1\. a) $\frac{d}{dx}\int_{x^3}^{5} \frac{\cos t}{t^2+2}\,dt = \frac{d}{dx}\left[-\int_{5}^{x^3} \frac{\cos t}{t^2+2}\,dt\right]$

$= -\left[\frac{\cos x^3}{(x^3)^2+2}\frac{d}{dx}(x^3) - 0\right] = \boxed{\frac{-3x^2\cos x^3}{x^6+2}}$

b) $\frac{d}{dx}\int_{\sin x}^{\cos x} t^2\,dt = \cos^2 x \frac{d}{dx}\cos x - \sin^2 x \frac{d}{dx}\sin x$

$= \boxed{\begin{array}{c} -\sin x\cos^2 x - \cos x \sin^2 x \\ \text{or} \\ -\sin x\cos x(\cos x + \sin x) \end{array}}$

c) $\frac{d}{dx}\int_{x^3}^{x^5} \cos 5t\,dt = \frac{d}{dx}x^5(\cos 5x^5) - \frac{d}{dx}x^3(\cos 5x^3)$

$= \boxed{5x^4\cos 5x^5 - 3x^2\cos 5x^3}$

2\. a) $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \csc^2\theta\,d\theta = \left[-\cot\theta\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = -\cot\frac{5\pi}{6} - \left(-\cot\frac{\pi}{6}\right)$

$= -(-\sqrt{3}) + \sqrt{3} = \boxed{2\sqrt{3}}$

b) $\int_{\frac{1}{3}}^{5} 4+\frac{1}{x}\,dx = \left[4x + \ln|x|\right]_{\frac{1}{3}}^{5} = 20 + \ln 5 - \left(\frac{4}{3} + \ln\frac{1}{3}\right)$

$= \frac{56}{3} + \ln 5 - (\ln 1 - \ln 3)$

$= \boxed{\frac{56}{3} + \ln 5 + \ln 3}$

c) $\int_1^2 4x^{-5} - 5x^4 \, dx = \left[-x^{-4} - x^5\right]_1^2 = -\frac{1}{16} - 32 - (-1-1)$

$= -\frac{1}{16} - 30 = \boxed{\frac{-481}{16}}$

d) $\int_4^9 \frac{t-3}{\sqrt{t}} \, dt = \int_4^9 t^{\frac{1}{2}} - 3t^{-\frac{1}{2}} \, dt = \left[\frac{2}{3} t^{\frac{3}{2}} - 6\sqrt{t}\right]_4^9$

$= \frac{2}{3}(27) - 18 - \left(\frac{2}{3}(8) - 12\right) = -\frac{16}{3} + 12 = \frac{20}{3}$

e) $\int_0^4 \frac{1-\sqrt{u}}{\sqrt{u}} \, du = \int_0^4 (u^{-\frac{1}{2}} - 1) \, du = \left[2\sqrt{u} - u\right]_0^4$

$= 4 - 4 - (0 - 0) = \boxed{0}$

3. a) $\int x \cos(2x^2) \, dx$ $\quad u = 2x^2$, $du = 4x\,dx$ $\quad = \frac{1}{4} \int 4x \cos 2x^2 \, dx$

$= \frac{1}{4} \int \cos u \, du = \frac{1}{4}\left[\sin u + C_1\right] = \frac{1}{4} \sin 2x^2 + C$

b) $\int \frac{9r^2 \, dr}{\sqrt{1-r^3}}$ $\quad u = 1 - r^3$, $du = -3r^2 \, dr$ $\quad = -\int \frac{-3(3r^2 \, dr)}{\sqrt{1-r^3}}$

$= -\int 3u^{-\frac{1}{2}} \, du = -6\sqrt{u} + C = \boxed{-6\sqrt{1-r^3} + C}$

c) $\int \frac{dx}{\sin^2 3x} = \int \csc^2 3x \, dx$ $\quad u = 3x, \; du = 3dx \quad$ $= \frac{1}{3}\int 3\csc^2 3x \, dx$

$= \frac{1}{3}\int \csc^2 u \, du = \frac{1}{3}\left[-\cot u + C_1\right] = -\frac{1}{3}\cot 3x + C$

d) $\int \frac{\sin(2t+1)}{\cos^2(2t+1)} dt = \int \sec(2t+1)\tan(2t+1)\, dt$ $\quad u = 2t+1, \; du = 2\,dt$

$= \frac{1}{2}\int 2\sec(2t+1)\tan(2t+1)\, dt = \frac{1}{2}\int \sec u \tan u \, du$

$= \frac{1}{2}\left[\sec u + C_1\right] = \frac{1}{2}\sec(2t+1) + C$

e) $\int \sqrt{\tan x}\, \sec^2 x \, dx$ $\quad u = \tan x, \; du = \sec^2 x\, d \quad$ $= \int u^{\frac{1}{2}} du$

$= \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{3}(\tan x)^{\frac{3}{2}} + C$

f) $\int \frac{dx}{\cot 3x} = \int \frac{\sin 3x}{\cos 3x} dx$ $\quad u = 3x, \; du = 3dx \quad$ $= \frac{1}{3}\int \frac{\sin 3x}{\cos 3x}(3)\, dx$

$= \frac{1}{3}\int \frac{\sin u}{\cos u} du$ $\quad v = \cos u, \; dv = -\sin u \, du \quad$ $= -\frac{1}{3}\int \frac{-\sin u}{\cos u} du$

$= -\frac{1}{3}\int \frac{dv}{v} = -\frac{1}{3}\ln|v| + C = -\frac{1}{3}\ln|\cos u| + C = -\frac{1}{3}\ln|\cos 3x| + C$

g) $\int \sec^4 x\,dx = \int \sec^2 x \sec^2 x\,dx = \int (1+\tan^2 x)\sec^2 x\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$

$= \int 1 + u^2\,du = u + \frac{u^3}{3} + c$

$= \boxed{\tan x + \frac{\tan^3 x}{3} + c}$

h) $\int \frac{dx}{x \ln x}$

$u = \ln x$
$du = \frac{1}{x}dx$

$= \int \frac{du}{u} = \ln|u| + c$

$= \boxed{\ln|\ln x| + c}$

4. a) $\int_0^{\frac{\pi}{4}} \tan^2 x \sec^2 x\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$
$x = \frac{\pi}{4},\ u = 1$
$x = 0,\ u = 0$

$= \int_0^1 u^2\,du = \left[\frac{u^3}{3}\right]_0^1 = \frac{1}{3} - 0$

$= \boxed{\frac{1}{3}}$

b) $\int_{-1}^{3} \frac{x}{x^2+1}\,dx$

$u = x^2+1$
$du = 2x\,dx$
$x = -1,\ u = 2$
$x = 3,\ u = 10$

$\frac{1}{2}\int_{-1}^{3} \frac{2x\,dx}{x^2+1} = \frac{1}{2}\int_2^{10} \frac{du}{u}$

$= \frac{1}{2}\Big[\ln|u|\Big]_2^{10} = \boxed{\frac{1}{2}(\ln 10 - \ln 2) = \frac{1}{2}\ln 5 = \ln\sqrt{5}}$

c) $\int_0^1 \sqrt{t^5 + 2t}\,(5t^4+2)\,dt$

$u = t^5 + 2t$
$du = 5t^4 + 2\,dt$
$t = 0,\ u = 0$
$t = 1,\ u = 3$

$= \int_0^3 u^{\frac{1}{2}}\,du$

$= \left[\frac{2}{3}u^{\frac{3}{2}}\right]_0^3 = \frac{2}{3}(3\sqrt{3})$

$= \boxed{2\sqrt{3}}$

d) $\displaystyle\int_0^2 \frac{e^x\,dx}{3+e^x}$

$u = 3+e^x$

$du = e^x\,dx$

$x=0,\ u=4$

$x=2,\ u=3+e^2$

$$= \int_4^{3+e^2} \frac{du}{u} = \Big[\ln|u|\Big]_4^{3+e^2}$$

$$= \boxed{\ln(3+e^2) - \ln 4 = \ln\left(\frac{3+e^2}{4}\right)}$$

e) $\displaystyle\int_1^4 \frac{dx}{\sqrt{x}(\sqrt{x}+1)^3}$

$u = \sqrt{x}+1$

$du = \frac{1}{2\sqrt{x}}$

$x=1,\ u=2$

$x=4,\ u=3$

$$= 2\int_1^4 \frac{dx}{2\sqrt{x}(\sqrt{x}+1)^3}$$

$$= 2\int_2^3 u^{-3}\,du = 2\left[-\frac{1}{2u^2}\right]_2^3 = 2\left[-\frac{1}{18} - \left(-\frac{1}{8}\right)\right]$$

$$= 2\left(\frac{5}{72}\right) = \boxed{\frac{5}{36}}$$