1\. a) $\lim_{x\to 3} \dfrac{x^2-9}{x^3-8x^2+15x} = \dfrac{0}{0}$ $\quad \lim_{x\to 3} \dfrac{2x}{3x^2-16x+15} = \dfrac{6}{-6} = \boxed{-1}$

b) $\lim_{x\to 2} \dfrac{x^2-7x+6}{x^2-4} = \dfrac{-4}{0}$ $\boxed{\text{DNE}}$

c) $\lim_{x\to 8} \dfrac{x^2-9x+8}{x^2-16} = \dfrac{0}{48} = \boxed{0}$

d) $\lim_{x\to 0} \dfrac{3\sin 7x}{2x} = \dfrac{0}{0} = \dfrac{3}{2} \lim_{x\to 0} \dfrac{\sin 7x}{x} = \dfrac{3}{2} \lim_{x\to 0} \dfrac{7\sin 7x}{7x} = \dfrac{21}{2} \lim_{x\to 0} \dfrac{\sin 7x}{7x}$

$= \dfrac{21}{2}(1) = \boxed{\dfrac{21}{2}}$

e) $\lim_{x\to 0} \dfrac{\sin^2 5x}{2x^2} = \dfrac{1}{2} \lim_{x\to 0} \left(\dfrac{\sin 5x}{x}\right)^2 = \dfrac{1}{2} \lim_{x\to 0} \left(\dfrac{5\sin 5x}{5x}\right)^2$

$= \dfrac{25}{2}\left[\lim_{x\to 0} \dfrac{\sin 5x}{5x}\right]^2 = \dfrac{25}{2}(1)^2 = \boxed{\dfrac{25}{2}}$

f) $\lim_{x\to 0} \dfrac{\sin^5 2x}{3x^5} = \dfrac{1}{3} \lim_{x\to 0} \left(\dfrac{\sin 2x}{x}\right)^5 = \dfrac{1}{3} \lim_{x\to 0} \left(\dfrac{2\sin 2x}{2x}\right)^5$

$= \dfrac{32}{3}\left[\lim_{x\to 0} \dfrac{\sin 2x}{2x}\right]^5 = \dfrac{32}{3}(1)^5 = \boxed{\dfrac{32}{3}}$

g) $\lim_{x\to 0} \dfrac{2x-3\sin x}{4x-2} = \dfrac{0}{-2} = \boxed{0}$

h) $\lim_{x\to 0} \dfrac{3x^3-2\sin 4x}{x^2-x} = \dfrac{0}{0} = \lim_{x\to 0} \dfrac{9x^2-8\cos 4x}{2x-1} = \dfrac{0-8}{0-1} = \boxed{8}$

i) $\lim_{x\to 0} \dfrac{2\sin^3 3x}{4x^3} = \dfrac{1}{2}\lim_{x\to 0}\left(\dfrac{\sin 3x}{x}\right)^3 = \dfrac{1}{2}\lim_{x\to 0}\left(\dfrac{3\sin 3x}{3x}\right)^3$

$= \dfrac{27}{2}\left[\lim_{x\to 0}\dfrac{\sin 3x}{3x}\right]^3 = \dfrac{27}{2}\left[1\right]^3 = \boxed{\dfrac{27}{2}}$

j) $\lim_{x\to\infty} \dfrac{7x^3}{e^x} = \dfrac{\infty}{\infty}$ faster $= \boxed{0}$

k) $\lim_{x\to\infty} \dfrac{\sqrt{x}}{\ln x} = \dfrac{\infty}{\infty}$ faster $= \boxed{\infty}$

l) $\lim_{x\to\infty} \dfrac{2^x}{x^{100}} = \dfrac{\infty}{\infty}$ faster $= \boxed{\infty}$

m) $\lim_{x\to\infty} \dfrac{\sqrt[3]{x}}{x} = \dfrac{\infty}{\infty}$ faster $= \boxed{0}$

n) $\lim_{x\to\infty} e^{-x^2+x}$ $\quad u = -x^2+x$, as $x\to\infty$, $u\to-\infty$

$\lim_{u\to-\infty} e^u = e^{-\infty} = \dfrac{1}{e^\infty} = \boxed{0}$

o) $\lim_{x\to-\infty} e^{-x^3-x-1}$ $\quad u = -x^3-x-1$, as $x\to-\infty$, $u\to\infty$

$\lim_{u\to\infty} e^u = e^\infty = \boxed{\infty}$

p) $\lim_{x\to\infty} \ln(x^2-1)$ $\quad u = x^2-1$, as $x\to\infty$, $u\to\infty$, $\lim_{u\to\infty}\ln u = \boxed{\infty}$

q) $\lim_{x\to-\infty} \ln(x^2+x+1)$ $\quad u = x^2+x+1$, $x\to-\infty$, $u\to\infty$

$\lim_{u\to\infty} \ln u = \boxed{\infty}$

r) $\lim_{x \to -\infty} \ln(x^3 - 5x + 7)$ $u = x^3 - 5x + 7$, as $x \to -\infty$, $u \to -\infty$

$\lim_{u \to -\infty} \ln u \to \boxed{DNE}$

s) $\lim_{x \to \infty} \ln(e^x - x^3)$ $u = e^x - x^3$, as $x \to \infty$, $u \to \infty$

$\lim_{u \to \infty} \ln u = \boxed{\infty}$

2. a) $\lim_{x \to -2^-} f(x) = (-2)^2 - (-2) - 1 = 5$

all separate pieces continuous for $(-\infty, \infty)$

$\lim_{x \to -2^+} f(x) = 4a - 2b$

$\lim_{x \to 1^-} f(x) = a + b$

$\lim_{x \to 1^+} f(x) = \ln 1 + 5(1) - 2 = 3$

$4a - 2b = 5$

$a + b = 3 \xrightarrow{\times 2} 2a + 2b = 6$

$+$

$6a = 11$

$\boxed{a = \frac{11}{6} \quad b = \frac{7}{6}}$

$\frac{11}{6} + b = 3$

b) $\lim_{x \to -2^-} f(x) = -6 - 8 = -14$

all separate pieces continuous for $(-\infty, \infty)$

$\lim_{x \to -2^+} f(x) = 4a - 2b$

$\lim_{x \to 1^-} f(x) = a + b$

$\lim_{x \to 1^+} f(x) = 1 - 1 - 1 = -1$

$a + b = -1 \xrightarrow{\times 2} 2a + 2b = -2$

$4a - 2b = -14$

$6a = -16$

$\boxed{a = -\frac{8}{3} \quad b = \frac{5}{3}}$

$-\frac{8}{3} + b = -1$

c) $\lim_{x \to -1^-} f(x) = \frac{1}{-1} - 2 = -3$

each piece is continuous on the piece of the domain it is defined on

$\lim_{x \to -1^+} f(x) = a - b$

$\lim_{x \to 2^-} f(x) = 4a + 2b$

$\lim_{x \to 2^+} f(x) = 3(2) - 2^3 = -2$

$$a - b = -3$$
$$4a + 2b = -2$$
$$\times 2 \quad 2a - 2b = -6$$
$$6a = -8$$

$$\boxed{a = -\frac{4}{3} \qquad b = \frac{5}{3}}$$

$$a - b = -3$$
$$-\frac{4}{3} - b = -3$$

d) $\lim_{x \to -1^-} f(x) = -3 + 1 = -2$

each piece is continuous on the piece of the domain it is defined on

$\lim_{x \to -1^+} f(x) = a - b$

$\lim_{x \to 2^-} f(x) = 4a + 2b$

$\lim_{x \to 2^+} f(x) = 2 - 2 + 5 = 5$

$$a - b = -2$$
$$4a + 2b = 5$$
$$\times 2 \quad 2a - 2b = -4$$
$$6a = 1$$

$$\boxed{a = \frac{1}{6} \qquad b = \frac{13}{6}}$$

$$a - b = -2$$
$$\frac{1}{6} - b = -2$$

3. a) $f(x) = \frac{x^2 - 4x + 3}{x^2 - 1} = \frac{(x-3)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$

$\boxed{\begin{array}{ll} x = 1 & \text{removable} \\ x = -1 & \text{infinite} \end{array}}$

b) $f(x) = \frac{x^2 - 4}{x^3 - 7x^2 + 10x} = \frac{(x+2)\cancel{(x-2)}}{x(x-5)\cancel{(x-2)}}$

$\boxed{\begin{array}{ll} x = 2 & \text{removable} \\ x = 0 & \text{infinite} \\ x = 5 & \text{infinite} \end{array}}$

c) $f(x) = \frac{x^3-9x}{x^3+2x^2-15x} = \frac{\cancel{x}(x+3)\cancel{(x-3)}}{\cancel{x}(x+5)\cancel{(x-3)}}$

$\boxed{\begin{array}{l} x=0 \text{ removable} \\ x=3 \text{ removable} \\ x=-5 \text{ infinite} \end{array}}$

4. a) $y=\sin(e^x)$ $\boxed{y' = \cos(e^x)[e^x]}$

b) $y=e^{\cos x}$ $\boxed{y' = e^{\cos x}(-\sin x)}$

c) $y=(10x-3)^6$ $y=u^6$ $u=10x-3$

$y' = 6(10x-3)^5(10) = \boxed{60(10x-3)^5}$

d) $y=(\cos x)^x$

$\ln y = x\ln\cos x$

$\frac{1}{y}\frac{dy}{dx} = \frac{x(-\sin x)}{\cos x} + \ln\cos x$

$\boxed{\frac{dy}{dx} = \left[-x\tan x + \ln\cos x\right](\cos x)^x}$

e) $y=\frac{x}{\tan x}$ $y' = \frac{\tan x(1) - x\sec^2 x}{\tan^2 x}$

$= \boxed{\frac{\tan x - x\sec^2 x}{\tan^2 x}}$

f) $y = (\tan^{-1}x)(3^x)$ $\boxed{y' = (\tan^{-1}x)(3^x)(\ln 3) + \frac{3^x}{1+x^2}}$

g) $y = x^{\ln x}$

$\ln y = (\ln x)(\ln x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{\ln x}{x} + \frac{\ln x}{x}$

$\boxed{\frac{dy}{dx} = \left(\frac{2\ln x}{x}\right)\left(x^{\ln x}\right)}$

h) $y = \sqrt[3]{3x-6} = (3x-6)^{\frac{1}{3}}$

$y' = \frac{1}{3}(3x-6)^{-\frac{2}{3}}(3) = \boxed{\frac{1}{(3x-6)^{\frac{2}{3}}}}$

i) $y = \sqrt{\sin^{-1}(x^2)} \rightarrow y = \sqrt{u}$ $u = \sin^{-1}(x^2)$

$u = \sin^{-1} v$ $v = x^2$

$\frac{dy}{du} = \frac{1}{2\sqrt{u}}$ $\frac{du}{dv} = \frac{1}{\sqrt{1-v^2}}$ $\frac{dv}{dx} = 2x$

$$\boxed{\frac{dy}{dx} = \frac{x}{\sqrt{\sin^{-1}(x^2)}\sqrt{1-x^4}}}$$

j) $y = x^2 \sec^{-1} x$

$$\boxed{y' = \frac{x^2}{|x|\sqrt{x^2-1}} + 2x \sec^{-1} x}$$

k) $y = \log_2(\sec x)$

$$y' = \frac{\sec x \tan x}{(\ln 2)(\sec x)} = \boxed{\frac{\tan x}{\ln 2}}$$

l) $y = (3x)^{\cot x}$

$$\ln y = (\cot x)(\ln 3x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3\cot x}{3x} + (-\csc^2 x)\ln 3x$$

$$\boxed{\frac{dy}{dx} = \left[\frac{\cot x}{x} - (\csc^2 x)(\ln 3x)\right](3x)^{\cot x}}$$

m) $y = (x^3-1)^{\pi}$

$$\boxed{y' = \pi(x^3-1)^{\pi-1}(3x^2)}$$

5. a) $y = \sqrt{x^2-x-2}$

$$y' = \frac{2x-1}{2\sqrt{x^2-x-2}}$$

$$y'(3) = \frac{5}{2\sqrt{9-3-2}} = \boxed{\frac{5}{4}}$$

b) $y = \sin 3x \qquad y' = 3\cos 3x \qquad y'\left(\frac{\pi}{4}\right) = 3\cos\frac{3\pi}{4} = 3\left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-3\sqrt{2}}{2}}$

c) $y = (2x-8)^3 \qquad y' = 3(2x-8)^2(2) \qquad y'(5) = 6(10-8)^2 = \boxed{24}$

d) $y = \ln(x^2-3x) \qquad y' = \frac{2x-3}{x^2-3x} \qquad y'(-1) = \frac{-2-3}{1+3} = \boxed{\frac{-5}{4}}$

e) $y = e^{3x} \qquad y' = 3e^{3x} \qquad y'(\ln 2) = 3e^{3\ln 2} = 3e^{\ln 2^3} = 3e^{\ln 8}$

$= 3(8) = \boxed{24}$

6. a) $y = (2x^2-3x+1)^4 \qquad y' = 4(2x^2-3x+1)^3(4x-3)$

$y'(2) = 4(3^3)(5) = \boxed{540}$

b) $y = \sin^{-1}(x^2) \qquad y' = \frac{2x}{\sqrt{1-x^4}} \qquad y'\left(\frac{1}{2}\right) = \frac{1}{\sqrt{1-\frac{1}{16}}}$

$= \frac{1}{\sqrt{\frac{15}{16}}} = \frac{1}{\frac{\sqrt{15}}{4}} = \boxed{\frac{4}{\sqrt{15}}}$

c) $y = \sqrt[3]{x^2+4} = (x^2+4)^{\frac{1}{3}} \qquad y' = \frac{1}{3}(x^2+4)^{-\frac{2}{3}}(2x)$

$= \frac{2x}{3(x^2+4)^{\frac{2}{3}}} \qquad y'(2) = \frac{4}{3(4)} = \boxed{\frac{1}{3}}$

d) $y = \frac{1}{x^2-8x}$ $\quad y' = -\frac{2x-8}{(x^2-8x)^2}$ $\quad y'(1) = -\frac{-6}{(-7)^2} = \boxed{\frac{6}{49}}$

e) $y = \ln(x^2-x+2)$ $\quad y' = \frac{2x-1}{x^2-x+2}$ $\quad y'(3) = \boxed{\frac{5}{8}}$

7. a) $y = \cos^3 x$ $\quad y' = 3\cos^2 x(-\sin x)$ $\quad y'\left(\frac{\pi}{3}\right) = 3\left(\frac{1}{2}\right)^2\left(-\frac{\sqrt{3}}{2}\right) = -\frac{3\sqrt{3}}{8}$

$m_{TL} = -\frac{3\sqrt{3}}{8}$ $\quad m_{NL} = \frac{8}{3\sqrt{3}}$

$y\left(\frac{\pi}{3}\right) = \left(\cos\frac{\pi}{3}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ $\quad \left(\frac{\pi}{3}, \frac{1}{8}\right)$

$\boxed{TL: y - \frac{1}{8} = -\frac{3\sqrt{3}}{8}\left(x - \frac{\pi}{3}\right) \quad NL: y - \frac{1}{8} = \frac{8}{3\sqrt{3}}\left(x - \frac{\pi}{3}\right)}$

b) $y = \sin x \cos x$ $\quad y' = \cos x \cos x + (\sin x)(-\sin x) = \cos^2 x - \sin^2 x = \cos 2x$

$y'\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{2} = 0$ $\quad m_{TL} = 0$ horizontal $\quad m_{NL}$ undefined vertical

$y\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{2}$ $\quad \left(\frac{\pi}{4}, \frac{1}{2}\right)$

$\boxed{TL: y = \frac{1}{2} \quad NL: x = \frac{\pi}{4}}$

c) $y = \sqrt{x^3-2}$ $y' = \dfrac{3x^2}{2\sqrt{x^3-2}}$ $y'(3) = \dfrac{27}{2\sqrt{25}} = \dfrac{27}{10}$

$y(3) = \sqrt{25} = 5$ $m_{TL} = \dfrac{27}{10}$ $m_{NL} = -\dfrac{10}{27}$

$(3,5)$

$\boxed{TL: y-5 = \frac{27}{10}(x-3) \quad NL: y-5 = -\frac{10}{27}(x-3)}$

d) $y = \left(2x - \frac{1}{x}\right)^3$ $y' = 3\left(2x-\frac{1}{x}\right)^2\left[2 + \frac{1}{x^2}\right]$

$y(1) = 1$ $y'(1) = 3(1^2)(2+1) = 9$ $m_{TL} = 9$ $m_{NL} = -\frac{1}{9}$

$(1,1)$

$\boxed{TL: y-1 = 9(x-1) \quad NL: y-1 = -\frac{1}{9}(x-1)}$

e) $y = \ln(x^2-1)$ $y' = \dfrac{2x}{x^2-1}$ $y'(3) = \dfrac{6}{8} = \dfrac{3}{4}$

$y(3) = \ln 8$ $(3, \ln 8)$ $m_{TL} = \frac{3}{4}$ $m_{NL} = -\frac{4}{3}$

$\boxed{TL: y-\ln 8 = \frac{3}{4}(x-3) \quad NL: y-\ln 8 = -\frac{4}{3}(x-3)}$

8. $A = 4\pi r^2$ $dA = 8\pi r\,dr$ $dA = 8\pi(2)(.2) = \boxed{3.2\pi \text{ cm}^2}$

9. $V = \frac{4}{3}\pi r^3$ $dV = 4\pi r^2\,dr$ $dV = 4\pi(3^2)(.1) = \boxed{3.6\pi \text{ cm}^3}$

10. $A = \dfrac{s^2\sqrt{3}}{4}$ $dA = \dfrac{2s\sqrt{3}}{4}\,ds$ $dA = \dfrac{2(5)\sqrt{3}(.1)}{4} = \boxed{\dfrac{\sqrt{3}}{4} \text{ in}^2}$

11. $V = \pi r^2 h$ $dV = \pi\left[r^2\,dh + 2rh\,dr\right]$ $dV = \pi\left[1^2(-.1) + 2(1)(3)(.1)\right]$

$= \pi(-.1 + .6) = \boxed{.5\pi \text{ ft}^3}$

12. $V = \frac{1}{3}\pi r^2 h$ with $h = 2 \rightarrow V = \frac{2}{3}\pi r^2$

$dV = \frac{4}{3}\pi r\, dr$ $\quad dV = \frac{4}{3}\pi(3)(-.1) = \boxed{-.4\pi \text{ in}^3}$

13. $f(x) = x^3 - 2x + 6$ $\quad f'(x) = 3x^2 - 2$ $\quad f''(x) = 6x$

$f(1) = 1 - 2 + 6 = 5$ $\quad f'(1) = 1$ $\quad f''(1) = 6 > 0$

$(1,5)$ $\quad$ concave up

TL: $y - 5 = 1(x-1)$

$f(1.1) \rightarrow y - 5 = 1(1.1 - 1)$

$\boxed{y = 5.1}$ $\boxed{\text{underestimate}}$

14. $f(x) = \frac{x+2}{x-3}$ $\quad f'(x) = \frac{(x-3)(1) - (x+2)(1)}{(x-3)^2}$ $\quad f''(x) = -\frac{-5(2)(x-3)}{(x-3)^4}$

$f(5) = \frac{7}{2}$ $\quad = \frac{10}{(x-3)^3}$

$(5, \frac{7}{2})$ $\quad = \frac{-5}{(x-3)^2}$ $\quad f'(5) = \frac{-5}{4}$

$f''(5) = \frac{10}{2^3} = \frac{5}{4} > 0$

TL: $y - \frac{7}{2} = -\frac{5}{4}(x-5)$ $\quad$ concave up

at $x = 4.9$, $y - \frac{7}{2} = -\frac{5}{4}(4.9 - 5)$

$y - \frac{7}{2} = -\frac{5}{4}\left(-\frac{1}{10}\right)$

$y - \frac{7}{2} = \frac{1}{8}$

$\boxed{y = \frac{29}{8} \quad \text{underestimate}}$

15. $f(x) = (2x+1)^5$

$f(3) = 7^5 = 16807$

$(3, 16807)$

$f'(x) = 5(2x+1)^4(2)$

$f'(3) = 10(7^4) = 24010$

$m = 24010$

$f''(x) = (10)(4)(2x+1)^3(2)$

$= 80(2x+1)^3$

$f''(3) = 27440 > 0$

concave up

$y - 16807 = 24010(x-3)$

$y - 16807 = 24010(3.1-3)$

$y = 2401 + 16807 = \boxed{19208 \text{ underestimate}}$

16. $f(x) = x^4 - 2x^2 - x - 1$

$f(1) = 1 - 2 - 1 - 1 = -3$

$f(3) = 81 - 18 - 3 - 1 = 59$

$\frac{59-(-3)}{3-1} = \frac{62}{2} = 31$

$f'(x) = 4x^3 - 4x - 1$

$f''(x) = 12x^2 - 4$

$f''(2) = 48 - 4 = 44 > 0$ concave up

$y + 3 = 31(x-1)$ at $x = 2$

$y + 3 = 31(2-1)$

$y + 3 = 31$

$\boxed{y = 28 \text{ overestimate}}$

17. $f(x) = (3x-2)^4$

$f(0) = (-2)^4 = 16$

$f(1) = 1^4 = 1$

$\frac{1-16}{1-0} = -15$

$f'(x) = 4(3x-2)^3(3)$

$f''(x) = 12(3)(3x-2)^2(3)$

$f''(\frac{1}{2}) = 108(-\frac{1}{2})^2 = 27 > 0$

concave up

at $x = \frac{1}{2}$

$y - 1 = -15(x-1)$

$y - 1 = -15(\frac{1}{2} - 1)$

$y - 1 = \frac{15}{2}$

$\boxed{y = \frac{17}{2} \text{ overestimate}}$

18. $f(x) = \dfrac{x-3}{x-2}$ $\qquad f'(x) = \dfrac{(x-2)(1)-(x-3)(1)}{(x-2)^2} = \dfrac{1}{(x-2)^2}$

$f(-2) = \dfrac{-5}{-4} = \dfrac{5}{4}$ $\qquad f''(x) = -\dfrac{2(x-2)}{(x-2)^4} = -\dfrac{2}{(x-2)^3}$

$f(0) = \dfrac{-3}{-2} = \dfrac{3}{2}$ $\qquad f''(-1) = -\dfrac{2}{(-3)^3} = \dfrac{2}{27} > 0$

$\dfrac{\frac{3}{2}-\frac{5}{4}}{0-(-2)} = \dfrac{\frac{1}{4}}{2} = \dfrac{1}{8}$ $\qquad$ concave up

$y - \frac{3}{2} = \frac{1}{8}(x-0)$

at $x=-1$ $\quad y - \frac{3}{2} = -\frac{1}{8} \rightarrow \boxed{y = \frac{11}{8} \text{ overestimate}}$

19. $\sqrt{102}$ $\quad f(x) = \sqrt{x}$ $\quad f'(x) = \dfrac{1}{2\sqrt{x}}$ $\quad x = 102 \quad a = 100$

$$\sqrt{102} \approx \sqrt{100} + \frac{1}{2\sqrt{100}}(102-100) = 10 + \frac{1}{10} = \boxed{\frac{101}{10}}$$

20. $\sqrt[3]{9}$ $\quad f(x) = x^{\frac{1}{3}}$ $\quad f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$ $\quad x = 9 \quad a = 8$

$$\sqrt[3]{9} \approx \sqrt[3]{8} + \frac{1}{3(8)^{2/3}}(9-8) = 2 + \frac{1}{12} = \boxed{\frac{25}{12}}$$

21. $\sqrt{62}$ $\quad f(x) = \sqrt{x}$ $\quad f'(x) = \dfrac{1}{2\sqrt{x}}$ $\quad x = 62 \quad a = 64$

$$\sqrt{62} \approx \sqrt{64} + \frac{1}{2\sqrt{64}}(62-64) = 8 - \frac{1}{8} = \boxed{\frac{63}{8}}$$

22. $\sqrt[3]{25}$ $\quad f(x) = x^{\frac{1}{3}}$ $\quad f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$ $\quad x = 25 \quad a = 27$

$$\sqrt[3]{25} \approx \sqrt[3]{27} + \frac{1}{3(27)^{2/3}}(25-27) = 3 - \frac{2}{3(9)} = 3 - \frac{2}{27}$$

$$= \boxed{\frac{79}{27}}$$

23. $\sqrt[4]{15}$ $\quad f(x) = x^{\frac{1}{4}} \quad f'(x) = \frac{1}{4}x^{-\frac{3}{4}} \quad x = 15 \quad a = 16$

$$\sqrt[4]{15} \approx \sqrt[4]{16} + \frac{1}{4(16)^{3/4}}(15-16) = 2 - \frac{1}{(4)(8)} = \boxed{\frac{63}{32}}$$

24. a) $f(x) = x^4 - 18x^2 + 4$

$f'(x) = 4x^3 - 36x = 0$

$4x(x^2 - 9) = 0$

$4x(x+3)(x-3) = 0$

$f(0) = 4$

$f(3) = -77$

$f(-3) = -77$

$x = 0, \quad x = -3, \quad x = 3$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = \infty$

$-\quad +\quad -\quad +$

$-3 \quad 0 \quad 3$

$f''(x) = 12x^2 - 36 = 0$

$12(x^2 - 3) = 0$

$x = \pm\sqrt{3}$

$+\quad -\quad +$

$-\sqrt{3} \quad \sqrt{3}$

$f(-\sqrt{3}) = -41$

$f(\sqrt{3}) = -41$

inc: $[-3, 0] \cup [3, \infty)$ $\quad$ dec: $(-\infty, -3] \cup [0, 3]$

local max of 4 at $x = 0$ $\quad$ no absolute max

absolute/local min of -77 at $x = 3$ and $x = -3$

concave up: $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

concave down: $(-\sqrt{3}, \sqrt{3})$

POI: $(-\sqrt{3}, -41)$ and $(\sqrt{3}, -41)$

b) $f(x) = x^3 - 6x^2 + 9x + 4$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

$f'(x) = 3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

$+ \quad | \quad - \quad | \quad +$

$1 \qquad 3$

$f(1) = 8 \qquad f(3) = 4$

$f''(x) = 6x - 12 = 0$

$x = 2$

$- \quad | \quad +$

2

$f(2) = 6$

inc: $(-\infty, 1] \cup [3, \infty)$ dec: $[1, 3]$

local max of 8 at $x = 1$ local min of 4 at $x = 3$

no absolute max or min

concave up: $(2, \infty)$ concave down $(-\infty, 2)$

POI: $(2, 6)$

c) $f(x) = 2x^3 + 21x^2 - 108x + 6$

$\lim_{x \to \infty} f(x) = \infty \qquad \lim_{x \to -\infty} f(x) = -\infty$

$f'(x) = 6x^2 + 42x - 108 = 0$

$6(x^2 + 7x - 18) = 0$

$6(x+9)(x-2) = 0$

$x = -9 \quad x = 2$

$+ \quad | \quad - \quad | \quad +$

$-9 \qquad 2$

$f(-9) = 1221 \qquad f(2) = -110$

$f''(x) = 12x + 42 = 0$

$x = -\frac{42}{12} = -\frac{7}{2}$

$- \quad | \quad +$

$-\frac{7}{2}$

$f\left(-\frac{7}{2}\right) = \frac{1111}{2}$

inc: $(-\infty, -9] \cup [2, \infty)$ dec: $[-9, 2]$ no absolute max/min

local max of 1221 at $x = -9$ local min of -110 at $x = 2$

conc up: $\left(-\frac{7}{2}, \infty\right)$ conc down: $\left(-\infty, -\frac{7}{2}\right)$

POI: $\left(-\frac{7}{2}, \frac{1111}{2}\right)$

d) $f(x) = x^4 - 6x^2 + 3$

$f'(x) = 4x^3 - 12x = 0$

$4x(x^2 - 3) = 0$

$x = 0 \quad x = \pm\sqrt{3}$

$-\quad + \quad - \quad +$

$-\sqrt{3} \quad 0 \quad \sqrt{3}$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = \infty$

$f(0) = 3$

$f(\sqrt{3}) = -6$

$f(-\sqrt{3}) = -6$

$f''(x) = 12x^2 - 12 = 0$

$12(x+1)(x-1) = 0$

$x = \pm 1$

$+ \quad - \quad +$

$-1 \quad 1$

$f(1) = -2$

$f(-1) = -2$

inc: $[-\sqrt{3}, 0] \cup [\sqrt{3}, \infty)$ dec: $(-\infty, -\sqrt{3}] \cup [0, \sqrt{3}]$

local max of 3 at $x = 0$, no absolute max

absolute/local min of -6 at $x = \sqrt{3}$ and $x = -\sqrt{3}$

conc up: $(-\infty, -1) \cup (1, \infty)$ conc down: $(-1, 1)$

POI: $(1, -2)$ and $(-1, -2)$

e) $f(x) = 2xe^x$ $\quad \lim_{x \to \infty} f(x) = \infty \quad \lim_{x \to -\infty} f(x) = 0$

$f'(x) = 2xe^x + 2e^x = 2e^x(x+1) = 0$ if $x = -1$

f': $- \quad +$

-1

$f(-1) = -\frac{2}{e}$

$f''(x) = 2xe^x + 2e^x + 2e^x$

$= 2e^x(x+2) = 0$

$x = -2$

f'': $- \quad +$

-2

$f(-2) = -\frac{4}{e^2}$

inc: $[-1, \infty)$ dec: $(-\infty, -1]$ local/absolute min of $-\frac{2}{e}$ at $x = -1$

no local/absolute max

conc up: $(-2, \infty)$ conc down: $(-\infty, -2)$

POI: $\left(-2, -\frac{4}{e^2}\right)$

25. a) $f(x) = \frac{1}{x}$ on $[1,2]$

f is continuous on $[1,2]$
f is differentiable on $(1,2)$
because rational functions are continuous and differentiable on their domains, in this case $(-\infty,0)\cup(0,\infty)$

$f'(x) = -\frac{1}{x^2}$

$$\frac{f(2)-f(1)}{2-1} = \frac{\frac{1}{2}-1}{1} = -\frac{1}{2}$$

$$-\frac{1}{2} = -\frac{1}{x^2} \rightarrow x^2 = 2 \rightarrow x = \pm\sqrt{2}$$

only $\boxed{x=\sqrt{2}}$ is on $[1,2]$

b) $f(x) = \frac{5}{x-1}$ on $[0,2]$ MVT does not apply because $f(x)$ is not continuous at $x=1$

c) $f(x) = \frac{2}{4-x}$ on $[0,2]$

f is continuous on $[0,2]$ and f is differentiable on $(0,2)$ because rational functions are continuous and differentiable on their domains, in this case $(-\infty,4)\cup(4,\infty)$

$$\frac{f(2)-f(0)}{2-0} = \frac{1-\frac{1}{2}}{2-0} = \frac{1}{4}$$

$$f'(x) = \frac{2}{(4-x)^2} = \frac{1}{4} \rightarrow 8 = (4-x)^2 \rightarrow 4-x = \pm 2\sqrt{2}$$

$$x = 4 \pm 2\sqrt{2}$$

only $\boxed{x = 4-2\sqrt{2}}$ is on $[0,2]$

d) $f(x) = x^3-3x+1$ on $[1,2]$

f is continuous on $[1,2]$ and differentiable on $(1,2)$ because it is a polynomial

$$\frac{f(2)-f(1)}{2-1} = \frac{3-(-1)}{1} = 4$$

$$f'(x) = 3x^2-3 = 4$$

$$3x^2 = 7$$
$$x^2 = \frac{7}{3}$$
$$\nearrow x = \pm\sqrt{\frac{7}{3}}$$

only $\boxed{x = \sqrt{\frac{7}{3}}}$ is on $[1,2]$

e) $f(x) = \sqrt{x+2}$ on $[-1, 2]$

f is continuous on $[-1,2]$ and differentiable on $(-1,2)$ because radical functions are differentiable and continuous on their domains in this case $[-2, \infty)$

$$\frac{f(2) - f(-1)}{2-(-1)} = \frac{2-1}{3} = \frac{1}{3}$$

$$f'(x) = \frac{1}{2\sqrt{x+2}} = \frac{1}{3} \rightarrow 3 = 2\sqrt{x+2}$$

$$\rightarrow \frac{3}{2} = \sqrt{x+2} \rightarrow \frac{9}{4} = x+2 \rightarrow \boxed{x = \frac{1}{4}}$$

26. $y = \sqrt{x} \rightarrow (x, \sqrt{x})$ and $(3, 0)$

$$d = \sqrt{(x-3)^2 + (\sqrt{x}-0)^2} = \sqrt{x^2 - 6x + 9 + x}$$

$$d = \sqrt{x^2 - 5x + 9}$$

$$d' = \frac{2x-5}{2\sqrt{x^2-5x+9}} = 0 \text{ if } x = \frac{5}{2}$$

$-$ $+$

$\frac{5}{2}$

$$y = \sqrt{\frac{5}{2}} \rightarrow \boxed{\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)}$$

27. $y = x^{\frac{3}{2}} + 1$ and $(2, 1)$ $(x, x^{\frac{3}{2}} + 1)$

$$d = \sqrt{(x-2)^2 + (x^{\frac{3}{2}} + 1 - 1)^2} \rightarrow d = \sqrt{x^2 - 4x + 4 + x^3}$$

$$d' = \frac{3x^2 + 2x - 4}{2\sqrt{x^3 + x^2 - 4x + 4}} = 0 \quad x = \frac{-2 \pm \sqrt{4 - 4(3)(-4)}}{6}$$

$$x = \frac{-2 \pm 2\sqrt{13}}{6} = \frac{-1 \pm \sqrt{13}}{3}$$

$+$ $-$ $+$

$\frac{-1-\sqrt{13}}{3}$ $\frac{-1+\sqrt{13}}{3}$

$$\boxed{\text{min at } \left(\frac{-1+\sqrt{13}}{3}, \left(\frac{-1+\sqrt{13}}{3}\right)^{\frac{3}{2}} + 1\right)}$$

28. $r = 4h$ $\quad \frac{dV}{dt} = 16$ $\quad r = 12$ $\quad \frac{dr}{dt} = ?$ $\quad V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2\left(\frac{r}{4}\right)$

$h = \frac{r}{4}$

$$V = \frac{1}{12}\pi r^3 \rightarrow \frac{dV}{dt} = \frac{1}{4}\pi r^2 \frac{dr}{dt}$$

$$16 = \frac{1}{4}\pi(12^2)\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{64}{144\pi} = \boxed{\frac{4}{9\pi} \text{ in/sec}}$$

29. $h = 3r$ $\quad \frac{dh}{dt} = 12$ $\quad h = 20$ $\quad \frac{dV}{dt} = ?$ $\quad V = \frac{1}{3}\pi r^2 h$

$r = \frac{h}{3}$

$$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2(h) = \frac{1}{27}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{1}{9}\pi(20^2)(12) \rightarrow \boxed{\frac{dV}{dt} = \frac{1600\pi}{3} \text{ in}^3\text{/sec}}$$

30. $r = 2h$ $\quad \frac{dV}{dt} = 40$ $\quad \frac{dr}{dt} = ?$ $\quad h = 30 \rightarrow r = 60$ $\quad V = \frac{1}{3}\pi r^2 h$

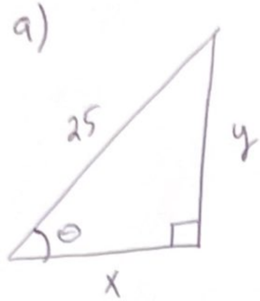
$h = \frac{r}{2}$

$$V = \frac{1}{3}\pi r^2\left(\frac{r}{2}\right) \rightarrow V = \frac{1}{6}\pi r^3$$

$$\frac{dV}{dt} = \frac{1}{2}\pi r^2 \frac{dr}{dt}$$

$$40 = \frac{1}{2}\pi(60^2)\frac{dr}{dt} \nearrow \frac{dr}{dt} = \frac{80}{3600\pi}\ \boxed{= \frac{1}{45\pi} \text{ in/sec}}$$

31. a)



$y = 7$

$\frac{dy}{dt} = -10$

$\frac{dx}{dt} = ?$

$x^2 + y^2 = 625$

$x^2 + 7^2 = 625$

$x = 24$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$24\frac{dx}{dt} = -7(-10)$

$\frac{dx}{dt} = \frac{70}{24} = \boxed{\frac{35}{12} \text{ ft/sec}}$

b) $A = \frac{1}{2}xy$ $\quad \frac{dA}{dt} = \frac{1}{2}\left[x\frac{dy}{dt} + y\frac{dx}{dt}\right]$

$\frac{dA}{dt} = \frac{1}{2}\left[24(-10) + 7\left(\frac{35}{12}\right)\right] = \boxed{-\frac{2635}{24} \text{ ft}^2\text{/sec}}$

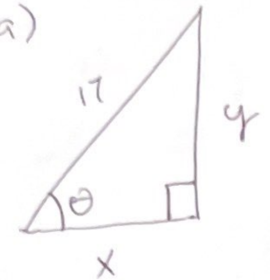
c) $\sin\theta = \frac{y}{25} \rightarrow 25\sin\theta = y$ $\quad \cos\theta = \frac{24}{25}$

$25\cos\theta\frac{d\theta}{dt} = \frac{dy}{dt}$

$25\left(\frac{24}{25}\right)\frac{d\theta}{dt} = -10$

$\frac{d\theta}{dt} = -\frac{10}{24} = \boxed{-\frac{5}{12} \text{ radians/sec}}$

32. a)



$x = 15$ $\qquad x^2 + y^2 = 289$

$\frac{dx}{dt} = 5$ $\qquad 225 + y^2 = 289$

$y = 8$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$x \frac{dx}{dt} + y \frac{dy}{dt} = 0$

$15(5) + 8 \frac{dy}{dt} = 0$ $\qquad \boxed{\frac{dy}{dt} = \frac{-75}{8} \text{ ft/sec}}$

b) $A = \frac{1}{2} xy$ $\qquad \frac{dA}{dt} = \frac{1}{2}\left[x \frac{dy}{dt} + y \frac{dx}{dt}\right]$

$\frac{dA}{dt} = \frac{1}{2}\left[15\left(-\frac{75}{8}\right) + 8(5)\right]$

$\boxed{\frac{dA}{dt} = \frac{-805}{16} \text{ ft}^2\text{/sec}}$

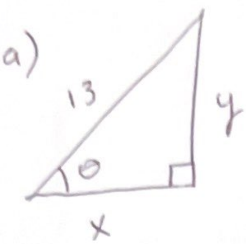
c) $\sin\theta = \frac{y}{17}$ $\qquad y = 17 \sin\theta$ $\qquad \cos\theta = \frac{15}{17}$

$\frac{dy}{dt} = 17 \cos\theta \frac{d\theta}{dt}$

$-\frac{75}{8} = 17\left(\frac{15}{17}\right) \frac{d\theta}{dt}$

$\boxed{\frac{d\theta}{dt} = -\frac{5}{8} \text{ radians/sec}}$

33. a)



$y=12$ $\quad x^2+y^2=169$ $\quad \frac{dy}{dt}=?$

$\frac{dx}{dt}=2$ $\quad x^2+144=169$

$x=5$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$x\frac{dx}{dt}+y\frac{dy}{dt}=0$$

$$5(2)+12\frac{dy}{dt}=0 \rightarrow \frac{dy}{dt}=\frac{-10}{12}=\boxed{\frac{-5}{6}\text{ ft/sec}}$$

b) $A=\frac{1}{2}xy \qquad \frac{dA}{dt}=\frac{1}{2}\left[x\frac{dy}{dt}+y\frac{dx}{dt}\right]$

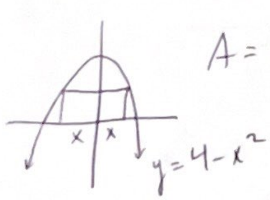
$$\frac{dA}{dt}=\frac{1}{2}\left[5\left(-\frac{5}{6}\right)+12(2)\right]=\boxed{\frac{119}{12}\text{ ft}^2/\text{sec}}$$

c) $\sin\theta=\frac{y}{13} \qquad y=13\sin\theta \qquad \cos\theta=\frac{5}{13}$

$$\frac{dy}{dt}=13\cos\theta\frac{d\theta}{dt}$$

$$-\frac{5}{6}=13\left(\frac{5}{13}\right)\frac{d\theta}{dt} \qquad \boxed{\frac{d\theta}{dt}=-\frac{1}{6}\text{ radian/sec}}$$

34.

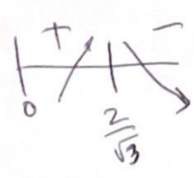


$A=lw \quad l=2x \quad w=4-x^2$ for $x>0$

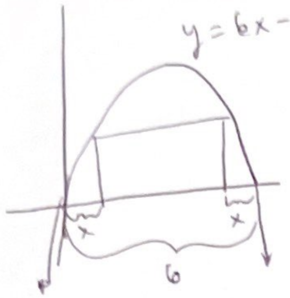
$A=2x(4-x^2)=8x-2x^3$

$A'=8-6x^2=0$

$x^2=\frac{4}{3} \rightarrow \boxed{x=\frac{2}{\sqrt{3}}}$



35.



$A = lw \quad l = 6 - 2x$

$w = 6x - x^2$ for $0 < x < 3$

$A = (6-2x)(6x - x^2)$

$A = 2x^3 - 18x^2 + 36x$

$A' = 6x^2 - 36x + 36 = 0$

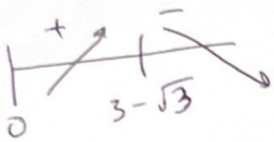
$x^2 - 6x + 6 = 0$

$x^2 - 6x + 9 = 3$

$(x-3)^2 = 3$

$x - 3 = \pm\sqrt{3}$

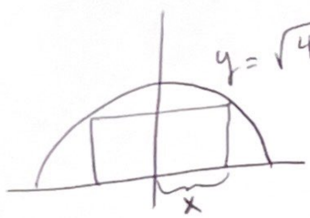
$x = 3 \pm \sqrt{3} \rightarrow x = 3 - \sqrt{3}$



$l = 6 - 2x = 6 - 2(3 - \sqrt{3}) = 6 - 6 + 2\sqrt{3}$

$= \boxed{2\sqrt{3}}$

36.



$A = lw \quad l = 2x \quad w = \sqrt{4-x^2}$

$A = 2x\sqrt{4-x^2}$ for $0 < x < 2$

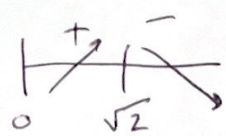
$A' = \dfrac{2x(-2x)}{2\sqrt{4-x^2}} + 2\sqrt{4-x^2}$

$\rightarrow A' = \dfrac{-2x^2 + 2(4-x^2)}{\sqrt{4-x^2}} = \dfrac{-4x^2+8}{\sqrt{4-x^2}} = 0$

$-4x^2 + 8 = 0$

$x^2 = 2$

$x = \sqrt{2}$



$\boxed{x = \sqrt{2}}$