx}\int_{\sin x}^{\tan x} \frac{t-3}{t+e^t}\,dt$

3. Write each integral as a limit and summation using Newton's notation:

a. $\int_0^3 (x^2 + 5x - 1)\,dx$

b. $\int_0^1 (x^2 + \sin x)\,dx$

c. $\int_1^2 (x+8)\,dx$

d. $\int_2^5 (x^2-3x)\,dx$

e. $\int_1^4 (e^x+\tan x)\,dx$

4. If $\int_1^3 f(x)\,dx = 4$, $\int_5^3 f(x)\,dx = -2$, $\int_1^3 h(x)\,dx = -6$, $\int_3^7 h(x)\,dx = 10$, $\int_7^6 h(x)\,dx = 5$, find:

a. $\int_1^3 \big(3f(x)-4h(x)\big)\,dx$

b. $\int_2^2 f(x)\,dx$

c. $\int_1^7 h(x)\,dx$

d. $\int_1^5 f(x)\,dx$

e. $\int_3^6 5h(x)\,dx$

f. $\int_3^1 h(x)\,dx$

5. If $\int_1^4 f(x)\,dx = 1$, $\int_7^4 f(x)\,dx = -2$, $\int_1^5 f(x)\,dx = 3$, $\int_1^5 h(x)\,dx = 10$, $\int_7^5 h(x)\,dx = -5$, find:

a. $\int_1^1 f(x)\,dx$

b. $\int_1^7 3f(x)\,dx$

c. $\int_1^5 4f(x)\,dx + \int_1^5 3h(x)\,dx$

d. $\int_1^7 h(x)\,dx$

e. $\int_4^5 f(x)\,dx$

f. $\int_4^1 f(x)\,dx$

6. Find the average value of each of the following functions on the given interval:

a. $f(x) = x^2+3$ on $[-1,2]$

b. $f(x) = 3x^2-2x-1$ on $[0,4]$

c. $f(x) = \sin x$ on $\left[0,\frac{2\pi}{3}\right]$

d. $f(x) = \tan 2x$ on $\left[0, \frac{\pi}{6}\right]$

e. $f(x) = x\sqrt{x^2 - 3}$ on $\left[\sqrt{3}, 2\right]$

7. Find each indefinite integral:

a. $\int (x^2 - 3x + 8)\, dx$

b. $\int (3x^4 - 5x^2 + x - 1)\, dx$

c. $\int \frac{5x^2-2x+1}{\sqrt[3]{x}}\, dx$

d. $\int \frac{7x^3+8\sqrt{x}-x^2}{\sqrt{x}}\, dx$

e. $\int \sqrt{6x}\, dx$

f. $\int x^3\sqrt{x^2 - 8}\, dx$

g. $\int 10x\sqrt{x^2 + 1}\, dx$

h. $\int \sqrt{4x + 7}\, dx$

i. $\int \tan 3x\, dx$

j. $\int \cot 8x\, dx$

k. $\int \frac{\cos x}{\sin^4 x}\, dx$

l. $\int \cos^5 \theta\, d\theta$

m. $\int \sin^7 x\, dx$

n. $\int \sin^2 x \cos^3 x\, dx$

o. $\int \frac{\sin x}{\cos^6 x}\, dx$

p. $\int \frac{x-3}{x^2-6x+1}\, dx$

q. $\int \frac{4x-8}{x^2-4x-1}\, dx$

r. $\int \frac{x+1}{(x^2+2x+5)^3}\, dx$

s. $\int \frac{x+4}{(x^2+8x+1)^5}\, dx$

t. $\int \frac{x^2+5x+1}{x-2} dx$

u. $\int \frac{x^3-x-7}{x+2} dx$

v. $\int \frac{x^4+x^3+5x^2-x+1}{x-3} dx$

w. $\int e^{\frac{x}{2}} + x^2 dx$

x. $\int e^{7x} + x dx$

y. $\int (x-7)^6 \, dx$

z. $\int (3x-4)^4 \, dx$

aa. $\int \frac{dx}{\sqrt[4]{5x+1}}$

bb. $\int \frac{2x}{\sqrt[3]{x^2-1}} dx$

cc. $\int \frac{7}{\sqrt{x-4}} dx$

dd. $\int \frac{2}{\sqrt[3]{5x+6}} dx$

8. Evaluate each definite integral:

a. $\int_1^4 (x^2 - x - 1) \, dx$

b. $\int_0^{\frac{\pi}{4}} \tan x \, dx$

c. $\int_0^{\frac{\pi}{6}} \sin 3x \, dx$

d. $\int_0^{\sqrt{3}} x\sqrt{x^2+1} \, dx$

e. $\int_0^5 (e^{2x} - x^2 + x) \, dx$

f. $\int_0^2 \frac{x^2-x-1}{x-3} dx$

g. $\int_1^{16} \sqrt[4]{x} \, dx$

h. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^3 x \, dx$