

$$1. a) \int_0^2 2x-3 \, dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n (2x_k - 3) \Delta x \quad \text{with } \Delta x = \frac{2}{n} \quad x_k = \frac{2k}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left[2\left(\frac{2k}{n}\right) - 3 \right] \left(\frac{2}{n}\right) = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{8k}{n^2} - \frac{6}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{8n(n+1)}{2n^2} - \frac{6n}{n} = 4 - 6 = \boxed{-2}$$

$$b) \int_0^3 x^2 - x \, dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n (x_k^2 - x_k) \Delta x \quad \text{with } \Delta x = \frac{3}{n} \quad x_k = \frac{3k}{n}$$

$$= \lim_{n \rightarrow \infty} \left[\left(\frac{3k}{n}\right)^2 - \frac{3k}{n} \right] \left(\frac{3}{n}\right) = \lim_{n \rightarrow \infty} \frac{27k^2}{n^3} - \frac{9k}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{27(n)(n+1)(2n+1)}{6n^3} - \frac{9n(n+1)}{2n^2} = 9 - \frac{9}{2} = \boxed{\frac{9}{2}}$$

$$c) \int_1^4 -x + 6 \, dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n (-x_k + 6) \Delta x \quad \text{with } \Delta x = \frac{3}{n} \quad x_k = 1 + \frac{3k}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(-1 - \frac{3k}{n} + 6\right) \left(\frac{3}{n}\right) = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{15}{n} - \frac{9k}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{15n}{n} - \frac{9n(n+1)}{2n^2} = 15 - \frac{9}{2} = \boxed{\frac{21}{2}}$$

$$d) \int_{-1}^1 x^2 - 2 \, dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n (x_k^2 - 2) \Delta x \quad \text{with } \Delta x = \frac{2}{n} \quad x_k = -1 + \frac{2k}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\left(-1 + \frac{2k}{n}\right)^2 - 2 \right) \left(\frac{2}{n}\right) = \lim_{n \rightarrow \infty} \sum_{k=1}^n -\frac{2}{n} - \frac{8k}{n^2} + \frac{8k^2}{n^3}$$

$$= \lim_{n \rightarrow \infty} -\frac{2n}{n} - \frac{8n(n+1)}{2n^2} + \frac{8n(n+1)(2n+1)}{6n^3} = -2 - 4 + \frac{8}{3} = \boxed{-\frac{10}{3}}$$

$$2. a) \int_4^7 f(x) dx = \int_0^7 f(x) dx - \int_0^4 f(x) dx = -2 - 3 = \boxed{-5}$$

$$b) \int_1^1 f(x) dx = \boxed{0}$$

$$c) \int_0^7 3f(x) + 2g(x) dx = 3 \int_0^7 f(x) dx + 2 \int_0^7 g(x) dx = 3(-2) + 2(11) = \boxed{16}$$

$$d) \int_0^3 g(x) - f(x) dx = \int_0^3 g(x) dx - \int_0^3 f(x) dx = 2 - \left[\int_0^7 f(x) dx - \int_3^7 f(x) dx \right]$$

$$= 2 - [-2 - (-5)] = 2 - 3 = \boxed{-1}$$

$$e) \int_2^2 g(x) dx = - \int_2^3 g(x) dx = \boxed{-1}$$

$$f) \int_0^3 5g(x) dx = 5 \int_0^3 g(x) dx = 5 \left[\int_0^3 g(x) dx - \int_2^3 g(x) dx \right] = 5[2 - 1] = \boxed{5}$$

$$g) \int_7^3 g(x) dx = - \int_3^7 g(x) dx = - \left[\int_0^7 g(x) dx - \int_0^3 g(x) dx \right] = -[11 - 2] = \boxed{-9}$$

$$3. a) \frac{d}{dx} \int_3^x \frac{1}{\ln^2 t} dt = \boxed{\frac{1}{\ln^2 x}}$$

$$b) \frac{d}{dx} \int_{-2}^{x^2} \sec(t^3) dt = \frac{d}{dx} [F(x^2) - F(-2)] \text{ if } F \text{ is an anti-derivative of } \sec(t^3)$$

$$= \boxed{2x \sec(x^6)}$$

$$c) \frac{d}{dx} \int_x^{\sin x} t^3 \cos(t^2) dt = \frac{d}{dx} [F(\sin x) - F(x)] \text{ if } F \text{ is an anti-derivative of } t^3 \cos(t^2)$$

$$= \boxed{(\sin^3 x)(\cos(\sin^2 x))(\cos x) - x^3 \cos(x^2)}$$

d) $\frac{d}{dx} \int_{2x^2}^{e^{2x}} \frac{1}{\sin t} dt = \frac{d}{dx} [F(e^{2x}) - F(2x^2)]$ if F is an antiderivative of $\frac{1}{\sin t}$

$$= \frac{2e^{2x}}{\sin(e^{2x})} - \frac{4x}{\sin(2x^2)}$$

4. a) $f(x) = x^2 - 3$ on $[-3, 0]$
 f cont for $\mathbb{R}$

$$\frac{1}{0 - (-3)} \int_{-3}^0 x^2 - 3 dx = \frac{1}{3} \left[\frac{x^3}{3} - 3x \right]_{-3}^0$$

$x^2 - 3 = 0$
 $x = \pm\sqrt{3}$ $x = \sqrt{3}$

$$= \frac{1}{3} [0 - (-9 + 9)] = \boxed{0}$$

b) $f(x) = \cos x$ on $[0, \frac{\pi}{2}]$
 f cont for $\mathbb{R}$
 $\cos x = \frac{2}{\pi}$

$$\frac{1}{\frac{\pi}{2} - 0} \int_0^{\frac{\pi}{2}} \cos x dx = \frac{2}{\pi} \left[\sin x \right]_0^{\frac{\pi}{2}}$$

$$= \frac{2}{\pi} [1 - 0] = \boxed{\frac{2}{\pi}}$$

$x = \cos^{-1}(\frac{2}{\pi})$

c) $f(x) = e^{2x}$ on $[1, 4]$
 f cont for $\mathbb{R}$

$$\frac{1}{4-1} \int_1^4 e^{2x} dx = \frac{1}{3} \left[\frac{1}{2} e^{2x} \right]_1^4$$

$$= \frac{1}{3} \left[\frac{1}{2} e^8 - \frac{1}{2} e^2 \right] = \boxed{\frac{1}{6} (e^8 - e^2)}$$

$e^{2x} = \frac{1}{6} (e^8 - e^2)$
 $x = \frac{1}{2} \ln \left[\frac{1}{6} (e^8 - e^2) \right]$

d) $f(x) = x \sqrt[3]{x-1}$ on $[0, 9]$

$$\frac{1}{9-0} \int_0^9 x \sqrt[3]{x-1} dx$$

$u = x-1$
 $du = dx$
 $x = u+1$
 $x=0, u=-1$
 $x=9, u=8$

$$= \frac{1}{9} \int_{-1}^8 (u+1) u^{\frac{1}{3}} du = \frac{1}{9} \int_{-1}^8 u^{\frac{4}{3}} + u^{\frac{1}{3}} du$$

$$= \frac{1}{9} \left[\frac{3}{7} u^{\frac{7}{3}} + \frac{3}{4} u^{\frac{4}{3}} \right]_{-1}^8 = \frac{1}{9} \left[\frac{3}{7} (128) + \frac{3}{4} (16) - \left(-\frac{3}{7} + \frac{3}{4} \right) \right]$$

$$= \frac{1}{9} \left[\frac{387}{7} + \frac{45}{4} \right] = \boxed{\frac{207}{28}}$$

don't bother finding where
 $f(x) = \frac{207}{28}$, it is too
 annoying

$$e) f(x) = \begin{cases} 3x^2+1 & \text{if } x < 3 \\ 3x^2-1 & \text{if } x \geq 3 \end{cases} \quad \text{on } [0, 6] \quad \frac{1}{6-0} \int_0^6 f(x) dx$$

$$= \frac{1}{6} \left[\int_0^3 (3x^2+1) dx + \int_3^6 (3x^2-1) dx \right] = \frac{1}{6} \left[\left[x^3+x \right]_0^3 + \left[x^3-x \right]_3^6 \right]$$

$$= \frac{1}{6} [30-0 + (210-24)] = \boxed{36}$$

$$\lim_{x \rightarrow 3^-} f(x) = 28$$

$$\lim_{x \rightarrow 3^+} f(x) = 26$$

so not continuous at $x=3$

$$f) f(x) = \begin{cases} e^x & \text{if } x \leq 1 \\ \frac{1}{x} & \text{if } x > 1 \end{cases} \quad \text{on } [0, 4] \quad \frac{1}{4-0} \int_0^4 f(x) dx$$

$$= \frac{1}{4} \left[\int_0^1 e^x dx + \int_1^4 \frac{1}{x} dx \right] = \frac{1}{4} \left[(e^x)_0^1 + (\ln|x|)_1^4 \right]$$

$$= \frac{1}{4} [e-1 + \ln 4 - \ln 1] = \boxed{\frac{1}{4}(e-1+\ln 4)}$$

$$\lim_{x \rightarrow 1^-} f(x) = e$$

$$\lim_{x \rightarrow 1^+} f(x) = 1$$

so not continuous at $x=1$

$$g) f(x) = \begin{cases} x^2+1 & \text{if } x \leq 2 \\ x^3-3 & \text{if } x > 2 \end{cases} \quad \text{on } [-1, 5] \quad \frac{1}{5-(-1)} \int_{-1}^5 f(x) dx$$

$$= \frac{1}{6} \left[\int_{-1}^2 (x^2+1) dx + \int_2^5 (x^3-3) dx \right] = \frac{1}{6} \left[\left[\frac{x^3}{3} + x \right]_{-1}^2 + \left[\frac{x^4}{4} - 3x \right]_2^5 \right]$$

$$= \frac{1}{6} \left[\frac{8}{3} + 2 - \left(-\frac{1}{3} - 1 \right) + \left[\frac{625}{4} - 15 - (4-6) \right] \right] = \boxed{\frac{199}{8}}$$

$$\lim_{x \rightarrow 2^-} f(x) = 5$$

$$x^2+1 = \frac{199}{8} \rightarrow x = \pm \sqrt{\frac{191}{8}}$$

$$\rightarrow x = \pm \sqrt{\frac{191}{8}}$$

only $\frac{\sqrt{191}}{2\sqrt{2}}$ on $[-1, 2]$ but not on $[-1, 5]$

$$\lim_{x \rightarrow 2^+} f(x) = 5$$

$$\text{and } x^3-3 = \frac{199}{8} \rightarrow x = \frac{\sqrt[3]{223}}{2}$$

$$\rightarrow x = \frac{\sqrt[3]{223}}{2} \text{ on } (2, 5]$$

$$h) f(x) = \begin{cases} \sin x & \text{if } x \leq \frac{\pi}{4} \\ \cos x & \text{if } x > \frac{\pi}{4} \end{cases} \text{ on } \left[0, \frac{\pi}{3}\right] \quad \frac{1}{\frac{\pi}{3}-0} \int_0^{\frac{\pi}{3}} f(x) dx$$

$$= \frac{3}{\pi} \left[\int_0^{\frac{\pi}{4}} \sin x dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos x dx \right] = \frac{3}{\pi} \left[(-\cos x)_0^{\frac{\pi}{4}} + (\sin x)_{\frac{\pi}{4}}^{\frac{\pi}{3}} \right]$$

$$= \frac{3}{\pi} \left[-\frac{\sqrt{2}}{2} - (-1) + \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \right) \right] = \boxed{\frac{3}{\pi} \left[-\sqrt{2} + 1 + \frac{\sqrt{3}}{2} \right]}$$

$$\lim_{x \rightarrow \frac{\pi}{4}^-} f(x) = \frac{\sqrt{2}}{2} \quad \lim_{x \rightarrow \frac{\pi}{4}^+} f(x) = \frac{\sqrt{2}}{2} \quad \dots \text{ but too annoying to find value so don't bother}$$

$$5. a) \int_1^3 (5x+1) dx = \left(\frac{6+16}{2} \right) (2) = \boxed{22}$$

$$b) \int_0^3 (-2x+6) dx = \frac{1}{2} (3)(6) = \boxed{9}$$

$$c) \int_{-1}^2 (-x+4) dx = \left(\frac{5+2}{2} \right) (3) = \boxed{\frac{21}{2}}$$

$$d) \int_0^2 \sqrt{4-x^2} dx = \frac{1}{4} \pi (2^2) = \boxed{\pi}$$

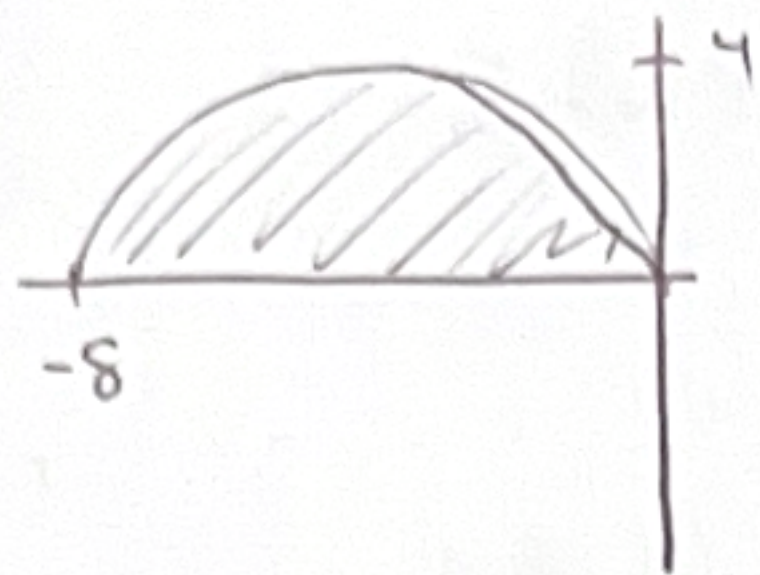
$$e) \int_0^3 \sqrt{9-x^2} dx = \frac{1}{4} \pi (3^2) = \boxed{\frac{9\pi}{4}}$$

$$f) \int_{-3}^1 \sqrt{1-x^2} dx = \frac{1}{2} \pi (1^2) = \boxed{\frac{\pi}{2}}$$

$$g) \int_{-2}^2 2 + \sqrt{4-x^2} dx \quad \begin{array}{c} \text{Diagram: A circle of radius 2 centered at the origin, with the upper half shaded. The area under the curve is the area of the rectangle plus the area of the semicircle.} \end{array} = \frac{1}{2} \pi (2^2) + 4(2) = \boxed{2\pi + 8}$$

$$h) \int_0^2 2 - \sqrt{4-x^2} dx \quad \begin{array}{c} \text{Diagram: A circle of radius 2 centered at the origin, with the lower half shaded. The area under the curve is the area of the rectangle minus the area of the semicircle.} \end{array} = (2)(2) - \frac{1}{4} \pi (2^2) = \boxed{4 - \pi}$$

$$i) \int_{-8}^0 \sqrt{16 - (x+4)^2} dx$$



$$\frac{1}{2} \pi (4^2) = \boxed{8\pi}$$

$$6. a) \int_0^2 x - 3 dx = \left[\frac{x^2}{2} - 3x \right]_0^2 = 2 - 6 - 0 = \boxed{-4}$$

$$b) \int_{-\pi}^{\pi} \sin x \cos x dx$$

$u = \sin x$
 $du = \cos x dx$
 $x = -\pi, u = 0$
 $x = \pi, u = 0$

$$\int_0^0 u du = \boxed{0}$$

$$c) \int_0^3 e^{2x} dx = \left[\frac{1}{2} e^{2x} \right]_0^3 = \boxed{\frac{1}{2} e^6 - \frac{1}{2}}$$

$$d) \int_{-1}^2 \sqrt{x+2} dx$$

$u = x+2$
 $du = dx$
 $x = -1, u = 1$
 $x = 2, u = 4$

$$\int_1^4 u^{\frac{1}{2}} du = \left[\frac{2}{3} u^{\frac{3}{2}} \right]_1^4 = \frac{2}{3} (8 - 1) = \boxed{\frac{14}{3}}$$

$$e) \int_1^6 \frac{1}{x+3} dx = \left[\ln|x+3| \right]_1^6 = \boxed{\ln 9 - \ln 4 \text{ or } \ln\left(\frac{9}{4}\right)}$$

$$f) \int_{-2}^3 e^{2x} - e^{3x} dx = \left[\frac{1}{2} e^{2x} - \frac{1}{3} e^{3x} \right]_{-2}^3 = \left(\frac{1}{2} e^6 - \frac{1}{3} e^9 - \left(\frac{1}{2} e^{-4} - \frac{1}{3} e^{-6} \right) \right)$$

$$g) \int_0^5 x \sqrt{x+4} dx$$

$u = x+4$
 $du = dx$
 $x = 0, u = 4$
 $x = 5, u = 9$
 $x = u - 4$

$$= \int_4^9 (u-4) u^{\frac{1}{2}} du = \int_4^9 u^{\frac{3}{2}} - 4u^{\frac{1}{2}} du$$

$$= \left[\frac{2}{5} u^{\frac{5}{2}} - \frac{8}{3} u^{\frac{3}{2}} \right]_4^9 = \frac{2}{5} (243) - \frac{8}{3} (27) - \left(\frac{2}{5} (32) - \frac{64}{3} \right)$$

$$= \boxed{\frac{506}{15}}$$

$$h) \int_1^3 \frac{2}{x+1} dx = \left[2 \ln|x+1| \right]_1^3 = 2 \ln 4 - 2 \ln 2 = \boxed{2 \ln 2}$$

$$i) \int_{-4}^{-1} \frac{2}{x-2} dx = \left[2 \ln|x-2| \right]_{-4}^{-1} = 2 \ln 3 - 2 \ln 6 = 2 \ln\left(\frac{1}{2}\right) = \boxed{-2 \ln 2}$$

$$j) \int_1^8 x^{\frac{2}{3}} dx = \left[\frac{3}{5} x^{\frac{5}{3}} \right]_1^8 = \frac{3}{5} [32 - 1] = \boxed{\frac{93}{5}}$$

$$k) \int_0^1 6x \sqrt{3x^2+1} dx \quad \begin{array}{l} u = 3x^2+1 \\ du = 6x dx \\ x=0, u=1 \\ x=1, u=4 \end{array} = \int_1^4 u^{\frac{1}{2}} du = \frac{1}{6} \left[\frac{2}{3} u^{\frac{3}{2}} \right]_1^4$$

$$= \frac{1}{9} [8 - 1] = \boxed{\frac{7}{9}}$$

$$l) \int_{-3}^0 \frac{1}{2} 2(2x-1)^2 dx \quad \begin{array}{l} u = 2x-1 \\ du = 2 dx \\ x=-3, u=-7 \\ x=0, u=-1 \end{array} = \frac{1}{2} \int_{-7}^{-1} u^2 du = \frac{1}{2} \left[\frac{u^3}{3} \right]_{-7}^{-1}$$

$$= \frac{1}{6} [-1 - (-343)] = \boxed{57}$$

$$m) \int_0^{\frac{\pi}{12}} \frac{1}{3} 3 \sec^2(3x) dx \quad \begin{array}{l} u = 3x \\ du = 3 dx \\ x=0, u=0 \\ x=\frac{\pi}{12}, u=\frac{\pi}{4} \end{array} = \frac{1}{3} \int_0^{\frac{\pi}{4}} \sec^2 u du = \frac{1}{3} \left[\tan u \right]_0^{\frac{\pi}{4}} = \frac{1}{3} (1 - 0) = \boxed{\frac{1}{3}}$$

$$n) \int_{-\frac{\pi}{2}}^{\frac{\pi}{3}} -\sin x \cos^3 x dx \quad \begin{array}{l} u = \cos x \\ du = -\sin x dx \\ x=-\frac{\pi}{2}, u=0 \\ x=\frac{\pi}{3}, u=\frac{1}{2} \end{array} = - \int_0^{\frac{1}{2}} u^3 du = - \left[\frac{u^4}{4} \right]_0^{\frac{1}{2}} = - \frac{1}{4} \left[\frac{1}{16} - 0 \right] = \boxed{-\frac{1}{64}}$$

$$o) \int_0^4 f(x) dx \text{ for } f(x) = \begin{cases} 3x^2 - 3 & \text{if } x \leq 3 \\ \frac{1}{x-2} & \text{if } x > 3 \end{cases}$$

$$= \int_0^3 (3x^2 - 3) dx + \int_3^4 \frac{1}{x-2} dx$$

$$= \left[x^3 - 3x \right]_0^3 + \left[\ln|x-2| \right]_3^4 = 18 - 0 + \ln 2 - \ln 1 = \boxed{18 + \ln 2}$$

$$7. a) \int x^3 - x^2 dx = \boxed{\frac{x^4}{4} - \frac{x^3}{3} + C}$$

$$b) \int \frac{1}{x} - \frac{1}{x^2} dx = \boxed{\ln|x| + \frac{1}{x} + C}$$

$$c) \int e^{3x} - \cos 2x dx = \boxed{\frac{1}{3} e^{3x} - \frac{1}{2} \sin 2x + C}$$

$$d) \int x \sin\left(\frac{1}{2}x^2\right) dx \quad \begin{matrix} u = \frac{1}{2}x^2 \\ du = x dx \end{matrix} = \int \sin u du = -\cos u + C = \boxed{-\cos\left(\frac{1}{2}x^2\right) + C}$$

$$e) \int 2x^3 (\sqrt[3]{x^2 - 1}) dx \quad \begin{matrix} u = x^2 - 1 \\ du = 2x dx \\ x^2 = u + 1 \end{matrix} = \int (u+1) u^{\frac{1}{3}} du$$

$$= \int u^{\frac{4}{3}} + u^{\frac{1}{3}} du = \frac{3}{7} u^{\frac{7}{3}} + \frac{3}{4} u^{\frac{4}{3}} + C$$

$$= \boxed{\frac{3}{7} (x^2 - 1)^{\frac{7}{3}} + \frac{3}{4} (x^2 - 1)^{\frac{4}{3}} + C}$$

$$f) 3 \int \frac{1}{3} \sec\left(\frac{1}{3}x\right) \tan\left(\frac{1}{3}x\right) dx \quad u = \frac{1}{3}x \quad = 3 \int \sec u \tan u du$$

$$du = \frac{1}{3} dx$$

$$= 3 \sec u + C = \boxed{3 \sec\left(\frac{1}{3}x\right) + C}$$

$$g) \int \frac{2}{3x-1} + \frac{4}{5x+1} dx = \frac{1}{3} \int \frac{2(3)}{3x-1} dx + \frac{1}{5} \int \frac{4(5)}{5x+1} dx$$

$$u = 3x-1$$

$$du = 3 dx$$

$$v = 5x+1$$

$$dv = 5 dx$$

$$= \frac{1}{3} \int \frac{2}{u} du + \frac{1}{5} \int \frac{4}{v} dv = \frac{2}{3} \ln|u| + \frac{4}{5} \ln|v| + C$$

$$= \boxed{\frac{2}{3} \ln|3x-1| + \frac{4}{5} \ln|5x+1| + C}$$

$$h) \int \frac{x^2 - 8x - 1}{x+1} dx \quad \begin{array}{r} -1 \quad 1 \quad -8 \quad -1 \\ \hline 1 \quad -9 \quad 8 \end{array} = \int x - 9 + \frac{8}{x+1} dx$$

$$= \boxed{\frac{x^2}{2} - 9x + 8 \ln|x+1| + C}$$

$$i) \int \frac{x^3 + x^2 + x + 1}{x-2} dx \quad \begin{array}{r} 2 \quad 1 \quad 1 \quad 1 \quad 1 \\ \hline 1 \quad 3 \quad 7 \quad 15 \end{array} = \int x^2 + 3x + 7 + \frac{15}{x-2} dx$$

$$= \boxed{\frac{x^3}{3} + \frac{3}{2}x^2 + 7x + 15 \ln|x-2| + C}$$

$$j) \int \frac{x^4 - x - 6}{x+5} dx \quad \begin{array}{r|rrrrrr} -5 & 1 & 0 & 0 & -1 & -6 \\ & & -5 & 25 & -125 & 630 \\ \hline & 1 & -5 & 25 & -126 & 624 \end{array}$$

$$= \int x^3 - 5x^2 + 25x - 126 + \frac{624}{x+5} dx$$

$$= \boxed{\frac{x^4}{4} - \frac{5}{3}x^3 + \frac{25}{2}x^2 - 126x + 624 \ln|x+5| + C}$$

$$k) \int \frac{2}{9+x^2} dx = \boxed{\frac{2}{3} \tan^{-1}\left(\frac{x}{3}\right) + C}$$

$$l) \int \frac{2x}{9+x^2} dx \quad \begin{array}{l} u=9+x^2 \\ du=2x dx \end{array} = \int \frac{du}{u} = \ln|u| + C = \boxed{\ln(9+x^2) + C}$$

$$m) \int \frac{3}{\sqrt{25-x^2}} dx = \boxed{3 \sin^{-1}\left(\frac{x}{5}\right) + C}$$

$$n) -\frac{1}{2} \int \frac{3x(-2)}{\sqrt{25-x^2}} dx \quad \begin{array}{l} u=25-x^2 \\ du=-2x dx \end{array} = -\frac{1}{2} \int 3u^{-\frac{1}{2}} du = -\frac{3(2)}{2} u^{\frac{1}{2}} + C$$

$$= \boxed{-3\sqrt{25-x^2} + C}$$

$$o) \frac{1}{2} \int \frac{2x^3}{1+x^2} dx \quad \begin{array}{l} u=1+x^2 \\ du=2x dx \\ x^2=u-1 \end{array} = \frac{1}{2} \int \frac{u-1}{u} du$$

$$= \frac{1}{2} \int 1 - \frac{1}{u} du = \frac{1}{2} [u - \ln|u| + C] = \boxed{\frac{1}{2}(1+x^2) - \frac{1}{2} \ln(1+x^2) + C}$$

$$p) \int \frac{5}{4+x^2} dx = \boxed{\frac{5}{2} \tan^{-1}\left(\frac{x}{2}\right) + C}$$

$$q) \int \frac{6}{\sqrt{100-x^2}} dx = \boxed{6 \sin^{-1}\left(\frac{x}{10}\right) + C}$$

$$r) \frac{1}{2} \int \frac{2(x-2)}{(x^2-4x-1)^3} dx \quad \begin{array}{l} u = x^2-4x-1 \\ du = 2x-4 \end{array} = \frac{1}{2} \int u^{-3} du$$

$$= \frac{1}{2} \left[\frac{u^{-2}}{-2} + C \right] = \boxed{-\frac{1}{4(x^2-4x-1)^2} + C}$$

$$s) \int \frac{x-x^3+1}{\sqrt{x}} dx = \int x^{\frac{1}{2}} - x^{\frac{5}{2}} + x^{-\frac{1}{2}} dx$$

$$= \boxed{\frac{2}{3} x^{\frac{3}{2}} - \frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{1}{2}} + C}$$

$$t) \int \frac{\cos x}{\sin^4 x} dx \quad \begin{array}{l} u = \sin x \\ du = \cos x dx \end{array} = \int u^{-4} du = \frac{u^{-3}}{-3} + C$$

$$= \boxed{-\frac{1}{3 \sin^3 x} + C}$$

$$u) \int \frac{\sin 2x}{\cos^3 x} dx = \int \frac{2 \sin x \cos x}{\cos^3 x} dx = 2 \int \frac{\sin x}{\cos^2 x} dx$$

$$= 2 \int \sec x \tan x dx = \boxed{2 \sec x + C}$$

$$v) \int \frac{\sin 2x}{\cos^4 x} dx = \int \frac{2 \sin x \cos x}{\cos^4 x} dx = -2 \int \frac{-\sin x}{\cos^3 x} dx$$

$$\begin{aligned} u &= \cos x \\ du &= -\sin x dx \end{aligned} = -2 \int u^{-3} du$$

$$= -2 \left[\frac{u^{-2}}{-2} + C_1 \right]$$

$$= \boxed{\frac{1}{\cos^2 x} + C}$$