Directions: Do the problems. No calculators until problem 28.

1. $\lim_{x\to\infty} \frac{5x^2-3x+1}{4x^2+2x+5} =$

a. 0

b. $\frac{4}{5}$

c. $\frac{3}{11}$

d. $\frac{5}{4}$

e. ∞

2. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-7x+12}{x-4}$ when $x \neq 4$, then $f(4)$ is

a. 1

b. $\frac{8}{7}$

c. -1

d. 0

e. Undefined

3. $\lim_{x\to 0} \frac{\sin(x)\cos(x)-\sin(x)}{x^2} =$

a. 2

b. $\frac{40}{3}$

c. 0

d. ∞

e. Undefined

4. $\lim_{x\to 0} \frac{\tan^3(2x)}{x^3} =$

a. -8

b. -2

c. 2

d. 8

e. The limit does not exist

5. Determine a value k such that $f(x)$ is continuous for all real numbers x where $f(x) = \begin{cases} 3kx - 5, & x > 2 \\ 4x - 5k. & x \leq 2 \end{cases}$

a. 1

b. $\frac{13}{11}$

c. $\frac{3}{11}$

d. $-\frac{3}{11}$

e. -3

6. If $3x^2 - 2xy + 3y = 1$, then when $x = 2$, $\frac{dy}{dx} =$

a. -12

b. -10

c. $-\frac{10}{7}$

d. 12

e. 32

7. If $f(x) = x^2\sqrt{3x+1}$, $f'(x) =$

a. $\frac{-3x^2-2x}{\sqrt{3x+1}}$

b. $\frac{9x^2+2x}{\sqrt{3x+1}}$

c. $\frac{-9x^2+4x}{2\sqrt{3x+1}}$

d. $\frac{15x^2+4x}{2\sqrt{3x+1}}$

e. $\frac{-9x^2-4x}{2\sqrt{3x+1}}$

8. If $f(x) = x^4 - 2x^2 + 5x$, then $f''(3) =$

a. 109

b. 78

c. 101

d. 3

e. 104

9. If $f(x) = (x^2 - 2x + 1)^5$, $f'(x) =$

a. $10(x-1)^9$

b. $5(x^2 - 2x + 1)^9$

c. $10x(x^2 - 2x + 1)^4$

d. $(2x-2)^5$

e. $32(x-1)^5$

10. If $f(x) = \frac{\ln(x)}{e^x}$, then $f'(1) =$

a. 0

b. e

c. 1

d. $\frac{1}{e}$

e. e^2

11. On $[0,2\pi)$, how many critical values does $f(x) = \sin(2x)$ have?

a. 0

b. 1

c. 2

d. 3

e. 4

12. If $f(x) = \ln(3x^2 - 6x)$, then at $x = 3$

a. f is increasing and concave up

b. f is increasing and concave down

c. f is decreasing and concave up

d. f is decreasing and concave down

e. f does not exist

13. If $f(x) = \frac{1}{4}x^4 - \frac{9}{2}x^2$, the f is concave up on…

a. There is no interval on which f is concave up

b. $(-\infty, -3) \cup (0,3)$

c. $(-3,0) \cup (3, \infty)$

d. $\left(-\sqrt{3}, \sqrt{3}\right)$

e. $\left(-\infty, -\sqrt{3}\right) \cup \left(\sqrt{3}, \infty\right)$

14. There is a local maximum of $f(x) = e^x(x^2 - 2x + 3)$ at

a. $x = -1$

b. $x = 1$

c. $x = 0$

d. $x = 3$

e. f does not have a local maximum

15. If $f(x) = \sin^3(x^2)$, then $f'\left(\sqrt{\frac{\pi}{4}}\right) =$

a. $\frac{3\sqrt{2\pi}}{4}$

b. $3\sqrt{\pi}$

c. $\sqrt{\pi}$

d. $\frac{\sqrt{2\pi}}{4}$

e. $3\sqrt{2\pi}$

16. $\lim_{x\to -2} \frac{2x^2+7x+6}{x+2} =$

a. 0

b. Undefined

c. -1

d. 1

e. -7

17. Use the mean value theorem to find all values c on $[0,2]$ that satisfy $f'(c) = \frac{f(b)-f(a)}{b-a}$ for the function $f(x) = \frac{x^2+2}{x-1}$.

a. 8

b. 0

c. -1

d. 1

e. The Mean Value Theorem does not apply because f is not continuous on $[0,2]$

18. $\lim_{x\to -\infty} \frac{3x-2}{\sqrt{x^2-1}} =$

a. 0

b. 1

c. 3

d. -3

e. Does not exist

19. If a spherical balloon with $v = \frac{4}{3}\pi r^3$ is slowly deflating at a rate of 10 cubic inches per minute, at what rate is the radius changing when r = 5 inches?

a. $-\frac{1}{10\pi}$ inches per minute

b. -10π inches per minute

c. $-\frac{\pi}{10}$ inches per minute

d. $-\pi$ inches per minute

e. $-\frac{1}{2\pi}$ inches per minute

20. If $f(x) = \tan^{-1}(x^2)$, then $f'(2) =$

a. 0

b. $\frac{4}{\sqrt{17}}$

c. $\frac{1}{\sqrt{17}}$

d. $\frac{4}{17}$

e. $\frac{1}{17}$

21. The line tangent to the curve $x^4 + 3xy^2 + 5y^3 = 9$ at the point (1,1) is given by:

a. $y - 1 = 3(x - 1)$

b. $y - 1 = -\frac{1}{3}(x - 1)$

c. $y = -\frac{1}{3}(x - 1)$

d. $y = -3(x - 1) + 1$

e. $y = \frac{1}{3}(x - 1) + 1$

22. If $f(x) = \sqrt{x^2 + 5x + 25}(x^2 + x + 1)^3$, then $f'(0) =$

a. 5

b. $\frac{1}{50}$

c. $\frac{31}{2}$

d. $\frac{1}{2}$

e. 15

23. Which of these is the correct derivative of $y = 4^x$?

a. $\frac{e^{4x}}{4}$

b. $\frac{4^x}{\ln 4}$

c. $\frac{4^x}{2 \ln 2}$

d. $\frac{e^4}{e^x}$

e. None of these

24. If f and g are inverses and the point $(3,1)$ is on f, and $f'(3) = 7$, then $g'(1) =$

a. 7

b. $\frac{1}{7}$

c. 21

d. $\frac{1}{21}$

e. $\frac{1}{3}$

25. Find all increasing and decreasing intervals, local maxima and minima, intervals of concavity, and points of inflection of $f(x) = x - 3\sqrt[3]{x}$. Show all steps used.

26. A window is being built in the shape of a rectangle with a semicircle sitting on top. The diameter of the semicircle is the same as the width of the rectangle. If I have twelve feet of material to make the frame of the window, how wide should the window be to have the maximum window area?

27. If g is the inverse function of $f(x) = 2x + \ln(x)$, write the equation of the line tangent to $g(x)$ at $x = 2$

28. If $f'(x) = 3 - e^{\sin 3x}$, which of the following must be true on the interval $[0,2\pi)$?

I. f has four points of inflection

II. f is increasing

III. f has a local maximum at $x = \frac{\pi}{2}$

a. I only

b. II only

c. I and II

d. II and III

e. I, II, and III

29. A particle moves along a line so that its velocity is given by $v(t) = -t^3 + 2t^2 + 2^{-t}$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

a. $(0,0.177)$ and $(1.256, \infty)$

b. $(0,1.256)$

c. $(0,2.057)$

d. $(0.177,1.256)$

e. $(0.177,1.256)$ and $(2.057, \infty)$

30. Line l is tangent to the graph of $y = e^x$ at the point (k, e^k). What is the positive value of k for which the y-intercept of l is $\frac{1}{2}$?

a. 0.405

b. 0.768

c. 1.500

d. 1.560

e. There is no such value of k

31. The derivative of the function f is given by $f'(x) = x^3 - 4\sin(x^2) + 1$. On the interval $(-2.5, 2.5)$, at which of the following values of x does f has a relative maximum?

a. -1.970 and 0

b. -1.467 and 1.075

c. -0.475, 0.542, and 1.396

d. -0.475 and 1.396

e. 0.542

32. The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeters per second, what is the radius of the sphere? The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

a. 0.141 cm

b. 0.244 cm

c. 0.250 cm

d. 0.489 cm

e. 0.977 cm

33. Use the Intermediate Value Theorem to show that the function $e^x + x = 0$ has a solution. Then, use Newton's Method to find the solution accurate to at least 9 decimals. Show each iteration