1. a) $\lim_{x\to\infty} \frac{e^{7x}}{7x^5} = \frac{\infty \text{ faster}}{\infty} = \boxed{\infty}$

b) $\lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty \text{ faster}} = \boxed{0}$

c) $\lim_{x\to\infty} \frac{x^3}{2^x} = \frac{\infty}{\infty \text{ faster}} = \boxed{0}$

d) $\lim_{x\to\infty} \frac{\sqrt{x}}{x^3} = \frac{\infty}{\infty \text{ faster}} = \boxed{0}$

e) $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{3x-5} = \lim_{x\to\infty} \frac{|x|}{3x-5} = \lim_{x\to\infty} \frac{x}{3x-5} \overset{LH.}{=} \lim_{x\to\infty} \frac{1}{3} = \boxed{\frac{1}{3}}$

f) $\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{3x-5} = \lim_{x\to-\infty} \frac{|x|}{3x} = \lim_{x\to-\infty} \frac{-x}{3x} = \lim_{x\to-\infty} -\frac{1}{3} = \boxed{-\frac{1}{3}}$

g) $\lim_{x\to\infty} \frac{5x+1}{\sqrt{16x^2-2}} = \lim_{x\to\infty} \frac{5x}{|4x|} = \lim_{x\to\infty} \frac{5x}{4x} = \lim_{x\to\infty} \frac{5}{4} = \boxed{\frac{5}{4}}$

h) $\lim_{x\to-\infty} \frac{5x+1}{\sqrt{16x^2-2}} = \lim_{x\to-\infty} \frac{5x}{|4x|} = \lim_{x\to-\infty} \frac{5x}{-4x} = \lim_{x\to-\infty} -\frac{5}{4} = \boxed{-\frac{5}{4}}$

i) $\lim_{x\to3} \frac{x^2-9}{x^2-7x+12} = \frac{0}{0}$ LH $\lim_{x\to3} \frac{2x}{2x-7} = \frac{6}{6-7} = \boxed{-6}$

j) $\lim_{x\to-3} \frac{x^2-9}{x^2-7x+12} = \frac{0}{42} = \boxed{0}$

k) $\lim_{x\to-5} \frac{x^3+7x^2+10x}{x^2-2x-35} = \frac{0}{0}$ LH $\lim_{x\to-5} \frac{3x^2+14x+10}{2x-2} = \frac{75-70+10}{-12} = \boxed{-\frac{5}{4}}$

l) $\lim_{x\to-4} \frac{x^2-3x-4}{x^2-4x} = \frac{24}{32} = \boxed{\frac{3}{4}}$

m) $\lim_{x\to4} \frac{x^2-3x-4}{x^2-4x} = \frac{0}{0}$ LH $\lim_{x\to4} \frac{2x-3}{2x-4} = \boxed{\frac{5}{4}}$

n) $\lim_{x\to6} \frac{x^2+36}{x^2-8x+12} = \frac{72}{0}$ $\boxed{\text{DNE}}$

o) $\lim_{x\to 0} \frac{\sin^2 x}{3x^2} = \frac{0}{0} = \frac{1}{3} \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = \frac{1}{3}\left[\lim_{x\to 0} \frac{\sin x}{x}\right]^2 = \frac{1}{3}(1)^2 = \boxed{\frac{1}{3}}$

p) $\lim_{x\to 0} \frac{\sin^5(4x^2)}{3x^{10}} = \frac{0}{0} = \frac{1}{3} \lim_{x\to 0}\left(\frac{\sin 4x^2}{x^2}\right)^5 = \frac{1}{3}\lim_{x\to 0}\left(\frac{4\sin 4x^2}{4x^2}\right)^5$

$= \frac{4^5}{3}\left[\lim_{x\to 0} \frac{\sin 4x^2}{4x^2}\right]^5 = \frac{1024}{3}(1^5) = \boxed{\frac{1024}{3}}$

q) $\lim_{x\to 0} \frac{x^3 - 5\sin 2x}{3x} = \frac{0}{0}$ LH $\lim_{x\to 0} \frac{3x^2 - 10\cos 2x}{3} = \frac{0 - 10(1)}{3} = \boxed{\frac{-10}{3}}$

r) $\lim_{x\to 0} \frac{1-\cos 7x}{4x} = \frac{0}{0}$ LH $\lim_{x\to 0} \frac{7\sin 7x}{4} = \frac{0}{4} = \boxed{0}$

s) $\lim_{x\to 0} \frac{\tan 12x}{5x} = \frac{0}{0}$ LH $\lim_{x\to 0} \frac{12\sec^2 12x}{5} = \frac{12(1)}{5} = \boxed{\frac{12}{5}}$

t) $\lim_{x\to 0} \frac{\tan^4(3x^2)}{5x^8} = \frac{1}{5}\lim_{x\to 0}\left(\frac{\tan 3x^2}{x^2}\right)^4 = \frac{1}{5}\lim_{x\to 0}\left(\frac{3\tan 3x^2}{3x^2}\right)^4$

$= \frac{3^4}{5}\left[\lim_{x\to 0}\frac{\tan 3x^2}{3x^2}\right]^4 = \frac{81}{5}(1^4) = \boxed{\frac{81}{5}}$

u) $\lim_{x\to 0} \frac{\sin 8x}{\sin 3x} = \frac{0}{0}$ LH $\lim_{x\to 0} \frac{8\cos 8x}{3\cos 3x} = \frac{8(1)}{3(1)} = \boxed{\frac{8}{3}}$

v) $\lim_{x\to 0} \frac{\tan^5(2x)}{\sin^5(3x)} = \left[\lim_{x\to 0} \frac{\tan 2x}{\sin 3x}\right]^5 = \left(\frac{0}{0}\right)^5$ LH $\left[\lim_{x\to 0} \frac{2\sec^2 2x}{3\cos 3x}\right]^5 = \left(\frac{2}{3}\right)^5 = \boxed{\frac{32}{243}}$

w) $\lim_{x\to 0} \frac{3x + \sin 2x - \cos x + 1}{x + 1 - \cos x} = \frac{0}{0}$ LH $\lim_{x\to 0} \frac{3 + 2\cos 2x + \sin x}{1 + \sin x} = \frac{3+2+0}{1+0} = \boxed{5}$

x) $\lim_{x\to 3^-} \frac{|x-3|}{x-3}$ $x<3$ so $x-3<0$ so $\lim_{x\to 3^-} \frac{-(x-3)}{x-3} = \lim_{x\to 3^-} -1 = \boxed{-1}$

y) $\lim_{x\to -4^+} \frac{|x+4|}{x+4}$ $x>-4$ so $x+4>0$ so $\lim_{x\to -4^+} \frac{x+4}{x+4} = \lim_{x\to -4^+} 1 = \boxed{1}$

z) $\lim_{x\to 2^+} \frac{2-x}{|x-2|}$ $x>2$ so $x-2>0$ so $\lim_{x\to 2^+} \frac{2-x}{x-2} = \lim_{x\to 2^+} \frac{-(x-2)}{x-2} = \lim_{x\to 2^+} -1 = \boxed{-1}$

aa) $\lim_{x\to 2} \dfrac{\frac{4}{x^2+1} - \frac{4}{x+3}}{2-x} = \lim_{x\to 2} \dfrac{\frac{4(x+3)-4(x^2+1)}{(x^2+1)(x+3)}}{2-x} = \lim_{x\to 2} \dfrac{-4x^2+4x+8}{(x^2+1)(x+3)(2-x)}$

$= \lim_{x\to 2} \dfrac{-4(x-2)(x+1)}{-(x^2+1)(x+3)(x-2)} = \lim_{x\to 2} \dfrac{4(x+1)}{(x^2+1)(x+3)} = \dfrac{4(3)}{(5)(5)} = \boxed{\dfrac{12}{25}}$

bb) $\lim_{x\to 3} \dfrac{\frac{2}{x^2+1} - \frac{1}{x+2}}{x-3} = \lim_{x\to 3} \dfrac{\frac{2(x+2)-1(x^2+1)}{(x^2+1)(x+2)}}{x-3} = \lim_{x\to 3} \dfrac{-x^2+2x+3}{(x^2+1)(x+2)(x-3)}$

$= \lim_{x\to 3} \dfrac{-(x-3)(x+1)}{(x^2+1)(x+2)(x-3)} = \lim_{x\to 3} \dfrac{-(x+1)}{(x^2+1)(x+2)} = \dfrac{-4}{(10)(5)} = \boxed{-\dfrac{2}{25}}$

cc) $\lim_{x\to 3} \dfrac{\frac{4}{x^2-3} - \frac{2}{x}}{x-3} = \lim_{x\to 3} \dfrac{4x-2(x^2-3)}{x(x^2-3)(x-3)} = \lim_{x\to 3} \dfrac{-2x^2+4x+6}{x(x^2-3)(x-3)}$

$= \lim_{x\to 3} \dfrac{-2(x-3)(x+1)}{x(x^2-3)(x-3)} = \lim_{x\to 3} \dfrac{-2(x+1)}{x(x^2-3)} = \dfrac{-2(4)}{3(6)} = \boxed{-\dfrac{4}{9}}$

dd) $\lim_{x\to 5} \dfrac{\frac{4}{x+3} - \frac{2}{x-1}}{2x-10} = \lim_{x\to 5} \dfrac{4(x-1)-2(x+3)}{2(x+3)(x-1)(x-5)} = \lim_{x\to 5} \dfrac{2x-10}{2(x+3)(x-1)(x-5)}$

$= \lim_{x\to 5} \dfrac{1}{(x+3)(x-1)} = \dfrac{1}{(8)(4)} = \boxed{\dfrac{1}{32}}$

ee) $\lim_{x\to 3} \dfrac{\frac{3}{x^2+3} - \frac{1}{x+1}}{x-3} = \lim_{x\to 3} \dfrac{3(x+1)-1(x^2+3)}{(x^2+3)(x+1)(x-3)} = \lim_{x\to 3} \dfrac{-x^2+3x}{(x^2+3)(x+1)(x-3)}$

$= \lim_{x\to 3} \dfrac{-x(x-3)}{(x^2+3)(x+1)(x-3)} = \lim_{x\to 3} \dfrac{-x}{(x^2+3)(x+1)} = \dfrac{-3}{(12)(4)} = \boxed{-\dfrac{1}{16}}$

ff) $\lim_{x\to \infty} e^{7x-2} = e^{\infty} = \boxed{\infty}$

gg) $\lim_{x\to \infty} e^{-x^3+x^2+x+1} = e^{-\infty} = \boxed{0}$

hh) $\lim_{x\to -\infty} e^{x^2-x} = e^{\infty} = \boxed{\infty}$

ii) $\lim_{x\to-\infty} e^{x^5+x+2} = e^{-\infty} = \boxed{0}$

jj) $\lim_{x\to\infty} \ln(5x-7) = \ln\infty = \boxed{\infty}$

kk) $\lim_{x\to-\infty} \ln(2x^2+8x-3) = \ln\infty = \boxed{\infty}$

ll) $\lim_{x\to-\infty} \ln(x^3+x^2+x+1) = \ln(-\infty)$ $\boxed{\text{DNE}}$

mm) $\lim_{x\to\infty} \ln\left(\frac{1}{x^2+1}\right) = \lim_{u\to0^+} \ln u = \boxed{-\infty}$

$u = \frac{1}{x^2+1}$ $\quad x\to\infty$, $u\to0^+$

nn) $\lim_{x\to-\infty} \ln\left(\frac{3}{x^2+1}\right)$ $\quad u=\frac{3}{x^2+1}$ $\quad x\to-\infty, u\to0^+$ $\quad \lim_{u\to0^+} \ln u = \boxed{-\infty}$

oo) $\lim_{x\to\infty} \ln\left(\frac{-3}{x^3-1}\right)$ $\quad u=\frac{-3}{x^3-1}$ $\quad x\to\infty, u\to0^-$ $\quad \lim_{u\to0^-} \ln u$ $\boxed{\text{DNE}}$

pp) $\lim_{x\to-\infty} \ln\left(\frac{5}{x^4-x}\right)$ $\quad u=\frac{5}{x^4-4}$ $\quad x\to-\infty, u\to0^+$ $\quad \lim_{u\to0^+} \ln u = \boxed{-\infty}$

qq) $\lim_{x\to0} x^2\sin\frac{1}{x}$ $\quad -1\le\sin\frac{1}{x}\le1$ $\quad -x^2\le x^2\sin\frac{1}{x}\le x^2$

$\lim_{x\to0} -x^2 = 0$

$\lim_{x\to0} x^2 = 0$

by S.T. $\lim_{x\to0} x^2\sin\frac{1}{x} = \boxed{0}$

rr) $\lim_{x\to0} \sqrt[3]{x}\cos\frac{2}{x^2}$ $\quad -1\le\cos\frac{2}{x^2}\le1$

if $x<0$

$-\sqrt[3]{x} \ge \sqrt[3]{x}\cos\frac{2}{x^2} \ge \sqrt[3]{x}$

$\lim_{x\to0^-} -\sqrt[3]{x} = 0$

$\lim_{x\to0^-} \sqrt[3]{x} = 0$

by S.T. $\lim_{x\to0^-} \sqrt[3]{x}\cos\frac{2}{x^2} = 0$

if $x>0$

$-\sqrt[3]{x} \le \sqrt[3]{x}\cos\frac{2}{x^2} \le \sqrt[3]{x}$

$\lim_{x\to0^+} -\sqrt[3]{x} = 0$

$\lim_{x\to0^+} \sqrt[3]{x} = 0$

by S.T. $\lim_{x\to0^+} \sqrt[3]{x}\cos\frac{2}{x^2} = 0$

so $\lim_{x\to0} \sqrt[3]{x}\cos\frac{2}{x^2} = \boxed{0}$

ss) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \frac{0}{0}$ LH $\lim_{x\to 4} \frac{\frac{1}{2\sqrt{x}}}{1} = \frac{1}{2\sqrt{4}} = \boxed{\frac{1}{4}}$

tt) $\lim_{x\to 7} \frac{\sqrt{x+2}+x-10}{x^2-49} = \frac{0}{0}$ LH $\lim_{x\to 7} \frac{\frac{1}{2\sqrt{x+2}}+1}{2x} = \frac{\frac{1}{6}+1}{14}$

$= \frac{\frac{7}{6}}{14} = \boxed{\frac{1}{12}}$

uu) $\lim_{x\to 1} \frac{x^{\frac{1}{5}}-1}{x-1} = \frac{0}{0}$ LH $\lim_{x\to 1} \frac{\frac{1}{5}x^{-\frac{4}{5}}}{1} = \boxed{\frac{1}{5}}$

vv) $\lim_{x\to 1} \frac{x^{\frac{2}{3}}+x-2}{x^2-5x+4} = \frac{0}{0}$ LH $\lim_{x\to 1} \frac{\frac{2}{3}x^{-\frac{1}{3}}+1}{2x-5} = \frac{\frac{2}{3}+1}{2-5} = \boxed{-\frac{5}{9}}$

ww) $\lim_{x\to 6} \frac{\sqrt[3]{x+2}-2}{2x-12} = \frac{0}{0}$ LH $\lim_{x\to 6} \frac{\frac{1}{3}(x+2)^{-\frac{2}{3}}}{2} = \lim_{x\to 6} \frac{1}{6(x+2)^{2/3}}$

$= \frac{1}{6(8)^{2/3}} = \frac{1}{6(4)} = \boxed{\frac{1}{24}}$

xx) $\lim_{x\to 3} \frac{(\sqrt[3]{x-2})^2-5\sqrt[3]{x-2}+4}{(\sqrt[3]{x-2})^2-3\sqrt[3]{x-2}+2}$ $u=\sqrt[3]{x-2}$, $x\to 3$, $u\to 1$ $\lim_{u\to 1} \frac{u^2-5u+4}{u^2-3u+2} = \lim_{u\to 1} \frac{(u-4)(u-1)}{(u-2)(u-1)}$

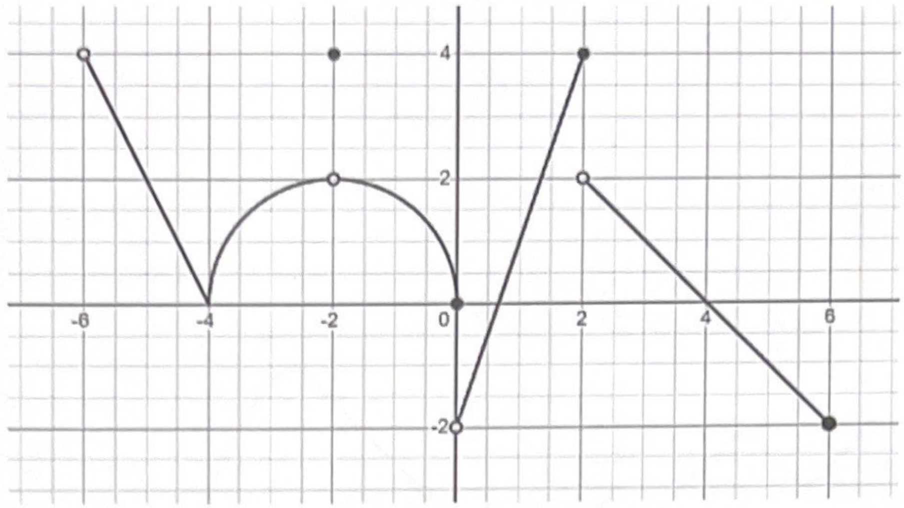
$= \lim_{u\to 1} \frac{u-4}{u-2} = \frac{-3}{-1} = \boxed{3}$

yy) $\lim_{x\to 5} \frac{x^3-125}{5\sin \pi x + x - 5} = \frac{0}{0}$ LH $\lim_{x\to 5} \frac{3x^2}{5\pi\cos \pi x + 1} = \boxed{\frac{75}{1-5\pi}}$

zz) $\lim_{x\to 0} \frac{1-3\cos x+3\cos^2 x-\cos^3 x}{2x^3} = \frac{1}{2}\lim_{x\to 0} \frac{(1-\cos x)^3}{x^3}$

$= \frac{1}{2}\left[\lim_{x\to 0} \frac{1-\cos x}{x}\right]^3 = \frac{1}{2}\left[0^3\right] = \boxed{0}$

2. Use the given graph to determine each of the following limits:



a. $\lim_{x \to -6^-} f(x)$ DNE

b. $\lim_{x \to -6^+} f(x)$ 4

c. $\lim_{x \to -6} f(x)$ DNE

d. $\lim_{x \to -5} f(x)$ 2

e. $\lim_{x \to -4} f(x)$ 0

f. $\lim_{x \to -2^-} f(x)$ 2

g. $\lim_{x \to -2^+} f(x)$ 2

h. $\lim_{x \to -2} f(x)$ 2

i. $\lim_{x \to 0^-} f(x)$ 0

j. $\lim_{x \to 0^+} f(x)$ −2

k. $\lim_{x \to 0} f(x)$ DNE

l. $\lim_{x \to 1} f(x)$ 1

m. $\lim_{x \to 2^-} f(x)$ 4

n. $\lim_{x \to 2^+} f(x)$ 2

o. $\lim_{x \to 2} f(x)$ DNE

p. $\lim_{x \to 3} f(x)$ 1

q. $\lim_{x \to 4} f(x)$ 0

r. $\lim_{x \to 5} f(x)$ −1

s. $\lim_{x \to 6^-} f(x)$ −2

t. $\lim_{x \to 6^+} f(x)$ DNE

u. $\lim_{x \to 6} f(x)$ DNE

3. Use the given graph to determine each of the following limits:

a. $\lim_{x \to -6^-} f(x)$ DNE

b. $\lim_{x \to -6^+} f(x)$ -4

c. $\lim_{x \to -6} f(x)$ DNE

d. $\lim_{x \to -5} f(x)$ 1

e. $\lim_{x \to -4} f(x)$ 4

f. $\lim_{x \to -3^-} f(x)$ 5

g. $\lim_{x \to -3^+} f(x)$ 5

h. $\lim_{x \to -3} f(x)$ 5

i. $\lim_{x \to -1^-} f(x)$ 3

j. $\lim_{x \to -1^+} f(x)$ 0

k. $\lim_{x \to -1} f(x)$ DNE

l. $\lim_{x \to 1} f(x)$ 2

m. $\lim_{x \to 2^-} f(x)$ 3

n. $\lim_{x \to 2^+} f(x)$ 2

o. $\lim_{x \to 2} f(x)$ DNE

p. $\lim_{x \to 3} f(x)$ 3

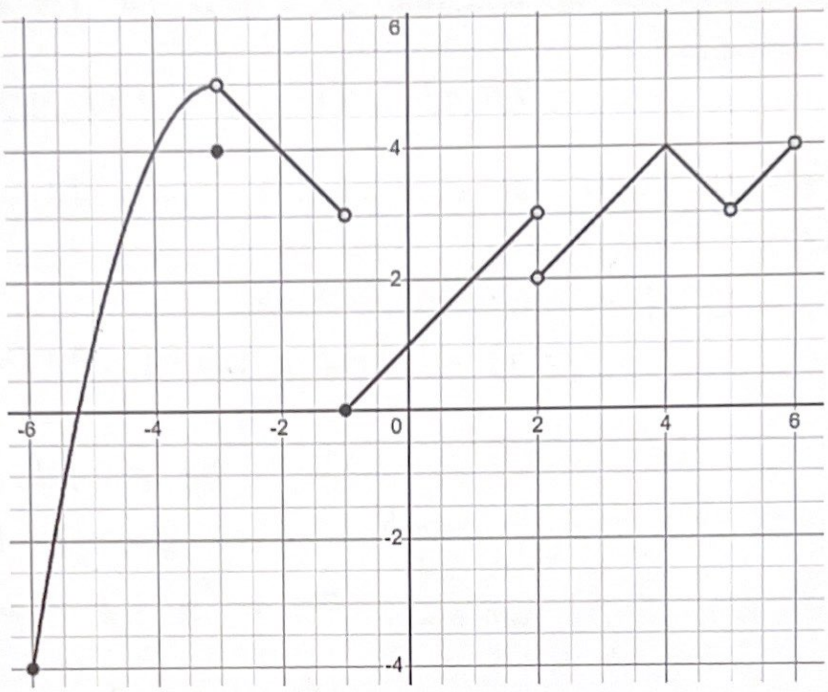
q. $\lim_{x \to 4} f(x)$ 4

r. $\lim_{x \to 5} f(x)$ 3

s. $\lim_{x \to 6^-} f(x)$ 4

t. $\lim_{x \to 6^+} f(x)$ DNE

u. $\lim_{x \to 6} f(x)$ DNE



4. $f(x) = x^3 + 2$ $\quad f(-2) = (-2)^3 + 2 = -6 < 0$

$f(-1) = (-1)^3 + 2 = 1 > 0$

$f(x)$ continuous on $[-2, -1]$ because it is a polynomial

so, by IVT, $f(x) = 0$ on $(-2, -1)$

5. $f(x) = x^2 - \cos x$ $\quad f(0) = 0 - 1 = -1 < 0$

$f(\pi) = \pi^2 - (-1) = \pi^2 + 1 > 0$

$f(x)$ continuous on $[0, \pi]$ because it is the difference of two continuous functions

so, by IVT $f(x) = 0$ (it has an x-int) on $(0, \pi)$

6. $f(x) = \frac{x-2}{x+1}$ $\quad f(1) = \frac{-1}{2} < 0$ $\quad f(x)$ has one discontinuity (infinite) at $x = -1$ which is not on $[1, 3]$

$f(3) = \frac{1}{4} > 0$ $\quad$ so $f(x)$ continuous on $[1, 3]$

so, by IVT $f(x) = 0$ (it has a root) on $(1, 3)$

7. $3 + \ln x = x^3 \rightarrow x^3 - \ln x - 3 = 0$ $\quad f(x) = x^3 - \ln x - 3$

$f(1) = 1 - \ln 1 - 3 = -2 < 0$ $\quad$ domain of $f(x)$ is $x > 0$,

$f(e) = e^3 - \ln e - 3 = e^3 - 4 > 0$ $\quad f(x)$ cont on $(0, \infty)$ and thus $[1, e]$ which is a subset of $(0, \infty)$

so, by IVT $f(x) = 0$ $(3 + \ln x = x^3)$ on $(1, e)$

8. $\sqrt{x+2} = 2x^2 + 1 \rightarrow 2x^2 + 1 - \sqrt{x+2} = 0$ $\quad f(x) = 2x^2 + 1 - \sqrt{x+2}$

$f(-1) = 2 + 1 - 1 = 2 > 0$ $\quad$ domain of $f(x)$ is $[-2, \infty)$

$f(0) = 0 + 1 - \sqrt{2} = 1 - \sqrt{2} < 0$ $\quad f(x)$ cont on $[-2, \infty)$ and thus on $[-1, 0]$ which is a subset of $[-2, \infty)$

so, by IVT $f(x) = 0$ $(\sqrt{x+2} = 2x^2 + 1)$ on $(-1, 0)$

9. a) $f(x) = \frac{x+5}{x^2+8x+15} = \frac{x+5}{(x+5)(x+3)}$ discontinuities at $x=-5$, $x=-3$

$\lim_{x\to -5} f(x) = \lim_{x\to -5} \frac{1}{x+3} = \frac{1}{-2}$ since $f(-5)$ does not exist but $\lim_{x\to -5} f(x)$ does exist, there is a $\boxed{\text{removable discontinuity at } x=-5}$

$\lim_{x\to -3} f(x) = \frac{2}{2(0)} = \frac{2}{0} \to \pm\infty$ since $f(-3)$ does not exist and $\lim_{x\to -3} f(x)$ is $\pm\infty$ (depending on which side you are approaching -3 from) $\boxed{\text{infinite discontinuity at } x=-3}$

b) $f(x) = \frac{2}{x-1}$ discontinuity at $x=1$

$\lim_{x\to 1} f(x) = \frac{2}{0} = \pm\infty$ Since $f(1)$ does not exist and $\lim_{x\to 1} f(x) = \pm\infty$ (depending on which side you approach 1 from), there is an $\boxed{\text{infinite discontinuity at } x=1}$

c) $f(x) = \frac{x-1}{x^3-2x^2+x} = \frac{x-1}{x(x-1)^2}$ discontinuities at $x=0$, $x=1$

$\lim_{x\to 0} f(x) = \frac{-1}{0} = \pm\infty$ since $f(0)$ does not exist and $\lim_{x\to 0} f(x) = \pm\infty$ (depending on which side you approach 0 from), there is an $\boxed{\text{infinte discontinuity at } x=0}$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{1}{x(x-1)} = \frac{1}{0} = \pm\infty$ Since $f(1)$ does not exist and $\lim_{x\to 1} f(x) = \pm\infty$ (depending on which side you approach 1 from) there is an $\boxed{\text{infinite discontinuity at } x=1}$

d) $f(x) = \dfrac{(x-1)\sin 3x}{x(x+2)}$ discontinuities at $x=0,\ x=-2$

$$\lim_{x\to 0} f(x) = \left[\lim_{x\to 0} \frac{x-1}{x+2}\right]\left[\lim_{x\to 0} \frac{\sin 3x}{x}\right] = -\frac{1}{2}\left[\lim_{x\to 0} \frac{3\cos 3x}{1}\right] = -\frac{1}{2}(3) = -\frac{3}{2}$$

$\hookrightarrow \frac{0}{0} \to LH$

since $f(0)$ does not exist but $\lim_{x\to 0} f(x) = -\frac{3}{2}$, there is a

$\boxed{\text{removable discontinuity at } x=0}$

$$\lim_{x\to -2} f(x) = \frac{(-3)\sin(-6)}{-2(0)} = \frac{-3\sin(-6)}{0} = \pm\infty$$

since $f(-2)$ does not exist but $\lim_{x\to -2} f(x) = \pm\infty$ (depending on which side you approach -2 from), there is an

$\boxed{\text{infinite discontinuity at } x=-2}$

e) $f(x) = \begin{cases} x+2 & \text{if } x \le -1 \\ x^2+2 & \text{if } -1 < x < 2 \\ \frac{6}{3-x} & \text{if } x > 2 \end{cases}$

$\longrightarrow$ cont on $\mathbb{R}$
$\longrightarrow$ cont on $\mathbb{R}$
$\longrightarrow$ discontinuity at $x=3$ on $(2,\infty)$

$\lim_{x\to 3} \frac{6}{3-x} = \frac{6}{0} = \pm\infty$ since $f(3)$ does not exist and $\lim_{x\to 3} f(x) = \pm\infty$ (depending on which side you are approaching 3 from), there is an

$\boxed{\text{infinite discontinuity at } x=3}$

$\lim_{x\to -1^-} f(x) = -1+2 = 1 \qquad \lim_{x\to -1^+} f(x) = (-1)^2+2 = 3$

since $\lim_{x\to -1^-} f(x) \ne \lim_{x\to -1^+} f(x)$, there is a $\boxed{\text{jump discontinuity at } x=-1}$

$\lim_{x\to 2^-} f(x) = (-2)^2+2 = 6 \qquad \lim_{x\to 2^+} f(x) = \frac{6}{3-2} = 6$ so $\lim_{x\to 2} f(x) = 6$

but $f(2)$ does not exist so there is a $\boxed{\text{removable discontinuity at } x=2}$

f) $f(x) = \begin{cases} \ln(-x) & \text{if } x \leq -1 \\ x^3+1 & \text{if } -1 < x < 2 \\ 5x-2 & \text{if } x \geq 2 \end{cases}$ cont on $(-\infty, 0)$ / cont on $\mathbb{R}$ / cont on $\mathbb{R}$

$\lim_{x \to -1^-} f(x) = \ln 1 = 0$ $\lim_{x \to -1^+} f(x) = (-1)^3 + 1 = 0$

$\lim_{x \to 2^-} f(x) = 2^3 + 1 = 9$ $\lim_{x \to 2^+} f(x) = 5(2) - 2 = 8$

so, $\lim_{x \to 2} f(x)$ does not exist but $f(2) = 8$

$\boxed{\text{jump discontinuity at } x = 2}$

g) $f(x) = \begin{cases} \frac{x}{6} & \text{if } x \leq -2 \\ \frac{1}{x-1} & \text{if } -2 < x < 3 \\ x^2-8 & \text{if } x \geq 3 \end{cases}$ cont on $\mathbb{R}$ / cont $(-\infty, 1) \cup (1, \infty)$ / cont on $\mathbb{R}$

$\lim_{x \to 1} f(x) = \frac{1}{1-1} = \frac{1}{0} = \pm\infty$ depending on which side of 1 you approach from, so there is an $\boxed{\text{infinite discontinuity at } x = 1}$

$\lim_{x \to -2^-} f(x) = -\frac{2}{6} = -\frac{1}{3}$ $\lim_{x \to -2^+} f(x) = -\frac{1}{-3}$ $f(-2) = -\frac{1}{3}$

$\lim_{x \to 3^-} f(x) = \frac{1}{3-1} = \frac{1}{2}$ $\lim_{x \to 3^+} f(x) = 9 - 8 = 1$

$\boxed{\text{jump discontinuity at } x = 3}$

10. a) $f(x) = \begin{cases} \frac{x^2-7x+12}{x-3} & \text{if } x \neq 3 \\ Ax+2 & \text{if } x = 3 \end{cases}$ $\lim_{x \to 3} f(x) = \frac{0}{0}$ LH

$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{2x-7}{1} = -1$

$f(3) = A(3) + 2 = -1$

$3A = -3$

$\boxed{A = -1}$

b) $f(x) = \begin{cases} \dfrac{x^2-6x}{2x} & \text{if } x \neq 0 \\ -A(x+6) & \text{if } x = 0 \end{cases}$

$\lim_{x \to 0} f(x) = \frac{0}{0} \quad LH \quad \lim_{x \to 0} \frac{2x-6}{2} = -3$

$f(0) = -A(0+6) = -3$

$-6A = -3$

$\boxed{A = \frac{1}{2}}$

c) $f(x) = \begin{cases} \dfrac{x^3-x^2}{1-\cos x} & \text{if } x \neq 0 \\ 2A(x-3) & \text{if } x = 0 \end{cases}$

$\lim_{x \to 0} f(x) = \frac{0}{0} \quad LH \quad \lim_{x \to 0} \frac{3x^2-2x}{\sin x} = \frac{0}{0}$

$LH \quad \lim_{x \to 0} \frac{6x-2}{\cos x} = \frac{-2}{1} = -2$

$f(0) = 2A(0-3) = -2$

$-6A = -2$

$\boxed{A = \frac{1}{3}}$

11. a) $f(x) = \begin{cases} x+2 & \text{if } x \leq -2 \\ ax^2+bx & \text{if } -2 < x < 3 \\ x^2-8 & \text{if } x \geq 3 \end{cases}$

$\lim_{x \to -2^-} f(x) = -2+2 = 0$

$\lim_{x \to -2^+} f(x) = 4a-2b$

$4a-2b = 0$ if $\lim_{x \to -2} f(x)$ exists

$\lim_{x \to 3^-} f(x) = 9a+3b$

$9a+3b = 1$ if $\lim_{x \to 3} f(x)$ exists

$\lim_{x \to 3^+} f(x) = 3^2-8 = 1$

$2a = b$

$9a+3(2a) = 1$

$15a = 1$

$\boxed{a = \frac{1}{15}}$

$\left(4\left(\frac{1}{15}\right)-2b\right) = 0$

$\boxed{b = \frac{2}{15}}$

b) $f(x) = \begin{cases} 3x-4 & \text{if } x \leq -2 \\ 2ax^2+3bx & \text{if } -2 < x < 1 \\ 3x^2-8 & \text{if } x \geq 1 \end{cases}$

$\lim_{x \to -2^-} f(x) = 3(-2)-4 = -10$

$\lim_{x \to -2^+} f(x) = 8a-6b$

$8A-6B = -10$ if $\lim_{x \to 2} f(x)$ exists

$\lim_{x \to 1^-} f(x) = 2a+3b$

$2A+3B = -5$ if $\lim_{x \to 1} f(x)$ exists

$\lim_{x \to 1^+} f(x) = 3-8 = -5$

$4A+6B = -10$
$8A-6B = -10$

$12A = -20$

$\boxed{a = -\frac{5}{3}}$

$2\left(-\frac{5}{3}\right)+3b = -5 \rightarrow 3b = -\frac{5}{3}$

$\boxed{b = -\frac{5}{9}}$

c) $f(x) = \begin{cases} 3x-4b & \text{if } x \le -2 \\ 2ax^2+bx & \text{if } -2<x<1 \\ ax^2+1 & \text{if } x \ge 1 \end{cases}$

$\lim_{x \to -2^-} f(x) = -6-4b$

$\lim_{x \to -2^+} f(x) = 8a-2b$

$-6-4b = 8a-2b$ if $\lim_{x \to 2} f(x)$ exists, $8a+2b = -6$

$\lim_{x \to 1^-} f(x) = 2a+b$ $\quad 2a+b = a+1$ if $\lim_{x \to 1} f(x)$ exists

$\lim_{x \to 1^+} f(x) = a+1$

$a+b = 1$

$4a+b = -3$

$-3a = 4$

$a = -\frac{4}{3}$

$-\frac{4}{3}+b = 1$

$b = \frac{7}{3}$

d) $f(x) = \begin{cases} 3x^2+b & \text{if } x \le -2 \\ 2ax^2+bx & \text{if } -2<x<1 \\ 3ax+\ln x & \text{if } x \ge 1 \end{cases}$

$\lim_{x \to -2^-} f(x) = 12+b$

$\lim_{x \to -2^+} f(x) = 8a-2b$

$12+b = 8a-2b$ if $\lim_{x \to -2} f(x)$ exists, $8a-3b = 12$

$\lim_{x \to 1^-} f(x) = 2a+b$ $\quad 2a+b = 3a \rightarrow a = b$

if $\lim_{x \to 1} f(x)$ exists

$\lim_{x \to 1^+} f(x) = 3a$

$8a-3a = 12$

$\frac{5a}{5} = \frac{12}{5}$

$a = \frac{12}{5} \quad b = \frac{12}{5}$