$f(x) = 2x^2 - 1 \quad f'(x) = 4x = 0$ at $x = 0$

$- \mid +$ at $0$

1\. a) Left: $\left[1(2(1)^2 - 1) + 1(2(2^2) - 1) + 1(2(3^2) - 1)\right] = 1 + 7 + 17 = \boxed{25}$

underestimate because $f(x)$ is increasing on $[1,4]$

Right: $1(2(2^2) - 1) + 1(2(3^2) - 1) + 1(2(4^2) - 1) = 7 + 17 + 31 = \boxed{55}$

overestimate because $f(x)$ is increasing on $[1,4]$

Midpoint: $1\left(2\left(\frac{3}{2}\right)^2 - 1\right) + 1\left(2\left(\frac{5}{2}\right)^2 - 1\right) + 1\left(2\left(\frac{7}{2}\right)^2 - 1\right)$

$= \frac{7}{2} + \frac{23}{2} + \frac{47}{2} = \boxed{\frac{77}{2}}$

Trapezoidal: $\dfrac{\left(\frac{4-1}{3}\right)}{2}\left[[2(1^2) - 1] + 2[2(2^2) - 1] + 2[2(3^2) - 1] + [2(4^2) - 1]\right]$

$= \frac{1}{2}\left[1 + 14 + 34 + 31\right] = \boxed{40}$

$f'' = 4 > 0$

concave up

overestimate

b) Left: $1(0 + 0 + 4) + 1\left(-\frac{1}{2} + 1 + 4\right) + 1\left(-\frac{1}{2}(4) + 2 + 4\right) + 1\left(-\frac{1}{2}(9) + 3 + 4\right)$

$= 4 + \frac{9}{2} + 4 + \frac{5}{2} = \boxed{15}$

$f'(x) = -x + 1 = 0$

$x = 1$

$+ \mid -$ at $1$

cannot determine because $f(x)$ is decreasing and increasing on interval

Right: $1\left(-\frac{1}{2} + 1 + 4\right) + 1\left(-\frac{1}{2}(4) + 2 + 4\right) + 1\left(-\frac{1}{2}(9) + 3 + 4\right) + 1\left(-\frac{1}{2}(16) + 4 + 4\right)$

$= \frac{9}{2} + 4 + \frac{5}{2} + 0 = \boxed{11}$

cannot determine because $f(x)$ is decreasing and increasing on interval

Midpoint: $\left(-\frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 4\right) + \left(-\frac{1}{2}\left(\frac{3}{2}\right)^2 + \frac{3}{2} + 4\right) + \left(-\frac{1}{2}\left(\frac{5}{2}\right)^2 + \frac{5}{2} + 4\right) + \left(-\frac{1}{2}\left(\frac{7}{2}\right)^2 + \frac{7}{2} + 4\right)$

$= -\frac{1}{8} + \frac{1}{2} + 4 - \frac{9}{8} + \frac{3}{2} + 4 - \frac{25}{8} + \frac{5}{2} + 4 - \frac{49}{8} + \frac{7}{2} + 4 = \boxed{\frac{27}{2}}$

Trap: $\dfrac{\left(\frac{4-0}{4}\right)}{2}\left[(0+0+4)+2\left(-\frac{1}{2}+1+4\right)+2(-2+2+4)+2\left(-\frac{9}{2}+3+4\right)+(-8+4+4)\right]$

$f''(x) = -1 < 0$

concave down

$= \frac{1}{2}(4+9+8+5+0) = \boxed{13}$

$\boxed{\text{underestimate}}$

C. Left: $1(-64-4+8)+1(-27-3+8)+1(-8-2+8)+1(-1-1+8)$

$= -60-22-2+6 = \boxed{-78}$

$f'(x) = 3x^2+1 = 0$

$x^2 = -\frac{1}{3}$ never

$\boxed{\text{underestimate because } f(x) \text{ is increasing}}$

$f' > 0$ for $(-\infty, \infty)$

Right: $1(-27-3+8)+1(-8-2+8)+1((-1)-1+8)+1(0+0+8)$

$= -22-2+6+8 = \boxed{-10}$

$\boxed{\text{overestimate because } f(x) \text{ is increasing}}$

Midpoint: $1\left(\left(-\frac{7}{2}\right)^3-\frac{7}{2}+8\right)+1\left(\left(-\frac{5}{2}\right)^3-\frac{5}{2}+8\right)+1\left(\left(-\frac{3}{2}\right)^3-\frac{3}{2}+8\right)+1\left(\left(-\frac{1}{2}\right)^3-\frac{1}{2}+8\right)$

$= -\frac{343}{8}-\frac{7}{2}+8-\frac{125}{8}-\frac{5}{2}+8-\frac{27}{8}-\frac{3}{2}+8-\frac{1}{8}-\frac{1}{2}+8 = -38$

Trap: $\dfrac{\left(\frac{0-(-4)}{4}\right)}{2}\left[(-64-4+8)+2(-27-3+8)+2(-8-2+8)+2(-1-1+8)+(0+0+8)\right]$

$= \frac{1}{2}(-60-44-4+12+8) = \boxed{-44}$

$\boxed{\text{underestimate}}$

$f''(x) = 6x = 0$

$x = 0$

− | +

0

concave down

$$\frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$$

d) Left: $\frac{1}{2}\left(\frac{1}{1^2}\right) + \frac{1}{2}\left(\frac{1}{(\frac{3}{2})^2}\right) + \frac{1}{2}\left(\frac{1}{2^2}\right) + \frac{1}{2}\left(\frac{1}{(\frac{5}{2})^2}\right)$

$$= \frac{1}{2} + \frac{1}{2}\left(\frac{4}{9}\right) + \frac{1}{8} + \frac{1}{2}\left(\frac{4}{25}\right) = \frac{1}{2} + \frac{2}{9} + \frac{1}{8} + \frac{2}{25} = \boxed{\frac{1669}{1800}}$$

$f'(x) = \frac{d}{dx} x^{-2} = -\frac{2}{x^3}$ und at $x=0$ $\quad + \nearrow \underset{0}{|} \searrow -$

overestimate because $f(x)$ is decreasing

Right: $\frac{1}{2}\left(\frac{1}{(\frac{3}{2})^2}\right) + \frac{1}{2}\left(\frac{1}{2^2}\right) + \frac{1}{2}\left(\frac{1}{(\frac{5}{2})^2}\right) + \frac{1}{2}\left(\frac{1}{3^2}\right)$

$$= \frac{2}{9} + \frac{1}{8} + \frac{2}{25} + \frac{1}{18} = \boxed{\frac{869}{1800}}$$

underestimate because $f(x)$ is decreasing

Midpoint: $\frac{1}{2}\left(\frac{1}{(\frac{5}{4})^2}\right) + \frac{1}{2}\left(\frac{1}{(\frac{7}{4})^2}\right) + \frac{1}{2}\left(\frac{1}{(\frac{9}{4})^2}\right) + \frac{1}{2}\left(\frac{1}{(\frac{11}{4})^2}\right)$

$$= \frac{8}{25} + \frac{8}{49} + \frac{8}{81} + \frac{8}{121} = \boxed{\frac{7781792}{12006225}}$$

Trapezoid: $\frac{\left(\frac{3-1}{4}\right)}{2}\left(\frac{1}{1^2} + 2\left(\frac{1}{(\frac{3}{2})^2}\right) + 2\left(\frac{1}{2^2}\right) + 2\left(\frac{1}{(\frac{5}{2})^2}\right) + \frac{1}{3^2}\right)$

$$= \frac{1}{4}\left(1 + \frac{8}{9} + \frac{1}{2} + \frac{8}{25} + \frac{1}{9}\right) = \boxed{\frac{141}{200}}$$

$f''(x) = \frac{6}{x^4}$ und $x=0$ $\quad + \underset{0}{|} +$

concave up, overestimate

2\. a) $\frac{d}{dx}\int_{-3}^{x} t^2(1+e^t)\,dt = \boxed{x^2(1+e^x)}$

b) $\frac{d}{dx}\int_{5}^{x} \cos(t^3)\,dt = \boxed{\cos(x^3)}$

c) $\frac{d}{dx}\int_x^7 e^{-t^2}\,dt = \frac{d}{dx}\left[-\int_7^x e^{-t^2}\,dt\right] = \boxed{-e^{-x^2}}$

d) $\frac{d}{dx}\int_5^{3x^2} \frac{t-3}{t-6}\,dt$ let F be an antiderivative of $\frac{t-3}{t-6}$

$\hookrightarrow \frac{d}{dx}\left[F(3x^2)-F(5)\right] = \boxed{\frac{3x^2-3}{3x^2-6}(6x)}$

e) $\frac{d}{dx}\int_4^{\sin x} \frac{t-4}{\sin t}\,dt$ let F be an antiderivative of $\frac{t-4}{\sin t}$

$\hookrightarrow \frac{d}{dx}\left[F(\sin x)-F(4)\right] = \boxed{\frac{\sin x-4}{\sin(\sin x)}(\cos x)}$

f) $\frac{d}{dx}\int_{3x}^{x^2-1} t+\tan t\,dt$ let F be an antiderivative of $t+\tan t$

$\hookrightarrow \frac{d}{dx}\left[F(x^2-1)-F(3x)\right]$

$\boxed{= \left[x^2-1+\tan(x^2-1)\right](2x) - \left[3x+\tan(3x)\right](3)}$

g) $\frac{d}{dx}\int_{\sin x}^{\tan x} \frac{t-3}{t+e^t}\,dt$ let F be an antiderivative of $\frac{t-3}{t+e^t}$

$\hookrightarrow \frac{d}{dx}\left[F(\tan x)-F(\sin x)\right]$

$= \boxed{\frac{\tan x-3}{\tan x+e^{\tan x}}(\sec^2 x) - \frac{\sin x-3}{\sin x+e^{\sin x}}(\cos x)}$

3.a) $\int_0^3 x^2+5x-1\,dx$ $\Delta x = \frac{3-0}{n} = \frac{3}{n}$ $x_k = 0+\frac{3k}{n} = \frac{3k}{n}$

$= \lim_{n\to\infty}\sum_{k=1}^{n}\left[(x_k)^2+5x_k-1\right]\Delta x = \boxed{\lim_{n\to\infty}\sum_{k=1}^{n}\left[\left(\frac{3k}{n}\right)^2+5\left(\frac{3k}{n}\right)-1\right]\left(\frac{3}{n}\right)}$

b) $\int_0^1 x^2 + \sin x \, dx = \lim_{n \to \infty} \sum_{k=1}^{n} \left[(x_k)^2 + \sin(x_k)\right] \Delta x$

$\Delta x = \frac{1-0}{n} = \frac{1}{n}$ $\quad x_k = 0 + \frac{k}{n} = \frac{k}{n}$

$$\boxed{\lim_{n \to \infty} \sum_{k=1}^{n} \left[\left(\frac{k}{n}\right)^2 + \sin\left(\frac{k}{n}\right)\right]\left(\frac{1}{n}\right)}$$

c) $\int_1^2 x + 8 \, dx = \lim_{n \to \infty} \sum_{k=1}^{n} (x_k + 8) \Delta x$

$\Delta x = \frac{2-1}{n} = \frac{1}{n}$ $\quad x_k = 1 + \frac{k}{n}$

$$\lim_{n \to \infty} \sum_{k=1}^{n} \left(1 + \frac{k}{n} + 8\right)\left(\frac{1}{n}\right) = \boxed{\lim_{n \to \infty} \sum_{k=1}^{n} \left(9 + \frac{k}{n}\right)\left(\frac{1}{n}\right)}$$

d) $\int_2^5 (x^2 - 3x) dx = \lim_{n \to \infty} \sum_{k=1}^{n} \left[(x_k)^2 - 3x_k\right] \Delta x$

$\Delta x = \frac{5-2}{n} = \frac{3}{n}$ $\quad x_k = 2 + \frac{3k}{n}$

$$\boxed{\lim_{n \to \infty} \sum_{k=1}^{n} \left[\left(2 + \frac{3k}{n}\right)^2 - 3\left(2 + \frac{3k}{n}\right)\right]\left(\frac{3}{n}\right)}$$

e) $\int_1^4 e^x + \tan x \, dx = \lim_{n \to \infty} \sum_{k=1}^{n} \left(e^{x_k} + \tan x_k\right) \Delta x$

$\Delta x = \frac{4-1}{n} = \frac{3}{n}$ $\quad x_k = 1 + \frac{3k}{n}$

$$= \boxed{\lim_{n \to \infty} \sum_{k=1}^{n} \left[e^{\left(1 + \frac{3k}{n}\right)} + \tan\left(1 + \frac{3k}{n}\right)\right]\left(\frac{3}{n}\right)}$$

4. a) $\int_1^3 3f(x) - 4h(x)\,dx = 3\int_1^3 f(x)\,dx - 4\int_1^3 h(x)\,dx = 3(4) - 4(-6) = \boxed{36}$

b) $\int_2^2 f(x)\,dx = \boxed{0}$

c) $\int_1^7 h(x)\,dx = \int_1^3 h(x)\,dx + \int_3^7 h(x)\,dx = -6 + 10 = \boxed{4}$

d) $\int_1^5 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^5 f(x)\,dx = 4 - \int_5^3 f(x)\,dx = 4 - (-2) = \boxed{6}$

e) $\int_3^6 5h(x)\,dx = 5\left[\int_3^7 h(x)\,dx - \int_6^7 h(x)\,dx\right] = 5\left[10 + \int_7^6 h(x)\,dx\right]$

$= 5\left[10 + 5\right] = \boxed{75}$

f) $\int_3^1 h(x)\,dx = -\int_1^3 h(x)\,dx = -(-6) = \boxed{6}$

5. a) $\int_1^1 f(x)\,dx = \boxed{0}$

b) $\int_1^7 3f(x)\,dx = 3\left[\int_1^4 f(x)\,dx + \int_4^7 f(x)\,dx\right] = 3\left[1 - \int_7^4 f(x)\,dx\right]$

$= 3\left[1 - (-2)\right] = \boxed{9}$

c) $\int_1^5 4f(x)\,dx + \int_1^5 3h(x)\,dx = 4(3) + 3(10) = \boxed{42}$

d) $\int_1^7 h(x)\,dx = \int_1^5 h(x)dx + \int_5^7 h(x)dx = 10 - \int_7^5 h(x)\,dx = 10-(-5) = \boxed{15}$

e) $\int_4^5 f(x)\,dx = \int_1^5 f(x)dx - \int_1^4 f(x)\,dx = 3-1 = \boxed{2}$

f) $\int_4^1 f(x)\,dx = -\int_1^4 f(x)\,dx = \boxed{-1}$

6. a) $\frac{1}{2-(-1)}\int_{-1}^{2} x^2+3\,dx = \frac{1}{3}\left[\frac{x^3}{3}+3x\right]_{-1}^{2} = \frac{1}{3}\left[\frac{8}{3}+6-\left(-\frac{1}{3}-3\right)\right] = \boxed{4}$

b) $\frac{1}{4-0}\int_0^4 3x^2-2x-1\,dx = \frac{1}{4}\left[x^3-x^2-x\right]_0^4 = \frac{1}{4}\left[64-16-4-0\right] = \boxed{11}$

c) $\frac{1}{\frac{2\pi}{3}-0}\int_0^{\frac{2\pi}{3}} \sin x\,dx = \frac{3}{2\pi}\left[-\cos x\right]_0^{\frac{2\pi}{3}} = \frac{3}{2\pi}\left[-\cos\frac{2\pi}{3}-(-\cos 0)\right]$

$= \frac{3}{2\pi}\left[\frac{1}{2}+1\right] = \boxed{\frac{9}{4\pi}}$

d) $\frac{1}{\frac{\pi}{6}-0}\int_0^{\frac{\pi}{6}} \tan 2x\,dx$

$u = 2x$
$du = 2\,dx$
$x=0,\ u=0$
$x=\frac{\pi}{6},\ u=\frac{\pi}{3}$

$= \frac{6}{\pi}\left(\frac{1}{2}\right)\int_0^{\frac{\pi}{6}} 2\tan 2x\,dx$

$= \frac{3}{\pi}\int_0^{\frac{\pi}{3}} \tan u\,du$

$= \frac{3}{\pi}\left[-\ln|\cos x|\right]_0^{\frac{\pi}{3}} = \frac{3}{\pi}\left(-\ln\frac{1}{2}-(-\ln 1)\right]$

$= \frac{3}{\pi}(\ln 2)+0 = \frac{3\ln 2}{\pi}$

e) $\frac{1}{2-\sqrt{3}} \int_{\sqrt{3}}^{2} x\sqrt{x^2-3}\, dx$

$u = x^2-3$
$du = 2x\,dx$
$x=\sqrt{3},\ u=0$
$x=2,\ u=1$

$= \left(\frac{1}{2-\sqrt{3}}\right)\left(\frac{1}{2}\right)\int_{\sqrt{3}}^{2} 2x\sqrt{x^2-3}\, dx$

$= \frac{1}{2}\left(\frac{1}{2-\sqrt{3}}\right)\int_0^1 u^{\frac{1}{2}}\, du = \frac{1}{2}\left(\frac{1}{2-\sqrt{3}}\right)\left[\left(\frac{2}{3}u^{\frac{3}{2}}\right)\right]_0^1$

$= \frac{1}{2}\left(\frac{1}{2-\sqrt{3}}\right)\left(\frac{2}{3}-0\right) = \boxed{\frac{1}{3}\left(\frac{1}{2-\sqrt{3}}\right)}$

7. a) $\int x^2-3x+8\, dx = \boxed{\frac{x^3}{3} - \frac{3}{2}x^2 + 8x + C}$

b) $\int 3x^4-5x^2+x-1\, dx = \boxed{\frac{3}{5}x^5 - \frac{5}{3}x^3 + \frac{x^2}{2} - x + C}$

c) $\int \frac{5x^2-2x+1}{\sqrt[3]{x}}\, dx = \int 5x^{\frac{5}{3}} - 2x^{\frac{2}{3}} + x^{-\frac{1}{3}}\, dx$

$= \boxed{\frac{15}{8}x^{\frac{8}{3}} - \frac{6}{5}x^{\frac{5}{3}} + \frac{3}{2}x^{\frac{2}{3}} + C}$

d) $\int \frac{7x^3+8\sqrt{x}-x^2}{\sqrt{x}}\, dx = \int 7x^{\frac{5}{2}} + 8 - x^{\frac{3}{2}}\, dx$

$= \boxed{2x^{\frac{7}{2}} + 8x - \frac{2}{5}x^{\frac{5}{2}} + C}$

e) $\int \sqrt{6x}\, dx = \sqrt{6}\int x^{\frac{1}{2}}\, dx = \sqrt{6}\left(\frac{2}{3}x^{\frac{3}{2}} + C_1\right) = \boxed{\frac{2\sqrt{6}}{3}x^{\frac{3}{2}} + C}$

f) $\int x^3 \sqrt{x^2-8}\, dx$ $\quad u = x^2-8$, $du = 2x\,dx$, $x^2 = u+8$ $\quad = \frac{1}{2}\int 2x(x^2)\sqrt{x^2-8}\, dx$

$= \frac{1}{2}\int (u+8)u^{\frac{1}{2}}\, du = \frac{1}{2}\int u^{\frac{3}{2}} + 8u^{\frac{1}{2}}\, du$

$= \frac{1}{2}\left[\frac{2}{5}u^{\frac{5}{2}} + \frac{16}{3}u^{\frac{3}{2}} + C_1\right] = \boxed{\frac{1}{5}(x^2-8)^{\frac{5}{2}} + \frac{8}{3}(x^2-8)^{\frac{3}{2}} + C}$

g) $\int 10x\sqrt{x^2+1}\, dx$ $\quad u = x^2+1$, $du = 2x\,dx$ $\quad = 5\int 2x\sqrt{x^2+1}\, dx$

$= 5\int u^{\frac{1}{2}}\, du = 5\left[\frac{2}{3}u^{\frac{3}{2}} + C_1\right] = \boxed{\frac{10}{3}(x^2+1)^{\frac{3}{2}} + C}$

h) $\int \sqrt{4x+7}\, dx$ $\quad u = 4x+7$, $du = 4\,dx$ $\quad = \frac{1}{4}\int 4\sqrt{4x+7}\, dx = \frac{1}{4}\int u^{\frac{1}{2}}\, du$

$= \frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}} + C_1\right) = \boxed{\frac{1}{6}(4x+7)^{\frac{3}{2}} + C}$

i) $\int \tan 3x\, dx$ $\quad u = 3x$, $du = 3\,dx$ $\quad = \frac{1}{3}\int 3\tan 3x\, dx = \frac{1}{3}\int \tan u\, du$

$= \frac{1}{3}(-\ln|\cos u| + C_1) = \boxed{-\frac{1}{3}\ln|\cos 3x| + C}$

j) $\int \cot 8x\, dx$ $\quad u = 8x$, $du = 8\,dx$ $= \frac{1}{8}\int 8\cot 8x\, dx = \frac{1}{8}\int \cot u\, du$

$= \frac{1}{8}\left[\ln|\sin u| + C_1\right] = \boxed{\frac{1}{8}\ln|\sin 8x| + C}$

k) $\int \frac{\cos x}{\sin^4 x}\, dx$ $\quad u = \sin x$, $du = \cos x\, dx$ $\quad = \int u^{-4}\, du = \frac{u^{-3}}{-3} + C$

$= \boxed{-\frac{1}{3\sin^3 x} + C}$

l) $\int \cos^5\theta \, d\theta = \int \cos^4\theta \cos\theta \, d\theta = \int (\cos^2\theta)^2 \cos\theta \, d\theta$

$= \int (1-\sin^2\theta)^2 \cos\theta \, d\theta$ $\quad$ $u = \sin\theta$, $du = \cos\theta \, d\theta$ $\quad$ $= \int (1-u^2)^2 \, du$

$= \int 1 - 2u^2 + u^4 \, du = u - \frac{2}{3}u^3 + \frac{u^5}{5} + C$

$= \boxed{\sin\theta - \frac{2}{3}\sin^3\theta + \frac{1}{5}\sin^5\theta + C}$

m) $\int \sin^7 x \, dx = \int (\sin^6 x)(\sin x)\, dx = \int (\sin^2 x)^3 \sin x \, dx = \int (1-\cos^2 x)^3 \sin x \, dx$

$u = \cos x$
* $du = -\sin x \, dx$ $\quad$ $= -\int (1-\cos^2 x)^3 (-\sin x) dx = -\int (1-u^2)^3 \, du$

$= -\int 1 - 3u^2 + 3u^4 - u^6 \, du = -\left[u - u^3 + \frac{3}{5}u^5 - \frac{u^7}{7} + C_1\right]$

$= \boxed{-\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{\cos^7 x}{7} + C}$

n) $\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x (\cos^2 x) \cos x \, dx = \int \sin^2 x (1-\sin^2 x)\cos x \, dx$

$u = \sin x$, $du = \cos x \, dx$ $\quad$ $= \int u^2(1-u^2)\, du = \int u^2 - u^4 \, du = \frac{u^3}{3} - \frac{u^5}{5} + C$

$= \boxed{\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C}$

o) $\int \frac{\sin x}{\cos^6 x} dx$ $\quad$ $u = \cos x$, $du = -\sin x \, dx$ $\quad$ $= -\int \frac{-\sin x \, dx}{\cos^6 x} = -\int u^{-6} du$

$-\left[\frac{u^{-5}}{-5} + C_1\right] = \boxed{\frac{1}{5\cos^5 x} + C}$

p) $\int \frac{x-3}{x^2-6x+1}\,dx$ $\quad u = x^2-6x+1$, $du = 2x-6\,dx$ $\quad \frac{1}{2}\int \frac{2(x-3)\,dx}{x^2-6x+1} = \frac{1}{2}\int \frac{du}{u}$

$$= \frac{1}{2}\left[\ln|u| + c_1\right] = \boxed{\frac{1}{2}\ln|x^2-6x+1| + C}$$

q) $\int \frac{4x-8}{x^2-4x-1}\,dx$ $\quad u = x^2-4x-1$, $du = 2x-4\,dx$ $\quad = \int \frac{2(2x-4)\,dx}{x^2-4x-1} = \int \frac{2\,du}{u}$

$$= 2\ln|u| + c = \boxed{2\ln|x^2-4x-1| + C}$$

r) $\int \frac{x+1}{(x^2+2x+5)^3}\,dx$ $\quad u = x^2+2x+5$, $du = 2x+2\,dx$ $\quad = \frac{1}{2}\int \frac{2(x+1)\,dx}{(x^2+2x+5)^3} = \frac{1}{2}\int u^{-3}\,du$

$$= \frac{1}{2}\left[\frac{u^{-2}}{-2} + c_1\right] = \boxed{-\frac{1}{4(x^2+2x+5)^2} + C}$$

s) $\int \frac{x+4}{(x^2+8x+1)^5}\,dx$ $\quad u = x^2+8x+1$, $du = 2x+8\,dx$ $\quad = \frac{1}{2}\int \frac{2(x+4)\,dx}{(x^2+8x+1)^5}$

$$= \frac{1}{2}\int u^{-5}\,du = \frac{1}{2}\left[\frac{u^{-4}}{-4} + c_1\right] = \boxed{-\frac{1}{8(x^2+8x+1)^4} + C}$$

t) $\int \frac{x^2+5x+1}{x-2}\,dx$

$$\begin{array}{r|rrr} 2 & 1 & 5 & 1 \\ & & 2 & 14 \\ \hline & 1 & 7 & 15 \end{array}$$

$= \int x + 7 + \frac{15}{x-2}\,dx$ $\quad u = x-2$, $du = dx$

$$= \frac{x^2}{2} + 7x + \int \frac{15\,du}{u} = \boxed{\frac{x^2}{2} + 7x + 15\ln|x-2| + C}$$

u) $\int \frac{x^3-x-7}{x+2}\,dx$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -1 & -7 \\ & & -2 & 4 & -6 \\ \hline & 1 & -2 & 3 & -13 \end{array}$$

$= \int x^2 - 2x + 3 - \frac{13}{x+2}\,dx$ $\quad u = x+2$, $du = dx$

$$= \frac{x^3}{3} - x^2 + 3x - \int \frac{13\,du}{u} = \boxed{\frac{x^3}{3} - x^2 + 3x - 13\ln|x+2| + C}$$

v) $\int \frac{x^4+x^3+5x^2-x+1}{x-3}\,dx$

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & 5 & -1 & 1 \\ & & 3 & 12 & 51 & 150 \\ \hline & 1 & 4 & 17 & 50 & 151 \end{array}$$

$u = x-3$
$du = dx$

$= \int x^3 + 4x^2 + 17x + 50 + \frac{151}{x-3}\,dx$

$= \frac{x^4}{4} + \frac{4}{3}x^3 + \frac{17}{2}x^2 + 50x + \int \frac{151}{u}\,du$

$= \boxed{\frac{x^4}{4} + \frac{4}{3}x^3 + \frac{17}{2}x^2 + 50x + 151\ln|x-3| + C}$

w) $\int e^{\frac{x}{2}} + x^2\,dx = \int e^{\frac{x}{2}}\,dx + \frac{x^3}{3} + C_1$

$u = \frac{x}{2}$
$du = \frac{1}{2}dx$

$= 2\int \frac{1}{2} e^{\frac{x}{2}} dx + \frac{x^3}{3} + C_1 = 2\int e^u\,du + \frac{x^3}{3} + C_1$

$= 2e^u + \frac{x^3}{3} + C = \boxed{2e^{\frac{x}{2}} + \frac{x^3}{3} + C}$

x) $\int e^{7x} + x\,dx = \int e^{7x}\,dx + \frac{x^2}{2} + C_1$

$u = 7x$
$du = 7\,dx$

$= \frac{1}{7}\int 7e^{7x}dx + \frac{x^2}{2} + C_1 = \frac{1}{7}\int e^u du + \frac{x^2}{2} + C_1 = \frac{1}{7}e^u + \frac{x^2}{2} + C$

$= \boxed{\frac{1}{7}e^{7x} + \frac{x^2}{2} + C}$

y) $\int (x-7)^6\,dx$

$u = x-7$
$du = dx$

$= \int u^6\,du = \frac{u^7}{7} + C$

$= \boxed{\frac{(x-7)^7}{7} + C}$

z) $\int (3x-4)^4\,dx \quad \begin{array}{l} u = 3x-4 \\ du = 3dx \end{array} \quad = \frac{1}{3}\int 3(3x-4)^4\,dx = \frac{1}{3}\int u^4\,du$

$= \frac{1}{3}\left[\frac{u^5}{5} + C_1\right] = \boxed{\frac{1}{15}(3x-4)^5 + C}$

aa) $\int \frac{dx}{\sqrt[4]{5x+1}}\,dx \quad \begin{array}{l} u = 5x+1 \\ du = 5\,dx \end{array} \quad = \frac{1}{5}\int (5)\left(\frac{dx}{(5x+1)^{\frac{1}{4}}}\right)$

$= \frac{1}{5}\int u^{-\frac{1}{4}}\,du = \frac{1}{5}\left[\frac{4}{3}u^{\frac{3}{4}} + C_1\right] = \boxed{\frac{4}{15}(5x+1)^{\frac{3}{4}} + C}$

bb) $\int \frac{2x}{\sqrt[3]{x^2-1}}\,dx \quad \begin{array}{l} u = x^2-1 \\ du = 2x\,dx \end{array} \quad = \int u^{-\frac{1}{3}}\,du = \frac{3}{2}u^{\frac{2}{3}} + C$

$= \boxed{\frac{3}{2}(x^2-1)^{\frac{2}{3}} + C}$

cc) $\int \frac{7}{\sqrt{x-4}}\,dx \quad \begin{array}{l} u = x-4 \\ du = dx \end{array} \quad = \int 7u^{-\frac{1}{2}}\,du = 14\sqrt{u} + C = \boxed{14\sqrt{x-4} + C}$

dd) $\int \frac{2}{\sqrt[3]{5x+6}}\,dx \quad \begin{array}{l} u = 5x+6 \\ du = 5\,dx \end{array} \quad = \frac{2}{5}\int \frac{5\,dx}{\sqrt[3]{5x+6}} = \frac{2}{5}\int u^{-\frac{1}{3}}\,du$

$= \frac{2}{5}\left[\frac{3}{2}u^{\frac{2}{3}} + C_1\right] = \boxed{\frac{3}{5}(5x+6)^{\frac{2}{3}} + C}$

8. a) $\int_1^4 x^2 - x - 1\,dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - x\right]_1^4 = \frac{64}{3} - 8 - 4 - \left(\frac{1}{3} - \frac{1}{2} - 1\right)$

$= \boxed{\frac{21}{2}}$

b) $\int_0^{\frac{\pi}{4}} \tan x\,dx = \left[-\ln|\cos x|\right]_0^{\frac{\pi}{4}} = -\ln\frac{\sqrt{2}}{2} - (-\ln 1) = \ln\frac{2}{\sqrt{2}} = \boxed{\ln\sqrt{2}}$

c) $\displaystyle\int_0^{\frac{\pi}{6}} \sin 3x\, dx$ $\quad u = 3x$, $du = 3dx$, $x=0, u=0$, $x=\frac{\pi}{6}, u=\frac{\pi}{2}$ $\quad = \frac{1}{3}\int_0^{\frac{\pi}{6}} 3\sin 3x\, dx = \frac{1}{3}\int_0^{\frac{\pi}{2}} \sin u\, du$

$$= \left[-\frac{1}{3}\cos u\right]_0^{\frac{\pi}{2}} = -\frac{1}{3}\cos\frac{\pi}{2} - \left(-\frac{1}{3}\cos 0\right) = \boxed{\frac{1}{3}}$$

d) $\displaystyle\int_0^{\sqrt{3}} x\sqrt{x^2+1}\, dx$ $\quad u = x^2+1$, $du = 2x\, dx$, $x=0, u=1$, $x=\sqrt{3}, u=4$ $\quad \frac{1}{2}\int_0^{\sqrt{3}} 2x\sqrt{x^2+1}\, dx = \frac{1}{2}\int_1^4 u^{\frac{1}{2}}\, du$

$$= \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^4 = \frac{1}{2}\left[\frac{16}{3} - \frac{2}{3}\right] = \boxed{\frac{7}{3}}$$

e) $\displaystyle\int_0^5 e^{2x} - x^2 + x\, dx = \frac{1}{2}\int_0^5 2e^{2x}\, dx - \frac{x^3}{3} + \frac{x^2}{2}\Big]_0^5$ $\quad u = 2x$, $du = 2dx$, $x=0, u=0$, $x=5, u=10$

$$= \frac{1}{2}\int_0^{10} e^u\, du - \frac{125}{3} + \frac{25}{2} - 0 = \frac{1}{2}\left[e^u\right]_0^{10} - \frac{125}{3} + \frac{25}{2}$$

$$= \frac{1}{2}e^{10} - \frac{1}{2} - \frac{125}{3} + \frac{25}{2} = \boxed{\frac{1}{2}e^{10} - \frac{89}{3}}$$

f) $\displaystyle\int_0^2 \frac{x^2-x-1}{x-3}\, dx$

$$\begin{array}{r|rrr} 3 & 1 & -1 & -1 \\ & & 3 & 6 \\ \hline & 1 & 2 & 5 \end{array}$$

$$= \int_0^2 x + 2 + \frac{5}{x-3}\, dx$$

$u = x-3$, $du = dx$, $x=0, u=-3$, $x=2, u=-1$

$$= \left[\frac{x^2}{2} + 2x\right]_0^2 + \int_{-3}^{-1} \frac{5\, du}{u} = 2+4-0+\left[5\ln|u|\right]_{-3}^{-1}$$

$$= 6 + 5\ln 1 - 5\ln 3 = \boxed{6 - 5\ln 3}$$

g) $\int_1^{16} x^{\frac{1}{4}}\,dx = \left[\frac{4}{5}x^{\frac{5}{4}}\right]_1^{16} = \frac{4}{5}(32) - \frac{4}{5}(1) = \boxed{\frac{124}{5}}$

h) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^3 x\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^2 x \cos x\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1-\sin^2 x)\cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$
$x = \frac{\pi}{6},\ u = \frac{1}{2}$
$x = \frac{\pi}{3},\ u = \frac{\sqrt{3}}{2}$

$$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} 1 - u^2\,du = \left[u - \frac{u^3}{3}\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{2} - \frac{3\sqrt{3}}{24} - \left(\frac{1}{2} - \frac{1}{24}\right)$$

$$= \boxed{\frac{3\sqrt{3}}{8} - \frac{11}{24}}$$