

$$1. \int \sqrt{7x} \, dx \quad \begin{array}{l} u = 7x \\ du = 7 \, dx \end{array} = \frac{1}{7} \int 7\sqrt{7x} \, dx$$

$$= \frac{1}{7} \int u^{\frac{1}{2}} \, du = \frac{1}{7} \left[\frac{2}{3} u^{\frac{3}{2}} + C \right]$$

$$= \left(\frac{2}{21} (7x)^{\frac{3}{2}} + C \right) = \left(\frac{2\sqrt{7}}{3} x^{\frac{3}{2}} + C \right)$$

$$2. \int 5^x \, dx = \left(\frac{5^x}{\ln 5} + C \right)$$

$$3. \int 2 + e^{5x} \, dx = \left(2x + \frac{1}{5} e^{5x} + C \right)$$

$$4. \int \frac{3e^{3x}}{e^{3x} + 1} \, dx \quad \begin{array}{l} u = e^{3x} + 1 \\ du = 3e^{3x} \, dx \end{array} = \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|e^{3x} + 1| + C \quad \text{or } \ln(e^{3x} + 1) + C$$

$$5. \int \frac{1}{x \ln x} \, dx \quad \begin{array}{l} u = \ln x \\ du = \frac{1}{x} \, dx \end{array} = \int \frac{du}{u} = \ln|u| + C$$

$$= \left(\ln|\ln x| + C \right)$$

$$6. \int 4\cos^2 x - 2 \, dx = 2 \int 2\cos^2 x - 1 \, dx = 2 \int \cos 2x \, dx$$

$$= 2 \left[\frac{1}{2} \sin 2x + C \right] = \left(\sin 2x + C \right)$$

$$7. \int \frac{e^x - 1}{e^x - x} dx \quad \begin{array}{l} u = e^x - x \\ du = e^x - 1 \, dx \end{array} = \int \frac{du}{u} = \ln|u| + C$$

$$= \boxed{\ln|e^x - x| + C}$$

$$8. \frac{1}{2} \int 2x \tan(x^2) dx \quad \begin{array}{l} u = x^2 \\ du = 2x \, dx \end{array} = \frac{1}{2} \int \tan u \, du$$

$$= -\frac{1}{2} \ln|\cos u| + C = \boxed{-\frac{1}{2} \ln|\cos(x^2)| + C}$$

$$9. \int x + e^{\frac{x}{3}} - \cot x \, dx = \boxed{\frac{x^2}{2} + 3e^{\frac{x}{3}} - \ln|\sin x| + C}$$

$$10. \int \frac{e^x}{e^x + 7} dx \quad \begin{array}{l} u = e^x + 7 \\ du = e^x \, dx \end{array} = \int \frac{du}{u} = \ln|u| + C = \boxed{\ln|e^x + 7| + C}$$

$$11. \int 2 - 4\sin^2 x \, dx = 2 \int 1 - 2\sin^2 x \, dx = 2 \int \cos 2x \, dx = \boxed{\sin 2x + C}$$

$$12. \int \frac{e^x}{\sqrt{e^x + 1}} dx \quad \begin{array}{l} u = e^x + 1 \\ du = e^x \, dx \end{array} = \int u^{-\frac{1}{2}} du = 2\sqrt{u} + C$$

$$= \boxed{2\sqrt{e^x + 1} + C}$$