

1. $f(x) = x^3$; $[1,5]$; $n = 4$ subintervals

$$\text{Left: } 1^3(1) + 2^3(1) + 3^3(1) + 4^3(1) = \boxed{100}$$

$$\text{Right: } 5^3(1) + 4^3(1) + 3^3(1) + 2^3(1) = \boxed{224}$$

$$\text{Mid: } \left(\frac{3}{2}\right)^3(1) + \left(\frac{5}{2}\right)^3(1) + \left(\frac{7}{2}\right)^3(1) + \left(\frac{9}{2}\right)^3(1) \\ = \boxed{153}$$

$$f(x) = -x^2 + 8x - 12; [2,6]; n = 4 \text{ subintervals}$$

$$\begin{aligned} \text{Left: } & [- (2^2) + 8(2) - 12](1) + [- (3^2) + 8(3) - 12](1) \\ & + [- (4^2) + 8(4) - 12](1) + [- (5^2) + 8(5) - 12](1) \\ & = 0 + 3 + 4 + 3 = \boxed{10} \end{aligned}$$

$$\begin{aligned} \text{Right: } & [- (6^2) + 8(6) - 12](1) + [- (5^2) + 8(5) - 12](1) \\ & + [- (4^2) + 8(4) - 12](1) + [- (3^2) + 8(3) - 12](1) \\ & = 0 + 3 + 4 + 3 = \boxed{10} \end{aligned}$$

$$\text{Mid} = \left[-\left(\frac{5}{2}\right)^2 + 8\left(\frac{5}{2}\right) - 12 \right](1) + \left[-\left(\frac{7}{2}\right)^2 + 8\left(\frac{7}{2}\right) - 12 \right](1) \\ + \left[-\left(\frac{9}{2}\right)^2 + 8\left(\frac{9}{2}\right) - 12 \right](1) + \left[-\left(\frac{11}{2}\right)^2 + 8\left(\frac{11}{2}\right) - 12 \right](1)$$

$$= \frac{7}{4} + \frac{15}{4} + \frac{15}{4} + \frac{7}{4} = \boxed{11}$$

3. $f(x) = -\frac{1}{4}x^2 + 10$; $[-6, 6]$; $n = 6$ subintervals

$$\begin{aligned} \text{Left: } & \left[-\frac{1}{4}(-6)^2 + 10 \right](2) + \left[-\frac{1}{4}(-4)^2 + 10 \right](2) + \left[-\frac{1}{4}(-2)^2 + 10 \right](2) \\ & + \left[-\frac{1}{4}(0^2) + 10 \right](2) + \left[-\frac{1}{4}(2^2) + 10 \right](2) + \left[-\frac{1}{4}(4^2) + 10 \right](2) \\ & = 2 + 12 + 18 + 20 + 18 + 12 = \boxed{82} \end{aligned}$$

$$\begin{aligned} \text{Right: } & \left[-\frac{1}{4}(6^2) + 10 \right](2) + \left[-\frac{1}{4}(4^2) + 10 \right](2) + \left[-\frac{1}{4}(2^2) + 10 \right](2) \\ & + \left[-\frac{1}{4}(0^2) + 10 \right](2) + \left[-\frac{1}{4}(-2)^2 + 10 \right](2) + \left[-\frac{1}{4}(-4)^2 + 10 \right](2) \\ & = 2 + 12 + 18 + 20 + 18 + 12 = \boxed{82} \end{aligned}$$

$$\text{Mid: } \left[-\frac{1}{4}(5^2) + 10\right](2) + \left[-\frac{1}{4}(3^2) + 10\right](2) + \left[-\frac{1}{4}(1^2) + 10\right](2) \\ + \left[-\frac{1}{4}(-1)^2 + 10\right](2) + \left[-\frac{1}{4}(-3)^2 + 10\right](2) + \left[-\frac{1}{4}(-5)^2 + 10\right](2)$$

$$= \frac{15}{2} + \frac{31}{2} + \frac{39}{2} + \frac{39}{2} + \frac{31}{2} + \frac{15}{2}$$

$$= \frac{170}{2} = \boxed{85}$$

4. $f(x) = \ln(ex)$; $[1,5]$; $n = 2$ subintervals

$$\text{Left: } [\ln(e)](2) + [\ln(3e)](2) = 2 + 2\ln 3 + 2 \\ = \boxed{4 + 2\ln 3}$$

$$\text{Right: } [\ln 5e](2) + [\ln 3e](2) = 2 + 2\ln 5 + 2\ln 3 + 2 \\ = 4 + \ln 25 + \ln 9 = \boxed{4 + \ln 225}$$

$$\text{Mid: } [\ln 2e](2) + [\ln 4e](2) \\ = 2\ln 2 + 2 + 2\ln 4 + 2 = \boxed{4 + \ln 64}$$