1. $$\frac{1}{3-1}\int_1^3 3x^2-4x+1\,dx = \frac{1}{2}\Big[x^3-2x^2+x\Big]_1^3$$

$$= \frac{1}{2}\Big[27-18+3-(1-2+1)\Big] = \boxed{6}$$

2. $$\frac{1}{\frac{\pi}{4}-0}\int_0^{\frac{\pi}{4}} \sin x\cos x\,dx$$

$u=\sin x$
$du=\cos x\,dx$
$x=0,\ u=0$
$x=\frac{\pi}{4},\ u=\frac{\sqrt{2}}{2}$

$$= \frac{4}{\pi}\int_0^{\frac{\sqrt{2}}{2}} u\,du$$

$$= \frac{4}{\pi}\left[\frac{u^2}{2}\right]_0^{\frac{\sqrt{2}}{2}} = \frac{4}{\pi}\left[\frac{\left(\frac{\sqrt{2}}{2}\right)^2}{2}-0\right] = \frac{4}{\pi}\left(\frac{1}{4}\right) = \boxed{\frac{1}{\pi}}$$

3. $$\frac{1}{1-(-1)}\int_{-1}^{1}\frac{2x}{1+x^4}\,dx$$

$u=x^2$
$du=2x\,dx$
$x=-$[redacted]$,\ u=1$
$x=1,\ u=1$

$$= \frac{1}{2}\int_1^1\frac{du}{1+u^2} = \boxed{0}$$

OR $$= \frac{1}{2}\Big[\tan^{-1}u\Big]_1^1 = \frac{1}{2}\left(\frac{\pi}{4}-\frac{\pi}{4}\right) =$$

4. $$\frac{1}{3-0}\int_0^3\sqrt{5x+1}\,dx$$

$u=5x+1$
$du=5\,dx$
$x=0,\ u=1$
$x=3,\ u=16$

$$= \frac{1}{3}\left(\frac{1}{5}\right)\int_{x=0}^{x=3} 5\sqrt{5x+1}\,dx$$

$$= \frac{1}{15}\int_1^{16}u^{\frac{1}{2}}\,du = \frac{1}{15}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^{16} = \frac{1}{15}\left[\frac{128}{3}-\frac{2}{3}\right] = \boxed{\frac{14}{5}}$$

5. $\frac{1}{1-(-3)}\int_{-3}^{1}(2x+3)^4\,dx$ $\quad$ $u=2x+3$, $du=2dx$, $x=-3,\ u=-3$, $x=1,\ u=5$ $\quad$ $=\frac{1}{4}\left(\frac{1}{2}\right)\int_{x=-3}^{x=1}2(2x+3)^4\,dx$

$$=\frac{1}{8}\int_{-3}^{5}u^4\,du=\frac{1}{8}\left[\frac{u^5}{5}\right]_{-3}^{5}=\frac{1}{8}\left[625-\left(\frac{-243}{5}\right)\right]=\boxed{\frac{421}{5}}$$

6. $\frac{1}{\frac{\pi}{3}-\frac{\pi}{6}}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\tan x\,dx=\frac{6}{\pi}\Big[-\ln|\cos x|\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}}=\frac{6}{\pi}\left[-\ln\cos\frac{\pi}{3}-\left(-\ln\cos\frac{\pi}{6}\right)\right]$

$$=\frac{6}{\pi}\left[-\ln\frac{1}{2}+\ln\frac{\sqrt{3}}{2}\right]=\frac{6}{\pi}\left(\ln\sqrt{3}\right)=\boxed{\frac{6\ln\sqrt{3}}{\pi}}$$

7. $\frac{1}{3-1}\int_{1}^{3}x^{-2}-x\,dx=\frac{1}{2}\left[-\frac{1}{x}-\frac{x^2}{2}\right]_{1}^{3}=\frac{1}{2}\left[-\frac{1}{3}-\frac{9}{2}-\left(-1-\frac{1}{2}\right)\right]$

$$=\boxed{-\frac{5}{3}}$$

8. $\frac{1}{2-(-1)}\int_{-1}^{2}e^{2x}\,dx=\frac{1}{3}\left(\frac{1}{2}\right)\int_{u=-2}^{u=4}2e^{2x}\,dx=\frac{1}{6}\int_{-2}^{4}e^u\,du=\frac{1}{6}\Big[e^u\Big]_{-2}^{4}$

$u=2x$, $du=2dx$, $x=-1,\ u=-2$, $x=2,\ u=4$

$$\boxed{=\frac{1}{6}\left[e^4-\frac{1}{e^2}\right]}$$

9. $\frac{1}{5-2}\int_2^5 (x-3)^6\,dx$ $\quad$ $u=x-3$, $du=dx$, $x=2, u=-1$, $x=5, u=2$ $\quad = \frac{1}{3}\int_{-1}^{2} u^6\,du = \frac{1}{3}\left[\frac{u^7}{7}\right]_{-1}^{2}$

$$= \frac{1}{3}\left[\frac{128}{7} - \left(-\frac{1}{7}\right)\right] = \boxed{\frac{43}{7}}$$

10. $\frac{1}{4-2}\int_2^4 \frac{x^2+5x+6}{x-1}\,dx = \frac{1}{2}\int_2^4 x+6+\frac{12}{x-1}\,dx$

$$\begin{array}{r|rrr} 1 & 1 & 5 & 6 \\ & & 1 & 6 \\ \hline & 1 & 6 & 12 \end{array}$$

$$= \frac{1}{2}\left[\frac{x^2}{2} + 6x + 12\ln|x-1|\right]_2^4$$

$$= \frac{1}{2}\left[8 + 24 + 12\ln 3 - (2 + 12 + 12\ln 1)\right]$$

$$= \frac{1}{2}\left[32 + 12\ln 3 - 14\right] = \boxed{9 + 6\ln 3}$$