1. a) $f(x) = \sin 2x \quad x = \frac{\pi}{8}$ $\quad f'(x) = 2\cos 2x$

$f\left(\frac{\pi}{8}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad f'\left(\frac{\pi}{8}\right) = 2\cos\frac{\pi}{4} = \sqrt{2}$

$$\boxed{TL: y - \frac{\sqrt{2}}{2} = \sqrt{2}\left(x - \frac{\pi}{8}\right) \qquad NL: y - \frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}\left(x - \frac{\pi}{8}\right)}$$

b) $f(x) = (2x + \sin x)^3 \quad x = 1$ $\quad f'(x) = 3(2x + \sin x)^2(2 + \cos x)$

$f(1) = (2 + \sin 1)^3$ $\quad f'(1) = 3(2 + \sin 1)^2(2 + \cos 1)$

$$\boxed{\begin{array}{l} TL: y - (2+\sin 1)^3 = 3(2+\sin 1)^2(2+\cos 1)(x-1) \\ NL: y - (2+\sin 1)^3 = -\dfrac{1}{3(2+\sin 1)^2(2+\cos 1)}(x-1) \end{array}}$$

c) $f(x) = \frac{2x+5}{x^2-1} \quad x = 2$ $\quad f'(x) = \frac{(x^2-1)(2) - (2x+5)(2x)}{(x^2-1)^2}$

$f(2) = \frac{9}{3} = 3$ $\quad f'(2) = \frac{6 - 36}{3^2} = \frac{-30}{9} = \frac{-10}{3}$

$$\boxed{TL: y - 3 = -\frac{10}{3}(x-2) \qquad NL: y - 3 = \frac{3}{10}(x-2)}$$

2. $f(x) = \sin^2 x \quad$ TL at $x = \frac{\pi}{6}$ $\quad f'(x) = 2\sin x\cos x = \sin 2x$

$f\left(\frac{\pi}{6}\right) = \sin^2\frac{\pi}{6} = \frac{1}{4}$ $\quad f'\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

TL at $x = \frac{\pi}{6}$: $y - \frac{1}{4} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$ at $x = \frac{\pi}{5}$, $y - \frac{1}{4} = \frac{\sqrt{3}}{2}\left(\frac{\pi}{5} - \frac{\pi}{6}\right)$

$f''(x) = 2\cos 2x$

$f''\left(\frac{\pi}{6}\right) = 2\cos\frac{\pi}{3} = 1 > 0$, $\boxed{\text{concave up, underestimate}}$ $\boxed{y = \frac{\sqrt{3}}{2}\left(\frac{\pi}{30}\right) + \frac{1}{4}}$

3. $f(x) = 3x^3 - 6x^2 - x - 6$ $\qquad f'(x) = 9x^2 - 12x - 1$

$f(3) = 81 - 54 - 3 - 6 = 18$ $\qquad f'(3) = 81 - 36 - 1 = 44$

TL to $f(x)$ at $x = 3$, $\quad y - 18 = 44(x-3)$

at 3.1, $\quad y - 18 = 44(3.1 - 3) \Rightarrow \boxed{y = 22.4}$

$f''(x) = 18x - 12 \Big|_{x=3} = 42 > 0$ $\boxed{\text{concave up, underestimate}}$

4. $f(x) = \tan^{-1} x$ $\qquad f'(x) = \dfrac{1}{1+x^2}$ $\qquad f'(-\sqrt{3}) = \dfrac{1}{1+(-\sqrt{3})^2} = \dfrac{1}{4}$

$f(-\sqrt{3}) = -\dfrac{\pi}{3}$

TL at $x = -\sqrt{3}$, $\quad y + \dfrac{\pi}{3} = \dfrac{1}{4}(x + \sqrt{3})$

at $x = -2$ $\quad y + \dfrac{\pi}{3} = \dfrac{1}{4}(-2 + \sqrt{3})$

$$\boxed{y = \frac{1}{4}(-2+\sqrt{3}) - \frac{\pi}{3}}$$

$f''(x) = -\dfrac{2x}{(1+x^2)^2} \Big|_{x=-\sqrt{3}} = \dfrac{2\sqrt{3}}{16} = \dfrac{\sqrt{3}}{8} > 0$ $\boxed{\text{concave up, underestimate}}$

5. $f(x) = \sin^2 x$ $\qquad \dfrac{f(\frac{\pi}{4}) - f(\frac{\pi}{6})}{\frac{\pi}{4} - \frac{\pi}{6}} = \dfrac{\frac{1}{2} - \frac{1}{4}}{\frac{\pi}{12}} = \dfrac{\frac{1}{4}}{\frac{\pi}{12}} = \dfrac{3}{\pi}$

$f(\frac{\pi}{4}) = \frac{1}{2}$

secant line ~~through~~ through $x = \frac{\pi}{4}$, $x = \frac{\pi}{6}$: $\quad y - \frac{1}{2} = \frac{3}{\pi}\left(x - \frac{\pi}{4}\right)$

at $x = \frac{\pi}{5}$, $\quad y - \frac{1}{2} = \frac{3}{\pi}\left(\frac{\pi}{5} - \frac{\pi}{4}\right)$

$y = \frac{3}{\pi}\left(-\frac{\pi}{20}\right) + \frac{1}{2} = -\frac{3}{20} + \frac{1}{2} = \boxed{\frac{7}{20}}$

$f'(x) = 2\sin x \cos x = \sin 2x$

$f''(x) = 2\cos 2x \Big|_{x=\frac{\pi}{5}}$ $\quad 2\cos\frac{2\pi}{5} > 0$ $\boxed{\text{concave up, overestimate}}$

6. $f(x) = 3x^3 - 6x^2 - x - 6$

$$\frac{f(2)-f(1)}{2-1} = \frac{24-24-2-6-(3-6-1-6)}{1} = 2$$

$f(1) = -10$ S.L. at $x=1$ $y+10 = 2(x-1)$

at $x = 1.5$, $y+10 = 2(1.5-1)$, $\boxed{y=-9}$

$f'(x) = 9x^2 - 12x - 1$

$f''(x) = 18x - 12 \big|_{x=1} = 6 > 0$, $\boxed{\text{concave up, overestimate}}$

7. $f(x) = \frac{x+1}{2x-3}$ $\quad \frac{f(3)-f(2)}{3-2} = \frac{\frac{4}{3}-3}{1} = -\frac{5}{3}$

$f(2)=3$ so S.L. is $y-3 = -\frac{5}{3}(x-2)$

at $x = 2.2$ $y-3 = -\frac{5}{3}(2.2-2)$

$y-3 = \left(-\frac{5}{3}\right)\left(\frac{1}{5}\right) \longrightarrow \boxed{y \approx \frac{8}{3}}$

$$f'(x) = \frac{2x-3-(x+1)(2)}{(2x-3)^2} = \frac{-5}{(2x-3)^2}$$

$$f''(x) = \frac{5(2)(2x-3)(2)}{(2x-3)^4} = \frac{20}{(2x-3)^3}\Big|_{x=2} = 20 > 0$$

$\boxed{\text{concave up, overestimate}}$

8. a) $\sqrt{102}$ $f(x) = \sqrt{x}$ $a = 100$ $x = 102$

$f'(x) = \frac{1}{2\sqrt{x}}$

$$\sqrt{102} \approx \sqrt{100} + \frac{1}{2\sqrt{100}}(102-100) = 10 + \frac{2}{20} = \boxed{10.1}$$

b) $\sqrt[3]{10}$ $f(x) = \sqrt[3]{x}$ $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$ $a=8$ $x=10$

$$\sqrt[3]{10} \approx \sqrt[3]{8} + \frac{1}{3}(8)^{-\frac{2}{3}}(10-8) = 2 + \frac{2}{12} = \boxed{\frac{13}{6}}$$

c) $\sqrt[4]{14}$ $x=14$ $a=16$ $f(x)=x^{\frac{1}{4}}$ $f'(x)=\frac{1}{4}x^{-\frac{3}{4}}$

$$\sqrt[4]{14} \approx 2+\frac{1}{4}(16)^{-\frac{3}{4}}(14-16) = 2+\frac{-2}{4(8)} = 2-\frac{1}{16} = \boxed{\frac{31}{16}}$$

d) $\sin\frac{\pi}{7}$ $x=\frac{\pi}{7}$ $a=\frac{\pi}{6}$ $f(x)=\sin x$ $f'(x)=\cos x$

$$\sin\frac{\pi}{7} \approx \sin\frac{\pi}{6}+\cos\frac{\pi}{6}\left(\frac{\pi}{7}-\frac{\pi}{6}\right) = \frac{1}{2}+\frac{\sqrt{3}}{2}\left(-\frac{\pi}{42}\right) = \boxed{\frac{1}{2}-\frac{\pi\sqrt{3}}{84}}$$

e) $\cos\frac{\pi}{8}$ $x=\frac{\pi}{8}$ $a=\frac{\pi}{6}$ $f(x)=\cos x$ $f'(x)=-\sin x$

$$\cos\frac{\pi}{8} \approx \cos\frac{\pi}{6}-\sin\left(\frac{\pi}{6}\right)\left(\frac{\pi}{8}-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}-\frac{1}{2}\left(-\frac{\pi}{24}\right) = \boxed{\frac{\sqrt{3}}{2}+\frac{\pi}{48}}$$

9. $A=4\pi r^2$ $dA=8\pi r\,dr \rightarrow dA=8\pi(6)(-.1)$

$r=6$ $dr=-.1$ $= \boxed{-4.8\pi\ m^2}$

10. $V=\frac{4}{3}\pi r^3$ $dV=4\pi r^2 dr = 4\pi(6^2)(.1) = \boxed{14.4\pi\ in^3}$

$r=6$ $dr=.1$

11. $V=\frac{1}{3}\pi r^2 h$ $dV=\frac{1}{3}\pi r^2 dh+\frac{1}{3}\pi(2rh)dr$

$h=4$ $dh=.2$ $dV=\frac{1}{3}\pi(r^2)(.2) \cancel{+\frac{1}{3}\pi(2)(r)(4)(0)}$

$dr=0$

$$\boxed{dV=\frac{\pi}{15}r^2\ ft^3}$$

12. $V=\frac{1}{3}\pi r^2 h$ $dV=\frac{1}{3}\pi r^2 dh+\frac{1}{3}\pi(2rh)dr$

$r=3$ $dr=-.1$ $dV=\cancel{\frac{1}{3}\pi(3^2)(0)}+\frac{1}{3}\pi(2)(3)(h)(-.1) = \boxed{-\frac{\pi}{5}h\ m^3}$

$dh=0$

13. $V = \frac{1}{3}\pi r^2 h$ $\qquad dV = \frac{1}{3}\pi r^2 dh + \frac{1}{3}\pi(2rh)dr$

$r = 6 \quad dr = .1$ $\qquad dV = \frac{1}{3}\pi(6^2)(.2) + \frac{1}{3}\pi(2)(6)(10)(.1)$

$h = 10 \quad dh = .2$

$$\frac{36(2)\pi}{30} + 4\pi = \boxed{\frac{32\pi}{5} \text{ in}^3}$$

14. a) $f(x) = 2x^3 - 3x^2 - 36x + 3$

$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x-3)(x+2) = 0$

$x = 3 \quad x = -2$

$(-)(-) \quad (-)(+) \quad (+)(+)$

$+ \quad -2 \quad - \quad 3 \quad +$

$f(-2) = -16 - 12 + 72 + 3 = 47$

$f(3) = 54 - 27 - 108 + 3 = -78$

$\boxed{\begin{array}{l} \text{inc: } (-\infty, -2] \cup [3, \infty) \\ \text{dec: } [-2, 3] \\ \text{local max of } 47 \text{ at } x = -2 \\ \text{local min of } -78 \text{ at } x = 3 \\ \text{no absolute max/min} \end{array}}$

$f''(x) = 12x - 6 = 0$ if $x = \frac{1}{2}$

$- \quad | \quad +$

down $\quad \frac{1}{2} \quad$ up

$f(\frac{1}{2}) = \frac{1}{4} - \frac{3}{4} - 18 + 3$

$= \boxed{-\frac{31}{2}}$

$\boxed{\begin{array}{l} \text{POI: } \left(\frac{1}{2}, -\frac{31}{2}\right) \\ \text{concave up: } \left(\frac{1}{2}, \infty\right) \\ \text{concave down } \left(-\infty, \frac{1}{2}\right) \end{array}}$

b) $f(x) = x^4 - 6x^3 + 12x^2 - 4$

$f'(x) = 4x^3 - 18x^2 + 24x = 0$

$2x(2x^2 - 9x + 12) = 0$

$x = 0$ no real sol.

$f(0) = -4$

$-$ | $+$ f'

0

$f''(x) = 12x^2 - 36x + 24 = 0$

$12(x^2 - 3x + 2) = 0$

$12(x-1)(x-2) = 0$

$x = 1 \quad x = 2$

$+$ | $-$ | $+$ f''

up   1   down   2   up

$f(1) = 1 - 6 + 12 - 4 = 3$

$f(2) = 16 - 48 + 48 - 4 = 12$

inc: $[0, \infty)$ dec: $(-\infty, 0]$ no ~~ab~~local max   local/abs min of -4 at $x=0$

conc up: $(-\infty, 1) \cup (2, \infty)$ conc down: $(1, 2)$ POI: $(1, 3), (2, 12)$

c) $f(x) = xe^{-x}$

$f'(x) = -xe^{-x} + e^{-x} = e^{-x}(1-x) = 0$ $x = 1$

$f(1) = \frac{1}{e}$

$+$ | $-$ f'

1

$f''(x) = xe^{-x} - e^{-x} - e^{-x} = e^{-x}(x-2) = 0$ $x = 2$

$f(2) = \frac{2}{e^2}$

$-$ | $+$ f''

down   2   up

inc: $(-\infty, 1]$ dec: $[1, \infty)$ abs/local max of $\frac{1}{e}$ at $x=1$ no abs/local min

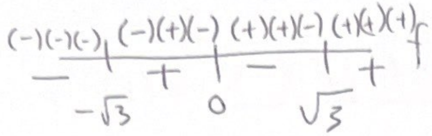
conc up: $(2, \infty)$ conc down: $(-\infty, 2)$ POI: $\left(2, \frac{2}{e^2}\right)$

d) $f(x) = \ln(x^3 - 3x)$

$x^3 - 3x > 0$

$x(x^2 - 3) > 0$

$x(x + \sqrt{3})(x - \sqrt{3}) = 0$



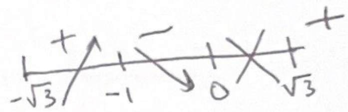
$x = 0 \quad x = -\sqrt{3} \quad x = \sqrt{3}$

domain $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

$f'(x) = \dfrac{3x^2 - 3}{x^3 - 3x} = 0$ if $3x^2 - 3 = 0$

$x = \pm 1$

$x = 1$ not in domain



$f(-1) = \ln 2$

undefined for $x = 0, -\sqrt{3}, \sqrt{3}$ (not in domain)

$f''(x) = \dfrac{(x^3 - 3x)(6x) - (3x^2 - 3)(3x^2 - 3)}{(x^3 - 3x)^2}$

$\lim_{x \to \infty} f(x) = \infty$

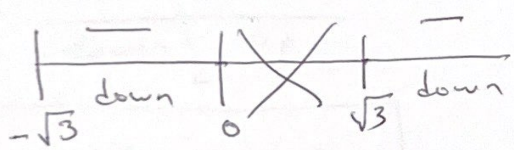
$= \dfrac{6x^4 - 18x^2 - (9x^4 - 18x^2 + 9)}{(x^3 - 3x)^2} = \dfrac{-3x^4 - 9}{(x^3 - 3x)^2} = 0 \quad x^4 = -3$

no real sol

und at

$x = 0, -\sqrt{3}, \sqrt{3}$

(not in domain)



increasing: $(-\sqrt{3}, -1] \cup (\sqrt{3}, \infty)$

decreasing: $[-1, 0)$

local max of $\ln 2$ at $x = -1$, no abs max

no local / abs minimum

conc up: never

conc down: $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

no POI

e) $f(x) = \dfrac{x^2+5}{x-2}$ domain: $(-\infty, 2) \cup (2, \infty)$

$$f'(x) = \frac{(x-2)(2x) - (x^2+5)(1)}{(x-2)^2} = \frac{2x^2-4x-x^2-5}{(x-2)^2} = \frac{x^2-4x-5}{(x-2)^2}$$

$$= \frac{(x-5)(x+1)}{(x-2)^2} = 0 \text{ if } x = 5,\ x = -1$$

und if $x = 2$ (not in domain)

f'	$(-)(-)$		$(-)(+)$		$(-)(+)$		$(+)(+)$
	$+$	-1	$-$	2	$-$	5	$+$

$f(-1) = -2$
$f(5) = 10$

$$f''(x) = \frac{(x-2)^2(2x-4) - (x-5)(x+1)(2)(x-2)}{(x-2)^4} = \frac{2x^2-8x+8-(2x^2-8x-10)}{(x-2)^3}$$

$$= \frac{18}{(x-2)^3} = 0 \text{ never}$$

und at $x = 2$ (not in domain)

f''	$-$		$+$
	down	2	up

Inc: $(-\infty, -1] \cup [5, \infty)$
dec: $[-1, 2) \cup (2, 5]$

local max of -2 at $x = -1$
local min of 10 at $x = 5$
no absolute max/min

conc up: $(2, \infty)$
conc down: $(-\infty, 2)$
no POI

15. $x(t) = (t^2 - t)e^{-t}$ for $t > 0$

a) $x(t) = 0$ if $t(t-1) = 0$

$t = 0$ $\quad$ $t = 1$

not in domain

$$v(t) = (t^2 - t)(-e^{-t}) + (2t-1)(e^{-t})$$

$$v(1) = (0)\left(-\frac{1}{e}\right) + (1)\left(\frac{1}{e}\right) = \boxed{\frac{1}{e}}$$

b) $v(3) = 6(-e^{-3}) + 5e^{-3} = \boxed{-e^{-3}}$

c) speed at $t=3$ $= |v(3)| = \boxed{e^{-3}}$

d) $a(t) = \frac{d}{dt} v(t)$ $\quad \rightarrow v(t) = e^{-t}(t - t^2 + 2t - 1)$

$$= e^{-t}(-t^2 + 3t - 1)$$

$$a(t) = e^{-t}(-2t+3) + -e^{-t}(-t^2 + 3t - 1)$$

$$= \boxed{e^{-t}(t^2 - 5t + 4)} \quad a(3) = e^{-3}(-2) = \boxed{-\frac{2}{e^3}}$$

e) $a(t) = e^{-t}(t^2 - 5t + 4) = 0$

if $(t-1)(t-4) = 0$

$$\boxed{t = 1 \quad t = 4}$$

16. $f(x) = \frac{4}{(2-x)^3}$ is continuous on $[0,1]$ which is a subset of $(-\infty, 2)$ which is a continuous piece of $f(x)$

$$f'(x) = \frac{-4(3)(2-x)^2(-1)}{(2-x)^6} = \frac{12}{(2-x)^4}$$

$f(x)$ is differentiable on $(0,1)$ which is a subset of $(-\infty, 2)$ for which $f'(x)$ exists

so, by MVT $\frac{f(1)-f(0)}{1-0} = f'(c)$

$$\frac{4-\frac{1}{2}}{1} = \frac{7}{2} = \frac{12}{(2-c)^4} \rightarrow (2-c)^4 = \frac{24}{7}$$

$$2-c = \pm\sqrt[4]{\frac{24}{7}}$$

$$c = 2 \pm \sqrt[4]{\frac{24}{7}}$$

$2+\sqrt[4]{\frac{24}{7}}$ not on $(0,1)$

$$\boxed{c = 2 - \sqrt[4]{\frac{24}{7}}}$$

17. $f(x) = \frac{2x}{(2-x)^2}$ is not continuous on $[1,3]$, infinite discontinuity at $x=2$

$\boxed{\text{MVT does not apply}}$

18. $f(x) = \frac{x-1}{x-3}$ is continuous on $[-2,1]$ which is a subset of $(-\infty, 3)$ which is a continuous piece of $f(x)$

$$f'(x) = \frac{x-3-(x-1)}{(x-3)^2} = -\frac{2}{(x-3)^2}$$

exists on $(-\infty,3)\cup(3,\infty)$ so $f(x)$ is differentiable on $(-2,1)$ which is a subset of $(-\infty,3)$

So, by MVT $\frac{f(1)-f(-2)}{1-(-2)} = \frac{-2}{(c-3)^2}$

$$\frac{0-\left(\frac{-3}{-5}\right)}{3} = -\frac{2}{(c-3)^2}$$

$$-\frac{1}{5} = \frac{-2}{(c-3)^2}$$

$$(c-3)^2 = 10$$

$$c-3 = \pm\sqrt{10}$$

$$c = 3 \pm \sqrt{10}$$

only $\boxed{c = 3-\sqrt{10}}$ is on $(-2,1)$

19. a) $f(x) = x^2 - \ln x$ $\quad f'(x) = 2x - \frac{1}{x} = 0$

$$2x^2 - 1 = 0$$
$$x^2 = \frac{1}{2}$$
$$x = \pm\frac{1}{\sqrt{2}}$$ only $\frac{1}{\sqrt{2}}$ in domain

$$f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2} - \ln\frac{1}{\sqrt{2}}$$

$$\boxed{y = \frac{1}{2} + \ln\sqrt{2}}$$

b) $f(x) = x^3 - 3x + 1$ $\quad f'(x) = 3x^2 - 3 = 0$

$$x = \pm 1$$

$f(1) = 1 - 3 + 1 = -1$

$f(-1) = -1 + 3 + 1 = 3$

$$\boxed{y = -1 \quad y = 3}$$

c) $xy - x^2y - xy^2 = 3$

$$x\frac{dy}{dx} + y - x^2\frac{dy}{dx} - 2xy - 2xy\frac{dy}{dx} - y^2 = 0$$

$$\frac{dy}{dx} = \frac{y^2 + 2xy - y}{x - x^2 - 2xy} = 0 \text{ if } y^2 + 2xy - y = 0$$

$$y(y + 2x - 1) = 0$$

$x(0) - x^2(0) - x(0) = 3 \leftarrow \cancel{y=0}$ $\quad y = 1 - 2x$

$0 = 3$ $\quad$ OR

NO $\quad x = \frac{1-y}{2}$

$$\left(\frac{1-y}{2}\right)y - \left(\frac{1-y}{2}\right)^2 y - \left(\frac{1-y}{2}\right)y^2 = 3$$

$$\frac{1}{2}y - \cancel{\frac{1}{2}y^2} - \frac{1}{4}y + \cancel{\frac{1}{2}y^2} - \frac{1}{4}y^3 - \frac{1}{2}y^2 + \frac{1}{2}y^3 = 3$$

$$\frac{1}{4}y^3 - \frac{1}{2}y^2 + \frac{1}{4}y = 3$$

$$y^3 - 2y^2 + y = 12$$

$$y^3 - 2y^2 + y - 12 = 0$$

3	1	-2	1	-12
		3	3	12
	1	1	4	0

$(y-3)(y^2+y+4) = 0$

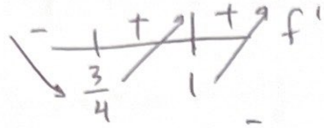
$y = 3$ $\quad$ no real sol

$$\boxed{y = 3}$$

20. a) $f(x) = 2x(x-1)^{\frac{1}{3}}$ $\qquad f'(x) = 2x\left(\frac{1}{3}\right)(x-1)^{-\frac{2}{3}} + 2(x-1)^{\frac{1}{3}}$

$f'(x) = 0$ if $x = \frac{3}{4}$

$$= \frac{2x}{3(x-1)^{\frac{2}{3}}} + \frac{2(x-1)^{\frac{1}{3}}(3)(x-1)^{\frac{2}{3}}}{3(x-1)^{\frac{2}{3}}}$$



$$= \frac{2x + 6(x-1)}{3(x-1)^{\frac{2}{3}}} = \frac{8x-6}{3(x-1)^{\frac{2}{3}}}$$

$f'(x)$ undefined if $x = 1$

no sign change of $f'(x)$ at $x=1$, not a cusp/corner $f(1)$ exists, not a discontinuity VTL: $\boxed{x = 1}$

b) $f(x) = (3-x)^{\frac{1}{3}}(x-2)^{\frac{1}{3}}$

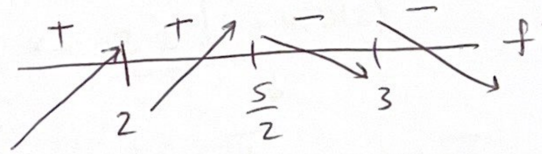
$f'(x) = (3-x)^{\frac{1}{3}}\left(\frac{1}{3}\right)(x-2)^{-\frac{2}{3}} + \left(\frac{1}{3}\right)(3-x)^{-\frac{2}{3}}(-1)(x-2)^{\frac{1}{3}}$

$$= \frac{(3-x)^{\frac{1}{3}}(3-x)^{\frac{2}{3}}}{3(x-2)^{\frac{2}{3}}(3-x)^{\frac{2}{3}}} + \frac{(-1)(x-2)^{\frac{1}{3}}(x-2)^{\frac{2}{3}}}{3(3-x)^{\frac{2}{3}}(x-2)^{\frac{2}{3}}}$$

$$= \frac{3-x-(x-2)}{3(x-2)^{\frac{2}{3}}(3-x)^{\frac{2}{3}}} = \frac{5-2x}{3(x-2)^{\frac{2}{3}}(3-x)^{\frac{2}{3}}}$$

$f'(x)$ undefied at $x=2$, $x=3$ $\qquad$ $f(2)$ and $f(3)$ exist, not discontinuities

$f'(x) = 0$ if $5-2x=0$ $\quad x = \frac{5}{2}$



$f'(x)$ does not change signs at $x=2$ or $x=3$, not corner/cusp

VTL: $\boxed{x=2}$ $\boxed{x=3}$

c) $x^2 - 2xy - y^2 = 4 \longrightarrow 2x - 2x\frac{dy}{dx} - 2y - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0$ if $x = y$

$$\frac{dy}{dx} = \frac{2y-2x}{-2x-2y} = \frac{y-x}{-x-y}$$

$\frac{dy}{dx}$ undefined if $y = -x$

$$x^2 - 2x(-x) - (-x)^2 = 4$$
$$x^2 + 2x^2 - x^2 = 4$$
$$2x^2 = 4$$
$$x = \pm\sqrt{2}$$

neither also satisfy $x = y$

VTL: $\boxed{\begin{array}{l} x = \sqrt{2} \\ x = -\sqrt{2} \end{array}}$

21. $y = x^{\frac{3}{2}} - 4 \quad (1, -4)$

distance $= z = \sqrt{\left[x^{\frac{3}{2}} - 4 - (-4)\right]^2 + (x-1)^2}$

$$z = \sqrt{\left(x^{\frac{3}{2}}\right)^2 + (x^2 - 2x + 1)} = \sqrt{x^3 + x^2 - 2x + 1}$$

$$z' = \frac{3x^2 + 2x - 2}{2\sqrt{x^3 + x^2 - 2x + 1}}$$

$$3x^2 + 2x - 2 = 0$$

$$x = \frac{-2 \pm \sqrt{4 + 4(3)(2)}}{6}$$

$$x = \frac{-2 \pm \sqrt{28}}{6} = \frac{-2 \pm 2\sqrt{7}}{6} = \frac{-1 \pm \sqrt{7}}{3}$$

only $\boxed{x = \frac{-1 + \sqrt{7}}{3}}$

22. $r = 6h \to h = \frac{r}{6}$

$\frac{dV}{dt} = 22$

$r = 10$

$\frac{dr}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 \left(\frac{r}{6}\right)$

$V = \frac{1}{18}\pi r^3$

$\frac{dV}{dt} = \frac{1}{6}\pi r^2 \frac{dr}{dt}$

$22 = \frac{1}{6}\pi (100) \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{33}{25\pi} \text{ in/sec}}$

23. $r = \frac{1}{2}h$

$\frac{dV}{dt} = 36$

$r = 6$

$\frac{dh}{dt} = ?$

$6 = \frac{1}{2}h \to h = 12$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$

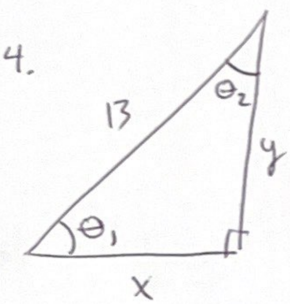
$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$36 = \frac{\pi (12^2) \frac{dh}{dt}}{4}$

$\boxed{\frac{dh}{dt} = \frac{1}{\pi} \text{ in/sec}}$

24.



a) $y = 10$

$\frac{dy}{dt} = -3$

$\frac{dx}{dt} = ?$

$x^2 + 100 = 169$

$x = \sqrt{69}$

$x^2 + y^2 = 169$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$2\sqrt{69} \frac{dx}{dt} + 2(10)(-3) = 0$

$\frac{dx}{dt} = \frac{60}{2\sqrt{69}} = \boxed{\frac{30}{\sqrt{69}} \text{ ft/sec}}$

b) $A = \frac{1}{2}xy$ $\quad \frac{dA}{dt} = \frac{1}{2}\left[x\frac{dy}{dt} + y\frac{dx}{dt}\right] = \frac{1}{2}\left[\sqrt{69}(-3) + 10\left(\frac{30}{\sqrt{69}}\right)\right]$

$= \frac{1}{2}\left[\frac{-3\sqrt{69}\sqrt{69}}{\sqrt{69}} + \frac{300}{\sqrt{69}}\right] = \frac{1}{2}\left[\frac{93}{\sqrt{69}}\right] = \boxed{\frac{93}{2\sqrt{69}} \text{ ft}^2\text{/sec}}$

c) $\sin\theta = \frac{y}{13} \to \cos\theta_1 \frac{d\theta_1}{dt} = \frac{1}{13}\frac{dy}{dt}$

$\frac{\sqrt{69}}{13}\frac{d\theta_1}{dt} = -\frac{3}{13}$

$\boxed{\frac{d\theta_1}{dt} = -\frac{3}{\sqrt{69}} \text{ rad/sec}}$

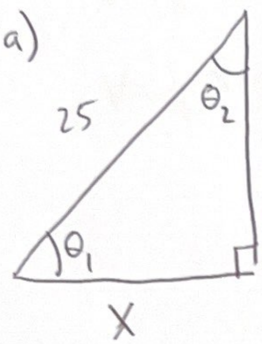
d) $\sin\theta_2 = \dfrac{x}{13}$

$\cos\theta_2 \dfrac{d\theta_2}{dt} = \dfrac{1}{13}\dfrac{dx}{dt}$

$\dfrac{10}{13}\dfrac{d\theta_2}{dt} = \dfrac{30}{13\sqrt{69}}$

$\boxed{\dfrac{d\theta_2}{dt} = \dfrac{3}{\sqrt{69}} \text{ rad/sec}}$

25. a)



$x = 7$

$\dfrac{dy}{dt} = -5$

$\dfrac{dx}{dt} = ?$

$x^2 + y^2 = 625$

$7^2 + y^2 = 625$

$y = 24$

$2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$

$2(7)\left(\dfrac{dx}{dt}\right) + 2(24)(-5) = 0$

$\boxed{\dfrac{dx}{dt} = \dfrac{120}{7} \text{ ft/sec}}$

b) $A = \frac{1}{2}xy \quad \dfrac{dA}{dt} = \dfrac{1}{2}\left[x\dfrac{dy}{dt} + y\dfrac{dx}{dt}\right] = \dfrac{1}{2}\left[7(-5) + (24)\left(\dfrac{120}{7}\right)\right]$

$= \dfrac{1}{2}\left(-35 + \dfrac{2880}{7}\right) = \boxed{\dfrac{2635}{14} \text{ ft}^2\text{/sec}}$

c) $\sin\theta_1 = \dfrac{y}{25}$

$\cos\theta_1 \dfrac{d\theta_1}{dt} = \dfrac{1}{25}\dfrac{dy}{dt}$

$\dfrac{7}{25}\dfrac{d\theta_1}{dt} = \dfrac{-5}{25}$

$\boxed{\dfrac{d\theta_1}{dt} = -\dfrac{5}{7} \text{ rad/sec}}$

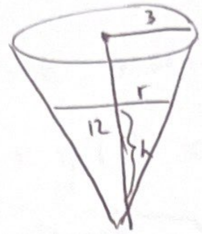
d) $\sin\theta_2 = \dfrac{x}{25}$

$\cos\theta_2 \dfrac{d\theta_2}{dt} = \dfrac{1}{25}\dfrac{dx}{dt}$

$\dfrac{24}{25}\dfrac{d\theta_2}{dt} = \dfrac{1}{25}\left(\dfrac{120}{7}\right)$

$\boxed{\dfrac{d\theta_2}{dt} = \dfrac{5}{7} \text{ rad/sec}}$

26. $\frac{dV}{dt} = 5$ $\quad h = 7$



$\frac{r}{h} = \frac{3}{12}$ $\quad h = 4r$

$r = \frac{h}{4}$

$h_{cone} = 12$

$r_{cone} = 3$

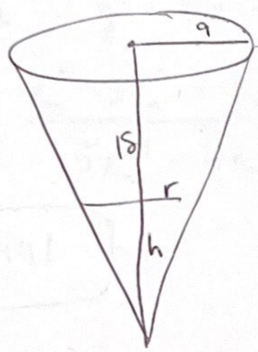
$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 (h) = \frac{\pi}{48} h^3 \longrightarrow \frac{dV}{dt} = \frac{\pi h^2}{16}\frac{dh}{dt}$

$5 = \frac{\pi(7^2)}{16}\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{80}{49\pi} \text{ ft/min}}$

27. $\frac{dV}{dt} = -3$



$\frac{r}{h} = \frac{9}{18}$ $\quad h = 2r \rightarrow r = \frac{h}{2}$

$h = 12$

$\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2(h)$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$-3 = \frac{144\pi}{4}\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = -\frac{1}{12\pi} \text{ ft/min}}$

28. $y = x^3 - 3x^2 + 1 \quad (2,1)$

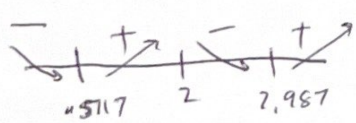
distance $= z = \sqrt{(x^3-3x^2+1-1)^2 + (x-2)^2}$ $\qquad z = \sqrt{x^6 - 6x^5 + 9x^4 + x^2 - 4x + 4}$

$\frac{dz}{dt} = \frac{6x^5 - 30x^4 + 36x^3 + 2x - 4}{2\sqrt{x^6 - 6x^5 + 9x^4 + x^2 - 4x + 4}} = 0$

$x \approx .51167038$

$x \approx 2$

$x = 2.9874987$



$z(.51167038) = 1.624662$

$z(2.9874987) \approx .99378211$

$\boxed{x \approx 2.9874987}$

29. a) $y(t) = \ln(t^2+1) - t + \sin t$

$v(t) = y'(t) \rightarrow v(1) = y'(1) \approx \boxed{.54030205}$

b) $v(4) = y'(4) \approx \boxed{-1.183055}$

c) speed $= |v(t)| \rightarrow |v(4)| = \boxed{1.183055}$

d) $y(t) = 0 \quad t \approx 2.5697983 \quad v(2.5697983) \approx \boxed{-1.165012}$

e) $a(t) = v'(t) \rightarrow v(t) = \dfrac{2t}{t^2+1} - 1 + \cos t$

$a(2.5697983) = v'(2.5697983) \approx \boxed{-.7349842}$

30. $y = x^5 - 3x^2 + x + 2 \qquad y' = 5x^4 - 6x + 1$

$$x_{n+1} = x_n - \frac{x_n^5 - 3x_n^2 + x_n + 2}{5x_n^4 - 6x_n + 1}$$

$\boxed{-.6441781941}$

$x_0 = -.5$
$x_1 = -.\overline{6}$
$x_2 = -.6446735395$
$x_3 = -.6441784371$
$x_4 = -.6441781941$
$x_5 =$ same as x_4

31. $y = 1 + x + x^3 - \sin x \qquad y' = 1 + 3x^2 - \cos x$

$$x_{n+1} = x_n - \frac{1 + x_n + x_n^3 - \sin x_n}{1 + 3x_n^2 - \cos x_n}$$

$\boxed{x = -.9519285787}$

$x_0 = -.5$
$x_1 = -1.47937696$
$x_2 = -1.115299447$
$x_3 = -.9744321808$
$x_4 = -.952438911$
$x_5 = -.9519288493$
$x_6 = -.9519285787$
$x_7 = x_6$

32. $V = \pi r^2 h = 1000$ $\qquad C = 19\pi r^2 + 32\pi r^2 + 15(2\pi r h)$

$h = \frac{1000}{\pi r^2}$ $\qquad C = 51\pi r^2 + 30\pi r\left(\frac{1000}{\pi r^2}\right)$

$$C = 51\pi r^2 + 30000 r^{-1}$$

$$C' = 102\pi r - \frac{30000}{r^2} = \frac{102\pi r^3 - 30000}{r^2} = 0$$

if $102\pi r^3 = 30000$

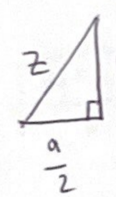
$$\boxed{r \approx 4.540709696 \text{ inches}}$$

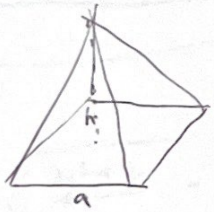
$C'' = 102\pi + \frac{60000}{r^3} > 0$, min

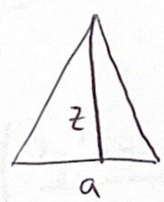
$$h = \frac{1000}{\pi(4.540709696)^2}$$

$$\boxed{h = 15.43841296 \text{ inches}}$$

33. $V = \frac{a^2 h}{3} = 500$ $\qquad h = \frac{1500}{a^2}$







← one piece of pyramid

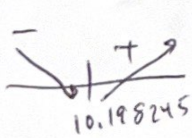
$z = \sqrt{\frac{a^2}{4} + h^2}$

$z^2 = \frac{a^2}{4} + h^2$ $\qquad$ Area $= \frac{1}{2} a \sqrt{\frac{a^2}{4} + h^2}$

surface area $=$ $S = a^2 + 4\left(\frac{1}{2} a \sqrt{\frac{a^2}{4} + h^2}\right) = a^2 + 2a\sqrt{\frac{a^2}{4} + \left(\frac{1500}{a^2}\right)^2}$

$$S = a^2 + 2a\sqrt{\frac{a^6 + 9000000}{4a^4}} = a^2 + \frac{\sqrt{a^6 + 9000000}}{a}$$

$$S' = 2a + \frac{\frac{a(6a^5)}{2\sqrt{a^6 + 9000000}} - \sqrt{a^6 + 9000000}}{a^2} = 2a + \frac{3a^6 - (a^6 + 9000000)}{a^2\sqrt{a^6 + 9000000}}$$



$$= \frac{2a^3\sqrt{a^6 + 9000000} + 2a^6 - 9000000}{a^2\sqrt{a^6 + 9000000}} = 0$$

$\boxed{a \approx 10.198245 \text{ cm}}$

$h = \frac{1500}{a^2} \approx \boxed{h = 14.42249433 \text{ cm}}$

34. $xy + 2xy^2 - 2x^3 = 1$ $\qquad x\frac{dy}{dx} + y + 2y^2 + 4xy\frac{dy}{dx} - 6x^2 = 0$

$$\frac{dy}{dx} = \frac{6x^2 - 2y^2 - y}{x + 4xy} = 0 \text{ if } 6x^2 - 2y^2 - y = 0$$

$$6x^2 = 2y^2 + y$$

$$x = \pm\sqrt{\frac{2y^2+y}{6}}$$

$\frac{dy}{dx}$ undefined if

$x + 4xy = 0$

$x(1+4y) = 0$

$x = 0 \qquad y = -\frac{1}{4}$

$\downarrow$

not in domain

$-\frac{1}{4}x + 2x\left(\frac{1}{16}\right) - 2x^3 = 1$

$-\frac{1}{4}x + \frac{1}{8}x - 2x^3 = 1$

$2x^3 + \frac{1}{8}x + 1 = 0$

VTL: $\boxed{x \approx -.7674621}$

$$y\sqrt{\frac{2y^2+y}{6}} + 2y^2\sqrt{\frac{2y^2+y}{6}} - 2\left(\frac{2y^2+y}{6}\right)^{\frac{3}{2}} = 1$$

HTL: $\boxed{\begin{array}{l} y \approx -1.369398 \\ y \approx .86939751 \end{array}}$

$$-y\sqrt{\frac{2y^2+y}{6}} + (-2y^2)\sqrt{\frac{2y^2+y}{6}} - 2\left(-\sqrt{\frac{2y^2+y}{6}}\right)^3 = 1$$

no solution

35. $xe^x + x^2 + y^2 = 1$

$$xe^x + e^x + 2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{xe^x + e^x + 2x}{2y} = -\frac{e^x(x+1) + 2x}{2y}$$

$\frac{dy}{dx}$ undefined if $y = 0$

$$e^x(x+1) + 2x = 0$$

$$x \approx -.2752084 = A$$

$$xe^x + x^2 = 1$$

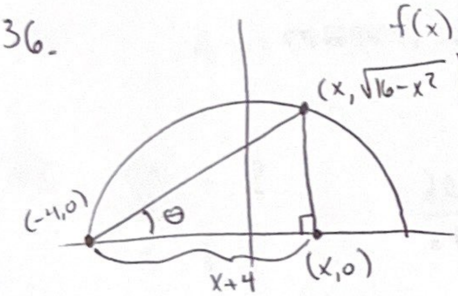
$$Ae^A + A^2 + y^2 = 1$$

$$\boxed{\text{VTL: } x \approx -1.167586 \quad x \approx .4781724}$$

$$\text{HTL: } \boxed{y = 1.06454563348 \quad y = -1.06454563348}$$

36. $f(x) = \sqrt{16 - x^2}$ $\qquad \frac{dx}{dt} = 3$

$x = 2$



$$\tan\theta = \frac{\sqrt{16-x^2}}{x+4}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{\frac{(x+4)(-2x)}{2\sqrt{16-x^2}} - \sqrt{16-x^2}}{(x+4)^2}\frac{dx}{dt}$$

$1 + \tan^2\theta = \sec^2\theta$

$$1 + \left(\frac{\sqrt{16-x^2}}{x+4}\right)^2$$

at $x = 2$

$$1 + \frac{12}{36} = 1 + \frac{1}{3}$$

$$\frac{d\theta}{dt} = \frac{\left(\frac{\frac{6(-4)}{2\sqrt{12}} - \sqrt{12}}{36}\right)(3)}{1 + \frac{1}{3}}$$

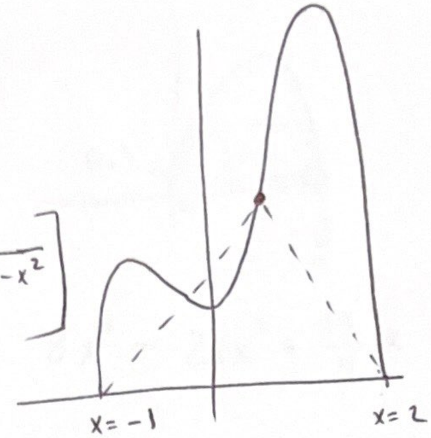
$$\approx \boxed{-.4330127019 \text{ rad/minute}}$$

37. $f(x) = (2x^2+1)\sqrt{2+x-x^2}$

a)

$A = \frac{1}{2}(3)f(x)$

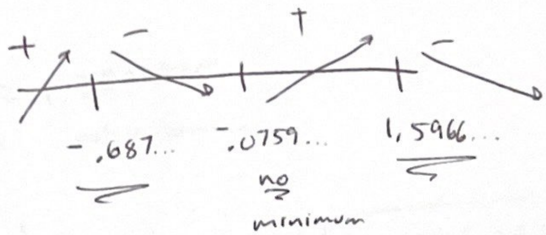
$A' = \frac{3}{2}\left[\frac{(2x^2+1)(1-2x)}{2\sqrt{2+x-x^2}} + 4x\sqrt{2+x-x^2}\right]$



$A' = 0 \quad x \approx -.6874417$

$x \approx -.0759208$

$x \approx 1.5966959$



$A = \frac{3}{2}(2x^2+1)\sqrt{2+x-x^2}$

$A(-.6874417) \approx 2.6741138$

$A(1.5966959) \approx \boxed{9.3619841}$

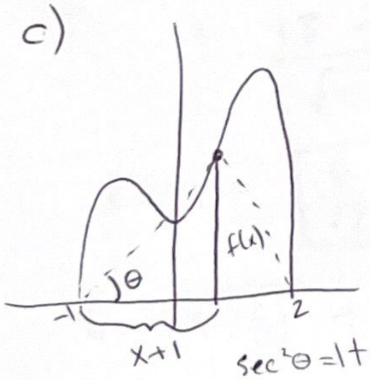
b) $\frac{dx}{dt} = 2$

$x = 1$

$\frac{dA}{dt} = \frac{3}{2}\left[\frac{(2x^2+1)(1-2x)}{2\sqrt{2+x-x^2}} + 4x\sqrt{2+x-x^2}\right]\frac{dx}{dt}$

$\frac{dA}{dt} = \frac{3}{2}\left[\frac{3(-1)}{2\sqrt{2}} + 4\sqrt{2}\right](2) = \boxed{\frac{39\sqrt{2}}{4} \approx 13.78858223 \text{ units}^2/\text{sec}}$

c)



$\tan\theta = \frac{(2x^2+1)\sqrt{2+x-x^2}}{x+1}$

$\sec^2\theta \frac{d\theta}{dt} = \left(\frac{dx}{dt}\right)\left[\frac{(x+1)\left[\frac{(2x^2+1)(1-2x)}{2\sqrt{2+x-x^2}} + 4x\sqrt{2+x-x^2}\right] - (2x^2+1)\sqrt{2+x-x^2}}{(x+1)^2}\right]$

$\sec^2\theta = 1 + \tan^2\theta$

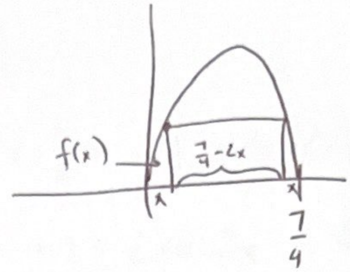
at $x = \frac{1}{2}$, $1 + \left(\frac{\left(\frac{1}{2}+1\right)\sqrt{\frac{9}{4}}}{\frac{3}{2}}\right)^2 = \frac{13}{4}$

$\frac{d\theta}{dt} = \frac{4}{13}(2)\left[\frac{\frac{3}{2}(0+3) - \left(\frac{3}{2}\right)\left(\frac{3}{2}\right)}{\frac{9}{4}}\right] = \boxed{\frac{8}{13} \text{ rad/sec}}$

38. $f(x) = 7x - 4x^2 = 0$

$x(7-4x) = 0$

$0 < x < \frac{7}{8}$ $\qquad x = 0 \quad x = \frac{7}{4}$

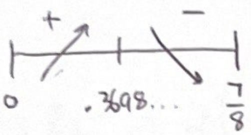


$$A = \left(\frac{7}{4} - 2x\right)\left(7x - 4x^2\right) = 8x^3 - 21x^2 + \frac{49}{4}x$$

$$A' = 24x^2 - 42x + \frac{49}{4} = 0$$

$$x \approx .36981851, \quad x \approx 1.3801815$$

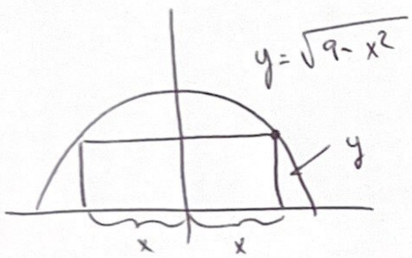
not in domain



$$l = \frac{7}{4} - 2x \Big|_{x = .36981851} = \boxed{1.01036298}$$

$$h = 7x - 4x^2 \Big|_{x = .36981851} = \boxed{2.041666649}$$

39.



$$A = 2x\sqrt{9-x^2}$$

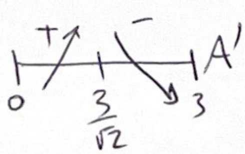
$$A' = \frac{2x(-2x)}{2\sqrt{9-x^2}} + 2\sqrt{9-x^2}$$

$$A' = \frac{-2x^2}{\sqrt{9-x^2}} + \frac{2\sqrt{9-x^2}\,\sqrt{9-x^2}}{\sqrt{9-x^2}} = \frac{-2x^2 + 2(9-x^2)}{\sqrt{9-x^2}}$$

$$= \frac{18 - 4x^2}{\sqrt{9-x^2}} = 0$$

$18 - 4x^2 = 0$

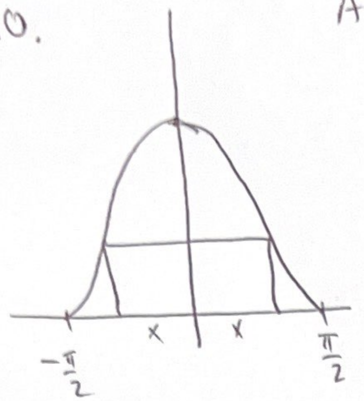
$x^2 = \frac{9}{2} \qquad x = \frac{3}{\sqrt{2}}$



$$l = 2x = \boxed{\frac{6}{\sqrt{2}}}$$

$$h = \sqrt{9 - \left(\frac{3}{\sqrt{2}}\right)^2} = \sqrt{9 - \frac{9}{2}} = \boxed{\frac{3}{\sqrt{2}}}$$

40.

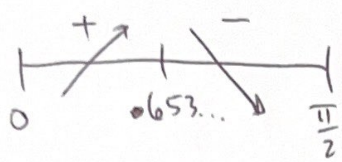


$A = 2x(1+\cos 2x)$

$A' = 2x(-2\sin 2x) + 2(1+\cos 2x) = 0$

$-4x\sin 2x + 2 + 2\cos 2x = 0$ on $(0, \frac{\pi}{2})$

$x \approx .65327119$



$l = 2x = 2(.65327119) = \boxed{1.30654238}$

$h = 1 + \cos(2(.65327119)) = \boxed{1.261189185}$