

$$1. \int \frac{x^2 - 5x + 7}{x-2} dx \quad \begin{array}{r} 2 \overline{) 1 \quad -5 \quad 7} \\ \underline{1 \quad -3 \quad 1} \end{array} = \int x - 3 + \frac{1}{x-2} dx$$

$$= \frac{x^2}{2} - 3x + C_1 + \int \frac{1}{x-2} dx \quad \begin{array}{l} u = x-2 \\ du = dx \end{array}$$

$$= \frac{x^2}{2} - 3x + C_1 + \int \frac{du}{u} = \boxed{\frac{x^2}{2} - 3x + \ln|x-2| + C}$$

$$2. \int \frac{x^3 - 5x + 2}{x-1} dx \quad \begin{array}{r} 1 \overline{) 1 \quad 0 \quad -5 \quad 2} \\ \underline{1 \quad 1 \quad -4} \\ 1 \quad 1 \quad -4 \quad -2 \end{array} = \int x^2 + x - 4 - \frac{2}{x-1} dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} - 4x + C_1 - \int \frac{2}{x-1} dx \quad \begin{array}{l} u = x-1 \\ du = dx \end{array} = \frac{x^3}{3} + \frac{x^2}{2} - 4x + C_1 - \int \frac{2du}{u}$$

$$= \boxed{\frac{x^3}{3} + \frac{x^2}{2} - 4x - 2\ln|x-1| + C}$$

$$3. \int \frac{2x^3 + 4x^2 - 2x - 1}{x+2} dx \quad \begin{array}{r} -2 \overline{) 2 \quad 4 \quad -2 \quad -1} \\ \underline{-4 \quad 0 \quad +4} \\ 2 \quad 0 \quad -2 \quad 3 \end{array}$$

$$= \int 2x^2 - 2 + \frac{3}{x+2} dx = \frac{2}{3}x^3 - 2x + C_1 + \int \frac{3}{x+2} dx \quad \begin{array}{l} u = x+2 \\ du = dx \end{array}$$

$$= \frac{2}{3}x^3 - 2x + C_1 + \int \frac{3du}{u} = \boxed{\frac{2}{3}x^3 - 2x + 3\ln|x+2| + C}$$

$$4. \int \frac{x^4 + x^2 + x}{x^2 + 4x} dx = \int \frac{x(x^3 + x + 1)}{x(x+4)} dx = \int \frac{x^3 + x + 1}{x+4} dx$$

$$\begin{array}{r} -4 \overline{) 1 \ 0 \ 1 \ 1} \\ \underline{-4 \ 16 \ -68} \\ 1 \ -4 \ 17 \ -67 \end{array} = \int x^2 - 4x + 17 - \frac{67}{x+4} dx$$

$$= \frac{x^3}{3} - 2x^2 + 17x + C_1 - \int \frac{67}{x+4} dx \quad \begin{array}{l} u = x+4 \\ du = dx \end{array}$$

$$= \frac{x^3}{3} - 2x^2 + 17x + C_1 - \int \frac{67 du}{u} = \boxed{\frac{x^3}{3} - 2x^2 + 17x - 67 \ln|x+4| + C}$$

$$5. \int \frac{2x^4 - x - 1}{x+3} dx \quad \begin{array}{r} -3 \overline{) 2 \ 0 \ 0 \ -1 \ -1} \\ \underline{-6 \ 18 \ -54 \ 165} \\ 2 \ -6 \ 18 \ -55 \ 164 \end{array}$$

$$= \int 2x^3 - 6x^2 + 18x - 55 + \frac{164}{x+3} dx \quad \begin{array}{l} u = x+3 \\ du = dx \end{array}$$

$$= \frac{x^4}{2} - 2x^3 + 9x^2 - 55x + C_1 + \int \frac{164 du}{u}$$

$$= \boxed{\frac{x^4}{2} - 2x^3 + 9x^2 - 55x + 164 \ln|x+3| + C}$$

$$6. \int \frac{x^5 + x^3 - 2x^2 - x - 1}{x-3} dx \quad \begin{array}{r} 3 \overline{) 1 \ 0 \ 1 \ -2 \ -1 \ -1} \\ \underline{3 \ 9 \ 30 \ 84 \ 249} \\ 1 \ 3 \ 10 \ 28 \ 83 \ 248 \end{array}$$

$$= \int x^4 + 3x^3 + 10x^2 + 28x + 83 + \frac{248}{x-3} dx \quad \begin{array}{l} u = x-3 \\ du = dx \end{array}$$

$$= \frac{x^5}{5} + \frac{3}{4}x^4 + \frac{10}{3}x^3 + 14x^2 + 83x + \int \frac{248 du}{u} = \boxed{\frac{x^5}{5} + \frac{3}{4}x^4 + \frac{10}{3}x^3 + 14x^2 + 83x + 248 \ln|x-3| + C}$$

$$7. \int \frac{x^3 - 5x - 2}{x+4} dx \quad \begin{array}{r} -4 \overline{) 1 \ 0 \ -5 \ -2} \\ \underline{-4 \ 16 \ -44} \\ 1 \ -4 \ 11 \ -46 \end{array}$$

$$= \int x^2 - 4x + 11 - \frac{46}{x+4} dx \quad \begin{array}{l} u = x+4 \\ du = dx \end{array}$$

$$= \frac{x^3}{3} - 2x^2 + 11x + C_1 - \int \frac{46 du}{u}$$

$$= \boxed{\frac{x^3}{3} - 2x^2 + 11x - 46 \ln|x+4| + C}$$

$$8. \int \frac{2x^2 + 6x - 3}{2x+2} dx = \int \frac{2x^2 + 6x - 3}{2(x+1)} dx = \frac{1}{2} \int \frac{2x^2 + 6x - 3}{x+1} dx$$

$$= \frac{1}{2} \int 2x + 4 - \frac{7}{x+1} dx \quad \begin{array}{l} u = x+1 \\ du = dx \end{array} \quad \begin{array}{r} -1 \overline{) 2 \ 6 \ -3} \\ \underline{-2 \ -4} \\ 2 \ 4 \ -7 \end{array}$$

$$= \frac{1}{2} \left[x^2 + 4x + C_1 \right] - \frac{1}{2} \int \frac{7 du}{u}$$

$$= \boxed{\frac{x^2}{2} + 2x - \frac{7}{2} \ln|x+1| + C}$$

$$9. \int \frac{x^4 - x^2 + 5x - 1}{x+2} dx \quad \begin{array}{r} -2 \overline{) 1 \ 0 \ -1 \ 5 \ -1} \\ \underline{-2 \ 4 \ -6 \ 2} \\ 1 \ -2 \ 3 \ -1 \ 1 \end{array}$$

$$= \int x^3 - 2x^2 + 3x - 1 + \frac{1}{x+2} dx \quad \begin{array}{l} u = x+2 \\ du = dx \end{array}$$

$$= \frac{x^4}{4} - \frac{2}{3}x^3 + \frac{3}{2}x^2 - x + C_1 + \int \frac{du}{u}$$

$$= \boxed{\frac{x^4}{4} - \frac{2}{3}x^3 + \frac{3}{2}x^2 - x + \ln|x+2| + C}$$

$$10. \int \frac{3x^5 - 7x^4 + 2x^3 + x^2 - 5x - 1}{x+2} dx \quad \begin{array}{r} -2 \overline{) 3 \ -7 \ 2 \ 1 \ -5 \ -1} \\ \underline{-6 \ 26 \ -56 \ 110 \ -210} \\ 3 \ -13 \ 28 \ -55 \ 105 \ -211 \end{array}$$

$$= \int 3x^4 - 13x^3 + 28x^2 - 55x + 105 - \frac{211}{x+2} dx \quad \begin{array}{l} u = x+2 \\ du = dx \end{array}$$

$$= \frac{3}{5}x^5 - \frac{13}{4}x^4 + \frac{28}{3}x^3 - \frac{55}{2}x^2 + 105x - \int \frac{211 du}{u}$$

$$= \boxed{\frac{3}{5}x^5 - \frac{13}{4}x^4 + \frac{28}{3}x^3 - \frac{55}{2}x^2 + 105x - 211 \ln|x+2| + C}$$