1. a) $\int x^3 + 6x + 1 \, dx = \boxed{\frac{x^4}{4} + 3x^2 + x + C}$

b) $\int (1-t)(2+t^2)\, dt = \int -t^3 + t^2 - 2t + 2 \, dt = \boxed{\frac{-t^4}{4} + \frac{t^3}{3} - t^2 - 2t + C}$

c) $\int (2-\sqrt{x})^2 \, dx = \int 4 - 4\sqrt{x} + x \, dx = \int 4 - 4x^{\frac{1}{2}} + x \, dx$

$= \boxed{4x - \frac{8}{3} x^{\frac{3}{2}} + \frac{x^2}{2} + C}$

d) $\int \frac{\sin x}{1-\sin^2 x} \, dx = \int \frac{\sin x}{\cos^2 x} \, dx = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \, dx$

$= \int \sec x \tan x \, dx = \boxed{\sec x + C}$

e) $\int 3e^u + \sec^2 u \, du = \boxed{3e^u + \tan u + C}$

f) $\int x(1+2x^4) \, dx = \int x + 2x^5 \, dx = \boxed{\frac{x^2}{2} + \frac{x^6}{3} + C}$

g) $\int 2e^x + 4\cos x \, dx = \boxed{2e^x + 4\sin x + C}$

h) $\int x\sqrt{x} \, dx = \int x^{\frac{3}{2}} \, dx = \boxed{\frac{2}{5} x^{\frac{5}{2}} + C}$

i) $\int \cos x - 2\sin x \, dx = \boxed{\sin x + 2\cos x + C}$

j) $\int \frac{y+5y^7}{y^3}\,dy = \int \frac{y}{y^3} + \frac{5y^7}{y^3} = \int y^{-2} + 5y^4\,dy$

$$\boxed{= -y^{-1} + y^5 + c \quad \text{OR} \quad -\frac{1}{y} + y^5 + c}$$

k) $\int \frac{1+4x}{\sqrt{2x^2+x+1}}\,dx \qquad u = 2x^2+x+1, \quad du = 4x+1\,dx \qquad = \int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}\,du$

$$= 2\sqrt{u} + C = \boxed{2\sqrt{2x^2+x+1} + C}$$

l) $\int (3x-2)^{20}\,dx \qquad u = 3x-2, \quad du = 3\,dx \qquad = \frac{1}{3}\int 3(3x-2)^{20}\,dx$

$$= \frac{1}{3}\int u^{20}\,du = \frac{1}{3}\left(\frac{u^{21}}{21} + c_1\right) = \boxed{\frac{1}{63}(3x-2)^{21} + C}$$

m) $\int x^2(x^3+5)^9\,dx \qquad u = x^3+5, \quad du = 3x^2\,dx \qquad \frac{1}{3}\int 3x^2(x^3+5)^9\,dx$

$$= \frac{1}{3}\int u^9\,du = \frac{1}{3}\left[\frac{u^{10}}{10} + c_1\right] = \boxed{\frac{1}{30}(x^3+5)^{10} + C}$$

n) $\int \sqrt{4-t}\,dt \qquad u = 4-t, \quad du = -1\,dt \qquad -\int -\sqrt{4-t}\,dt = -\int u^{\frac{1}{2}}\,du$

$$= -\frac{2}{3}u^{\frac{3}{2}} + C = \boxed{-\frac{2}{3}(4-t)^{3/2} + C}$$

o) $\int \frac{e^x}{e^x+1}\,dx \qquad u = e^x+1, \quad du = e^x\,dx \qquad = \int \frac{du}{u} = \ln|u| + c = \boxed{\ln|e^x+1| + c \text{ or } \ln(e^x+1) + c}$

p) $\int \frac{\tan^{-1}x}{1+x^2}\,dx$ $\quad u = \tan^{-1}x$, $\quad du = \frac{1}{1+x^2}\,dx$ $\quad = \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{1}{2}\left[\tan^{-1}x\right]^2 + C}$

q) $\int \frac{(\ln x)^2}{x}\,dx$ $\quad u = \ln x$, $\quad du = \frac{1}{x}\,dx$ $\quad \int u^2\,du = \frac{u^3}{3} + C = \boxed{\frac{(\ln x)^3}{3} + C}$

r) $\int \frac{x}{\sqrt{1-4x^2}}\,dx$ $\quad u = 1-4x^2$, $\quad du = -8x\,dx$ $\quad = -\frac{1}{8}\int \frac{-8x\,dx}{\sqrt{1-4x^2}}$

$= -\frac{1}{8}\int \frac{du}{\sqrt{u}} = -\frac{1}{8}\int u^{-\frac{1}{2}}\,du = -\frac{1}{8}\left[2\sqrt{u} + C_1\right]$

$= \boxed{-\frac{1}{4}\sqrt{1-4x^2} + C}$

s) $\int \frac{z^2}{\sqrt[3]{1+z^3}}\,dz$ $\quad u = 1+z^3$, $\quad du = 3z^2\,dz$ $\quad = \frac{1}{3}\int \frac{3z^2\,dz}{(1+z^3)^{1/3}} = \frac{1}{3}\int u^{-\frac{1}{3}}\,du$

$= \frac{1}{2}u^{\frac{2}{3}} + C = \boxed{\frac{1}{2}(1+z^3)^{\frac{2}{3}} + C}$

t) $\int \sec^3 x \tan x\,dx = \int \sec^2 x \sec x \tan x\,dx$ $\quad u = \sec x$, $\quad du = \sec x \tan x\,dx$

$= \int u^2\,du = \frac{u^3}{3} + C = \boxed{\frac{\sec^3 x}{3} + C}$

u) $\int \sin^3\theta \cos\theta\,d\theta$ $\quad u = \sin\theta$, $\quad du = \cos\theta\,d\theta$ $\quad = \int u^3\,du = \frac{u^4}{4} + C = \boxed{\frac{\sin^4\theta}{4} + C}$

v) $\int \sqrt{3+x^2}\,(x^3)\,dx$ $\quad u = 3+x^2$, $du = 2x\,dx$, $x^2 = u-3$ $\quad = \frac{1}{2}\int x^2\sqrt{3+x^2}\,(2x)\,dx$

$$= \frac{1}{2}\int (u-3)u^{\frac{1}{2}}\,du = \frac{1}{2}\int u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\,du = \frac{1}{2}\left[\frac{2}{5}u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + C_1\right]$$

$$= \boxed{\frac{1}{5}(3+x^2)^{\frac{5}{2}} - (3+x^2)^{\frac{3}{2}} + C}$$

w) $\int x\sqrt{x-1}\,dx$ $\quad u = x-1$, $du = dx$, $x = u+1$ $\quad = \int (u+1)(u^{\frac{1}{2}})\,du = \int u^{\frac{3}{2}} + u^{\frac{1}{2}}\,du$

$$= \frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + C = \boxed{\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C}$$

x) $\int \frac{x}{\sqrt[4]{x+2}}\,dx$ $\quad u = x+2$, $du = dx$, $x = u-2$ $\quad = \int \frac{u-2}{u^{1/4}}\,du = \int u^{\frac{3}{4}} - 2u^{-\frac{1}{4}}\,du$

$$= \frac{4}{7}u^{\frac{7}{4}} - \frac{8}{3}u^{\frac{3}{4}} + C \quad \boxed{= \frac{4}{7}(x+2)^{\frac{7}{4}} - \frac{8}{3}(x+2)^{\frac{3}{4}} + C}$$

y) $\int \frac{x^2}{\sqrt{1-x}}\,dx$ $\quad u = 1-x$, $du = -dx$, $x = 1-u$ $\quad = -\int -\frac{x^2}{\sqrt{1-x}}\,dx = -\int \frac{(1-u)^2}{u^{\frac{1}{2}}}\,du$

$$= -\int \frac{1-2u+u^2}{u^{\frac{1}{2}}}\,du = \int -u^{-\frac{1}{2}} + 2u^{\frac{1}{2}} - u^{\frac{3}{2}}\,du$$

$$= -2\sqrt{u} + \frac{4}{3}u^{\frac{3}{2}} - \frac{2}{5}u^{\frac{5}{2}} + C = \boxed{-2\sqrt{1-x} + \frac{4}{3}(1-x)^{\frac{3}{2}} - \frac{2}{5}(1-x)^{\frac{5}{2}} + C}$$

z) $\int x^5 \sqrt{x^3+1}\, dx$ $\quad$ $u = x^3+1$, $du = 3x^2 dx$, $x^3 = u-1$ $\quad = \frac{1}{3}\int x^3 \sqrt{x^3+1}\; 3x^2\, dx$

$= \frac{1}{3}\int (u-1)u^{\frac{1}{2}}\, du = \frac{1}{3}\int u^{\frac{3}{2}} - u^{\frac{1}{2}}\, du$

$= \frac{1}{3}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + C_1\right] = \boxed{\frac{2}{15}(x^3+1)^{\frac{5}{2}} - \frac{2}{9}(x^3+1)^{\frac{3}{2}} + C}$

2. a) $\int_0^2 6x^2-4x+5\, dx = \left[2x^3 - 2x^2 + 5x\right]_0^2 = 2(2^3) - 2(2^2) + 5(2) - [0]$

$= \boxed{18}$

b) $\int_{-3}^{0} 5y^4 - 6y^2 + 14\, dy = \left[y^5 - 2y^3 + 14y\right]_{-3}^{0} = 0 - \left[(-3)^5 - 2(-3)^3 + 14(-3)\right]$

$= 243 - 54 + 42 = \boxed{231}$

c) $\int_{-2}^{2} (3u+1)^2 = \int_{-2}^{2} 9u^2 + 6u + 1\, du = \left[3u^3 + 3u^2 + u\right]_{-2}^{2}$

$= 3(2^3) + 3(2^2) + 2 - \left[3(-2)^3 + 3(-2)^2 - 2\right] = \boxed{52}$

d) $\int_0^9 \sqrt{2t}\, dt = \sqrt{2}\int_0^9 t^{\frac{1}{2}}\, dt = \sqrt{2}\left[\frac{2}{3}t^{\frac{3}{2}}\right]_0^9 = \sqrt{2}\left[\frac{2}{3}(27) - 0\right] = \boxed{18\sqrt{2}}$

e) $\int_1^{64} \frac{1+x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = \int_1^{64} x^{-\frac{1}{2}} + x^{-\frac{1}{6}}\, dx = \left[2\sqrt{x} + \frac{6}{5}x^{\frac{5}{6}}\right]_1^{64} = 16 + \frac{6(32)}{5} - \left(2 + \frac{6}{5}\right)$

$= \boxed{\frac{256}{5}}$

f) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec\theta \tan\theta \, d\theta = \left[\sec\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \boxed{2-\sqrt{2}}$

g) $\displaystyle\int_0^7 \sqrt{4+3x}\, dx$

$\downarrow$

$\displaystyle= \frac{1}{3}\int_0^7 3\sqrt{4+3x}\, dx$

$u = 4+3x$
$du = 3\,dx$
$x=7,\ u = 4+3(7) = 25$
$x=0,\ u = 4+3(0) = 4$

$\displaystyle= \frac{1}{3}\int_4^{25} u^{\frac{1}{2}}\, du = \frac{1}{3}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_4^{25}$

$\displaystyle= \frac{2}{9}\left[125 - 8\right] = \boxed{26}$

h) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin\theta}{\cos^2\theta}\, d\theta = \int_0^{\frac{\pi}{3}} \sec\theta\tan\theta\, d\theta = \left[\sec\theta\right]_0^{\frac{\pi}{3}} = 2-1 = \boxed{1}$

i) $\displaystyle\int_{-2}^{1} x^2\sqrt{x+3}\, dx$

$u = x+3$
$du = dx$
$x = u-3$
$x=-2,\ u=1$
$x=1,\ u=4$

$\displaystyle= \int_1^4 (u-3)^2(u^{\frac{1}{2}})\, du$

$\displaystyle= \int_1^4 u^{\frac{5}{2}} - 6u^{\frac{3}{2}} + 9u^{\frac{1}{2}}\, du = \left[\frac{2}{7}u^{\frac{7}{2}} - \frac{12}{5}u^{\frac{5}{2}} + 6u^{\frac{3}{2}}\right]_1^4$

$\displaystyle= \frac{2}{7}(128) - \frac{12}{5}(32) + 48 - \left(\frac{2}{7} - \frac{12}{5} + 6\right) = \boxed{\frac{136}{35}}$

j) $\displaystyle\int_0^{13} \frac{dx}{(1+2x)^{\frac{2}{3}}}$

$u = 1+2x$
$du = 2\,dx$
$x = 13,\ u=27$
$x=0,\ u=1$

$\displaystyle\frac{1}{2}\int_0^{13} \frac{2\,dx}{(1+2x)^{\frac{2}{3}}} = \frac{1}{2}\int_1^{27} u^{-\frac{2}{3}}\, du$

$\displaystyle= \left[\frac{3}{2}\sqrt[3]{u}\right]_1^{27} = \frac{9}{2} - \frac{3}{2} = \boxed{3}$

k) $\int_{\frac{1}{6}}^{\frac{1}{2}} \csc(\pi t)\cot(\pi t)\,dt$

$u = \pi t$
$du = \pi\,dt$
$t = \frac{1}{2},\ u = \frac{\pi}{2}$
$t = \frac{1}{6},\ u = \frac{\pi}{6}$

$\frac{1}{\pi}\int_{\frac{1}{6}}^{\frac{1}{2}} \csc(\pi t)\cot(\pi t)\,\pi\,dt$

$$= \frac{1}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \csc u \cot u\,du = \frac{1}{\pi}\Big[-\csc u\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{1}{\pi}\Big[-1-(-2)\Big] = \boxed{\frac{1}{\pi}}$$

l) $\int_0^{\sqrt{3}} x^5\sqrt{x^2+1}\,dx$

$u = x^2+1$
$du = 2x\,dx$
$x^2 = u-1$
$x = \sqrt{3},\ u = 4$
$x = 0,\ u = 1$

$\frac{1}{2}\int_0^{\sqrt{3}} x^5\sqrt{x^2+1}\;2\,dx$

$$= \frac{1}{2}\int_1^4 (u-1)^2\left(u^{\frac{1}{2}}\right)du = \int_1^4 \frac{1}{2}u^{\frac{5}{2}} - u^{\frac{3}{2}} + \frac{1}{2}u^{\frac{1}{2}}\,du$$

$$= \left[\frac{1}{7}u^{\frac{7}{2}} - \frac{2}{5}u^{\frac{5}{2}} + \frac{1}{3}u^{\frac{3}{2}}\right]_1^4 = \frac{128}{7} - \frac{64}{5} + \frac{8}{3} - \left(\frac{1}{7} - \frac{2}{5} + \frac{1}{3}\right)$$

$$= \boxed{\frac{848}{105}}$$