Applications of the Derivative Extra Review

Directions: Do what it says. Don't use a calculator.

1. Find the equations of the tangent line and the normal line to each of the following functions at the given point.

a. $f(x) = \sin 2x;\ x = \frac{\pi}{8}$

b. $f(x) = (2x + \sin x)^3;\ x = 1$

c. $f(x) = \frac{2x+5}{x^2-1};\ x = 2$

2. Approximate the value of $f\left(\frac{\pi}{5}\right)$ by using the tangent line of $f(x) = \sin^2 x$ at $x = \frac{\pi}{6}$. Is your approximation an underestimate or overestimate? Why?

3. Approximate the value of $f(3.1)$ by using the tangent line of $f(x) = 3x^3 - 6x^2 - x - 6$ at $x = 3$. Is your approximation an underestimate or overestimate? Why?

4. Approximate the value of $f(-2)$ by using the tangent line of $f(x) = \tan^{-1}(x)$ at $x = -\sqrt{3}$. Is your approximation an underestimate or overestimate? Why?

5. Approximate the value of $f\left(\frac{\pi}{5}\right)$ by using the secant line of $f(x) = \sin^2 x$ on $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$. Is your approximation an underestimate or overestimate? Why?

6. Approximate the value of $f(1.5)$ by using the secant line of $f(x) = 3x^3 - 6x^2 - x - 6$ on $[1,2]$. Is your approximation an underestimate or overestimate? Why?

7. Approximate the value of $f(2.2)$ by using the secant line of $f(x) = \frac{x+1}{2x-3}$ on $[2,3]$. Is your approximation an underestimate or overestimate? Why?

8. Use linearization to approximate each of the following:

a. $\sqrt{102}$

b. $\sqrt[3]{10}$

c. $\sqrt[4]{14}$

d. $\sin\frac{\pi}{7}$

e. $\cos\frac{\pi}{8}$

9. Approximate the change in the surface area of a sphere if its radius decreases from 6 to 5.9 meters.

10. Approximate the change in the volume of a sphere if its radius increases from 6 to 6.1 inches.

11. Approximate the change in the volume of a cone if the height increases from 4 to 4.2 feet and there is no change in the radius of the cone.

12. Approximate the change in the volume of a cone if the radius decreases from 3 to 2.9 meters and there is no change in the height of the cone.

13. Approximate the change in the volume of a cone if the radius increases from 6 to 6.1 inches and the height increases from 10 to 10.2 inches.

14. Find all intervals on which each of the following functions are increasing and decreasing. Determine all local and absolute maxima and minima. Find all intervals on which the functions are concave up and concave down. Find all points of inflection.

a. $f(x) = 2x^3 - 3x^2 - 36x + 3$

b. $f(x) = x^4 - 6x^3 + 12x^2 - 4$

c. $f(x) = xe^{-x}$

d. $f(x) = \ln(x^3 - 3x)$

e. $f(x) = \frac{x^2+5}{x-2}$

15. A particle moves along the x-axis for $t > 0$ according to the position equation $x(t) = (t^2 - t)e^{-t}$.

a. Find the velocity of the particle the first time it is at the origin.

b. Find the velocity of the particle at $t = 3$.

c. Find the speed of the particle at $t = 3$.

d. Find the acceleration of the particle at $t = 3$.

e. Find all values of t for which the acceleration of the particle is 0.

16. Determine if the following function satisfies the hypotheses of the Mean Value Theorem on the interval $[0,1]$. If so, find all values c on the interval $(0,1)$ that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = \frac{4}{(2-x)^3}$$

17. Determine if the following function satisfies the hypotheses of the Mean Value Theorem on the interval $[1,3]$. If so, find all values c on the interval $(1,3)$ that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = \frac{2x}{(2-x)^2}$$

18. Determine if the following function satisfies the hypotheses of the Mean Value Theorem on the interval $[-2,1]$. If so, find all values c on the interval $(-2,1)$ that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = \frac{x-1}{x-3}$$

19. Determine the equations of all horizontal tangent lines for each of the following equations.

a. $f(x) = x^2 - \ln x$

b. $f(x) = x^3 - 3x + 1$

c. $xy - x^2y - xy^2 = 3$

20. Determine the equations of all vertical tangent lines for each of the following equations.

a. $f(x) = 2x(x-1)^{\frac{1}{3}}$

b. $f(x) = \sqrt[3]{3-x}\sqrt[3]{x-2}$

c. $x^2 - 2xy - y^2 = 4$

21. What is the x-value of the point on the graph of $y = x^{\frac{3}{2}} - 4$ that is closest to the point $(1,-4)$?

22. Sand is falling into a conical pile in which the radius is always six times the height. If sand is falling at a rate of 22 cubic inches per second, at what rate is the radius of the pile changing when the radius is 10 inches?

23. Sand is falling into a conical pile in which the radius is always half the height. If sand is falling at a rate of 36 cubic inches per second, at what rate is the height of the pile changing when the radius is 6 inches?

24. A 13 foot long metal pole is leaning against a wall that meets the ground at a right angle. At the instant when the top of the pole is 10 feet above the ground, the pole is sliding down the wall at 3 feet per second.

a. Find the rate of change of the base of the pole sliding away from the wall.

b. Find the rate of change of the area of the triangle formed by the wall, the ground, and the pole

c. Find the rate of change of the angle made by the ground and the pole.

d. Find the rate of change of the angle made by the wall and the pole.

25. A 25 foot long ladder is leaning against a wall that meets the ground at a right angle. At the instant when the bottom of the ladder is 7 feet away from the wall, the pole is sliding down the wall at 5 feet per second.

a. Find the rate of change of the base of the ladder sliding away from the wall.

b. Find the rate of change of the area of the triangle formed by the wall, the ground, and the ladder

c. Find the rate of change of the angle made by the ground and the ladder.

d. Find the rate of change of the angle made by the wall and the ladder.

26. Water is flowing into a conical tank that is oriented with the vertex down at a rate of 5 cubic feet per minute. The height of the tank is 12 feet and the radius is 3 feet. Find the rate of change of the height of the water in the tank when the height of the water is 7 feet.

27. Water is flowing out of a conical tank that is oriented with the vertex down at a rate of 3 cubic feet per minute. The height of the tank is 18 feet and the radius is 9 feet. Find the rate of change of the height of the water in the tank when the height of the water is 12 feet.

New Directions: Do what it says. Use a calculator. Round your final answer to three or more decimals.

28. What is the x-value of the point on the graph of $y = x^3 - 3x^2 + 1$ that is closest to the point $(2,1)$?

29. A particle moves along a line for $t > 0$ according to its position equation $y(t) = \ln(t^2 + 1) - t + \sin t$.

a. Find the velocity of the particle at $t = 1$.

b. Find the velocity of the particle at $t = 4$.

c. Find the speed of the particle at $t = 4$.

d. Find the velocity of the particle the first time it is at the origin.

e. Find the acceleration of the particle the first time it is at the origin.

30. Use Newton's Method to approximate the root of $y = x^5 - 3x^2 + x + 2$ on the interval $[-1,0]$.

31. Use Newton's Method to approximate the root of $y = 1 + x + x^3 - \sin x$ on the interval $[-1,0]$.

32. A cylindrical can is being manufactured. The cost of the lateral portion of the cylinder is 15 cents per square inch. The material for the top of the can costs 19 cents per square inch while the base of the can costs 32 cents per square inch. If the can must have a capacity of 1000 cubic inches, what dimensions for the can, radius and height, yield a minimum cost to the manufacturer?

33. A square pyramid is to have a volume of 500 cubic cm as given by the formula $V = \frac{a^2h}{3}$ where a is the length of one side of the base and h is the height of the pyramid measured directly from the center of the base to the vertex of the pyramid. What dimensions (a and h) minimize the surface area of the pyramid?

34. Find all horizontal and vertical tangent lines of $xy + 2xy^2 - 2x^3 = 1$.

35. Find all horizontal and vertical tangent lines of $xe^x + x^2 + y^2 = 1$.

36. A right triangle is drawn beneath the curve $y = \sqrt{16 - x^2}$ such that angle θ is located at the point $(-4,0)$, a second vertex is at the point $\left(x, \sqrt{16 - x^2}\right)$ and the third vertex which makes the right angle is located at $(x, 0)$. The point at $\left(x, \sqrt{16 - x^2}\right)$ moves along the curve such that x-value is increasing at 3 units per minute. How fast is angle θ changing when $x = 2$?

37. A triangle is drawn using the curve $f(x) = (2x^2 + 1)\sqrt{2 + x - x^2}$ with its base stretching from one endpoint of the function to the other and its vertex angle on $f(x)$.

a. Find the area of the largest such triangle that can be drawn.

b. If a particle starts at the point $(-1,0)$ and moves along $f(x)$ towards the point $(2,0)$ such that the value of x is increasing at 2 units per second. At what rate is the area of the triangle formed by the end points of the function and the particle changing when $x = 1$?

c. Given the same situation described above, at what rate is angle θ, the angle at the point $(-1,0)$ changing when $x = \frac{1}{2}$?

38. A rectangle is inscribed under the curve $y = 7x - 4x^2$. Find the dimensions that maximize the area of the rectangle.

39. A rectangle is inscribed under the curve $y = \sqrt{9 - x^2}$. Find the dimensions that maximize the area of the rectangle.

40. A rectangle is inscribed under one arch of the curve $y = 1 + \cos 2x$. Find the dimensions that maximize the area of the rectangle.