Convert each integral into Newton's notation and then evaluate the integral using a summation and a limit.

1. $\int_0^5 (2x+1)\,dx$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \left[2\left(\frac{5k}{n}\right)+1\right]\left(\frac{5}{n}\right) = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{10k}{n}+1\right)\left(\frac{5}{n}\right)$

$\Delta x = \frac{5}{n}$

$x_k = \frac{5k}{n}$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{50k}{n^2} + \frac{5}{n} = \lim_{n\to\infty} \frac{50n(n+1)}{2n^2} + \frac{5n}{n} = 25+5 = \boxed{30}$

2. $\int_1^3 5x\,dx$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \left[5\left(1+\frac{2k}{n}\right)\right]\frac{2}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(5+\frac{10k}{n}\right)\left(\frac{2}{n}\right)$

$\Delta x = \frac{3-1}{n} = \frac{2}{n}$

$x_k = 1+\frac{2k}{n}$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{10}{n} + \frac{20k}{n^2} = \lim_{n\to\infty} \frac{10n}{n} + \frac{20n(n+1)}{2n^2}$

$= 10 + 10 = \boxed{20}$

3. $\int_0^2 x^2\,dx$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{2k}{n}\right)^2\left(\frac{2}{n}\right) = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{8k^2}{n^3} = \lim_{n\to\infty} \frac{8n(n+1)(2n+1)}{6n^3}$

$\Delta x = \frac{2}{n}$

$x_k = \frac{2k}{n}$ $= \frac{16}{6} = \boxed{\frac{8}{3}}$

4. $\int_1^2 (x^2-1)\,dx$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(1+\frac{k}{n}\right)^2 - 1\right]\frac{1}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(1+\frac{2k}{n}+\frac{k^2}{n^2}\right)-1\right]\frac{1}{n}$

$\Delta x = \frac{2-1}{n} = \frac{1}{n}$

$x_k = 1+\frac{k}{n}$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{2k}{n}+\frac{k^2}{n^2}\right)\frac{1}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2k}{n^2} + \frac{k^2}{n^3}$

$= \lim_{n\to\infty} \frac{2n(n+1)}{2n^2} + \frac{n(n+1)(2n+1)}{6n^3} = \frac{2}{2} + \frac{2}{6} = \boxed{\frac{4}{3}}$

5. $\int_0^2 (x^2+x+2)\,dx$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(\frac{2k}{n}\right)^2 + \frac{2k}{n} + 2\right]\frac{2}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{8k^2}{n^3} + \frac{4k}{n} + \frac{4}{n}$

$\Delta x = \frac{2}{n}$

$x_k = \frac{2k}{n}$ $= \lim_{n\to\infty} \frac{8n(n+1)(2n+1)}{6n^3} + \frac{4n(n+1)}{2} + \frac{4n}{n} = \frac{16}{6} + \frac{4}{2} + 4 = \boxed{\frac{26}{3}}$

6. $\int_1^2 (3x^2-x-1)\,dx$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \left[3\left(1+\frac{k}{n}\right)^2 - \left(1+\frac{k}{n}\right) - 1\right]\left(\frac{1}{n}\right)$

$\Delta x = \frac{2-1}{n} = \frac{1}{n}$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \left[3\left(1+\frac{2k}{n}+\frac{k^2}{n^2}\right) - 1 - \frac{k}{n} - 1\right]\left(\frac{1}{n}\right)$

$x_k = 1+\frac{k}{n}$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \left(3+\frac{6k}{n}+\frac{3k^2}{n^2} - 1 - \frac{k}{n} - 1\right)\left(\frac{1}{n}\right) = \lim_{n\to\infty} \sum_{k=1}^{n} \left(1+\frac{5k}{n}+\frac{3k^2}{n^2}\right)\left(\frac{1}{n}\right)$

$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n} + \frac{5k}{n^2} + \frac{3k^2}{n^3} = \lim_{n\to\infty} \frac{n}{n} + \frac{5n(n+1)}{2n^2} + \frac{3n(n+1)(2n+1)}{6n^3} = 1 + \frac{5}{2} + \frac{6}{6} = \boxed{\frac{9}{2}}$