Extra Review of Limits and Continuity

Directions: Do what it says. Don't use a calculator.

1. Find each limit:

a. $\lim_{x\to\infty} \frac{e^{7x}}{7x^5}$

b. $\lim_{x\to\infty} \frac{\ln x}{x}$

c. $\lim_{x\to\infty} \frac{x^3}{2^x}$

d. $\lim_{x\to\infty} \frac{\sqrt{x}}{x^3}$

e. $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{3x-5}$

f. $\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{3x-5}$

g. $\lim_{x\to\infty} \frac{5x+1}{\sqrt{16x^2-2}}$

h. $\lim_{x\to-\infty} \frac{5x+1}{\sqrt{16x^2-2}}$

i. $\lim_{x\to3} \frac{x^2-9}{x^2-7x+12}$

j. $\lim_{x\to-3} \frac{x^2-9}{x^2-7x+12}$

k. $\lim_{x\to-5} \frac{x^3+7x^2+10x}{x^2-2x-35}$

l. $\lim_{x\to-4} \frac{x^2-3x-4}{x^2-4x}$

m. $\lim_{x\to4} \frac{x^2-3x-4}{x^2-4x}$

n. $\lim_{x\to6} \frac{x^2+36}{x^2-8x+12}$

o. $\lim_{x\to0} \frac{\sin^2 x}{3x^2}$

p. $\lim_{x\to0} \frac{\sin^5(4x^2)}{3x^{10}}$

q. $\lim_{x\to0} \frac{x^3-5\sin 2x}{3x}$

r. $\lim_{x\to 0} \frac{1-\cos 7x}{4x}$

s. $\lim_{x\to 0} \frac{\tan 12x}{5x}$

t. $\lim_{x\to 0} \frac{\tan^4(3x^2)}{5x^8}$

u. $\lim_{x\to 0} \frac{\sin 8x}{\sin 3x}$

v. $\lim_{x\to 0} \frac{\tan^5(2x)}{\sin^5(3x)}$

w. $\lim_{x\to 0} \frac{3x+\sin 2x-\cos x+1}{x+1-\cos x}$

x. $\lim_{x\to 3^-} \frac{|x-3|}{x-3}$

y. $\lim_{x\to -4^+} \frac{|x+4|}{x+4}$

z. $\lim_{x\to 2^+} \frac{2-x}{|x-2|}$

aa. $\lim_{x\to 2} \frac{\frac{4}{x^2+1}-\frac{4}{x+3}}{2-x}$

bb. $\lim_{x\to 3} \frac{\frac{2}{x^2+1}-\frac{1}{x+2}}{x-3}$

cc. $\lim_{x\to 3} \frac{\frac{4}{x^2-3}-\frac{2}{x}}{x-3}$

dd. $\lim_{x\to 5} \frac{\frac{4}{x+3}-\frac{2}{x-1}}{2x-10}$

ee. $\lim_{x\to 3} \frac{\frac{3}{x^2+3}-\frac{1}{x+1}}{x-3}$

ff. $\lim_{x\to\infty} e^{7x-2}$

gg. $\lim_{x\to\infty} e^{-x^3+x^2+x+1}$

hh. $\lim_{x\to-\infty} e^{x^2-x}$

ii. $\lim_{x\to-\infty} e^{x^5+x+2}$

jj. $\lim_{x\to\infty} \ln(5x-7)$

kk. $\lim_{x\to-\infty} \ln(2x^2+8x-3)$

ll. $\lim_{x\to-\infty} \ln(x^3+x^2+x+1)$

mm. $\lim_{x\to\infty} \ln\left(\frac{1}{x^2+1}\right)$

nn. $\lim_{x\to-\infty} \ln\left(\frac{3}{x^2+1}\right)$

oo. $\lim_{x\to\infty} \ln\left(\frac{-3}{x^3-1}\right)$

pp. $\lim_{x\to-\infty} \ln\left(\frac{5}{x^4-x}\right)$

qq. $\lim_{x\to0} x^2 \sin\left(\frac{1}{x}\right)$

rr. $\lim_{x\to0} \sqrt[3]{x}\cos\left(\frac{2}{x^2}\right)$

ss. $\lim_{x\to4} \frac{\sqrt{x}-2}{x-4}$

tt. $\lim_{x\to7} \frac{\sqrt{x+2}+x-10}{x^2-49}$

uu. $\lim_{x\to1} \frac{x^{\frac{1}{5}}-1}{x-1}$

vv. $\lim_{x\to1} \frac{x^{\frac{2}{3}}+x-2}{x^2-5x+4}$

ww. $\lim_{x\to6} \frac{\sqrt[3]{x+2}-2}{2x-12}$

xx. $\lim_{x\to3} \frac{\left(\sqrt[3]{x-2}\right)^2-5\sqrt[3]{x-2}+4}{\left(\sqrt[3]{x-2}\right)^2-3\sqrt[3]{x-2}+2}$

yy. $\lim_{x\to5} \frac{x^3-125}{5\sin(\pi x)+x-5}$

zz. $\lim_{x\to0} \frac{1-3\cos x+3\cos^2 x-\cos^3 x}{2x^3}$

2. Use the given graph to determine each of the following limits:

a. $\lim_{x\to -6^-} f(x)$

b. $\lim_{x\to -6^+} f(x)$

c. $\lim_{x\to -6} f(x)$

d. $\lim_{x\to -5} f(x)$

e. $\lim_{x\to -4} f(x)$

f. $\lim_{x\to -2^-} f(x)$

g. $\lim_{x\to -2^+} f(x)$

h. $\lim_{x\to -2} f(x)$

i. $\lim_{x\to 0^-} f(x)$

j. $\lim_{x\to 0^+} f(x)$

k. $\lim_{x\to 0} f(x)$

l. $\lim_{x\to 1} f(x)$

m. $\lim_{x\to 2^-} f(x)$

n. $\lim_{x\to 2^+} f(x)$

o. $\lim_{x\to 2} f(x)$

p. $\lim_{x\to 3} f(x)$

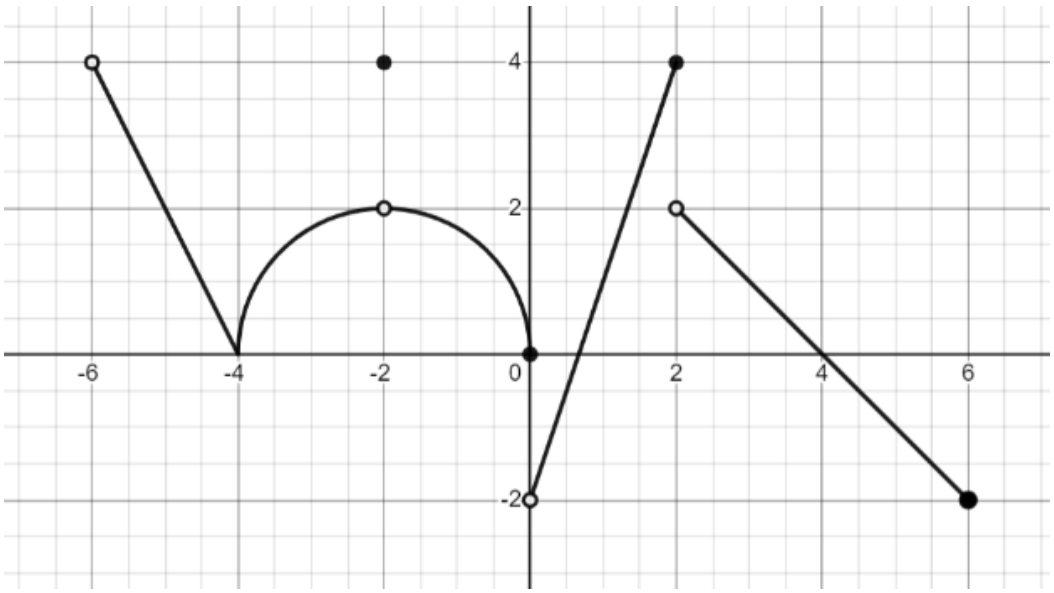
q. $\lim_{x\to 4} f(x)$

r. $\lim_{x\to 5} f(x)$

s. $\lim_{x\to 6^-} f(x)$

t. $\lim_{x\to 6^+} f(x)$

u. $\lim_{x\to 6} f(x)$



3. Use the given graph to determine each of the following limits:

a. $\lim_{x\to -6^-} f(x)$

b. $\lim_{x\to -6^+} f(x)$

c. $\lim_{x\to -6} f(x)$

d. $\lim_{x\to -5} f(x)$

e. $\lim_{x\to -4} f(x)$

f. $\lim_{x\to -3^-} f(x)$

g. $\lim_{x\to -3^+} f(x)$

h. $\lim_{x\to -3} f(x)$

i. $\lim_{x\to -1^-} f(x)$

j. $\lim_{x\to -1^+} f(x)$

k. $\lim_{x\to -1} f(x)$

l. $\lim_{x\to 1} f(x)$

m. $\lim_{x\to 2^-} f(x)$

n. $\lim_{x\to 2^+} f(x)$

o. $\lim_{x\to 2} f(x)$

p. $\lim_{x\to 3} f(x)$

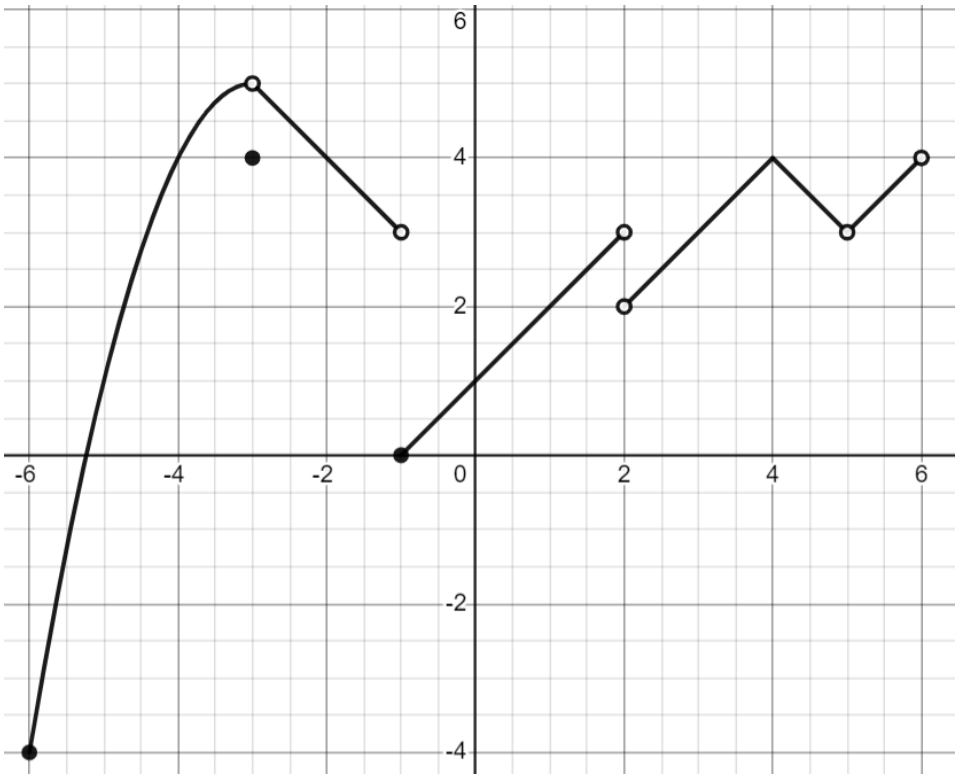
q. $\lim_{x\to 4} f(x)$

r. $\lim_{x\to 5} f(x)$

s. $\lim_{x\to 6^-} f(x)$

t. $\lim_{x\to 6^+} f(x)$

u. $\lim_{x\to 6} f(x)$



4. Use the Intermediate Value Theorem to determine if there is a solution to the equation $x^3 + 2 = 0$ on the interval $[-2, -1]$.

5. Use the Intermediate Value Theorem to determine if $f(x) = x^2 - \cos x$ has an x-intercept on the interval $[0, \pi]$.

6. Use the Intermediate Value Theorem to determine if $f(x) = \frac{x-2}{x+1}$ has a root on the interval $[1,3]$.

7. Use the Intermediate Value Theorem to determine if there is a solution to the equation $3 + \ln x = x^3$ on the interval $[1, e]$.

8. Use the Intermediate Value Theorem to determine if the equation $\sqrt{x+2} = 2x^2 + 1$ has a solution on the interval $[-1,0]$.

9. Use limits to classify all discontinuities for each of the following functions:

a. $f(x) = \frac{x+5}{x^2+8x+15}$

b. $f(x) = \frac{2}{x-1}$

c. $f(x) = \frac{x-1}{x^3-2x^2+x}$

d. $f(x) = \frac{(x-1)\sin 3x}{x(x+2)}$

e. $$f(x) = \begin{cases} x+2 & if\ x \le -1 \\ x^2+2 & if\ -1 < x < 2 \\ \frac{6}{3-x} & if\ x > 2 \end{cases}$$

f. $$f(x) = \begin{cases} \ln(-x) & if\ x \le -1 \\ x^3+1 & if\ -1 < x < 2 \\ 5x-2 & if\ x \ge 2 \end{cases}$$

g. $$f(x) = \begin{cases} \frac{x}{6} & if\ x \le -2 \\ \frac{1}{x-1} & if\ -2 < x < 3 \\ x^2-8 & if\ x \ge 3 \end{cases}$$

10. Determine all possible values of A for each of the following functions such that it is continuous for all real numbers.

a. $$f(x) = \begin{cases} \dfrac{x^2 - 7x + 12}{x - 3} & if\ x \neq 3 \\ Ax + 2 & if\ x = 3 \end{cases}$$

b. $$f(x) = \begin{cases} \dfrac{x^2 - 6x}{2x} & if\ x \neq 0 \\ -A(x + 6) & if\ x = 0 \end{cases}$$

c. $$f(x) = \begin{cases} \dfrac{x^3 - x^2}{1 - \cos x} & if\ x \neq 0 \\ 2A(x - 3) & if\ x = 0 \end{cases}$$

11. Determine all possible values of a and b such that each of the following functions is continuous for all real numbers.

a. $$f(x) = \begin{cases} x + 2 & if\ x \leq -2 \\ ax^2 + bx & if\ -2 < x < 3 \\ x^2 - 8 & if\ x \geq 3 \end{cases}$$

b. $$f(x) = \begin{cases} 3x - 4 & if\ x \leq -2 \\ 2ax^2 + 3bx & if\ -2 < x < 1 \\ 3x^2 - 8 & if\ x \geq 1 \end{cases}$$

c. $$f(x) = \begin{cases} 3x - 4b & if\ x \leq -2 \\ 2ax^2 + bx & if\ -2 < x < 1 \\ ax^2 + 1 & if\ x \geq 1 \end{cases}$$

d. $$f(x) = \begin{cases} 3x^2 + b & if\ x \leq -2 \\ 2ax^2 + bx & if\ -2 < x < 1 \\ 3ax + \ln x & if\ x \geq 1 \end{cases}$$