1. $\lim_{x\to\infty} \frac{5x^2-3x+1}{4x^2+2x+5} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{10x-3}{8x+2} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{10}{8} = \boxed{\frac{5}{4}}$

2. If $f(x)$ is continuous at $x=4$, $f(4) = \lim_{x\to 4} f(x)$

so $\lim_{x\to 4} \frac{x^2-7x+12}{x-4} = \frac{0}{0} \to \lim_{x\to 4} \frac{2x-7}{1} = 8-7 = 1$

so $\boxed{f(4) = 1}$

3. $\lim_{x\to 0} \frac{\sin x\cos x - \sin x}{x^2} = \frac{0}{0} \quad \lim_{x\to 0} \frac{\sin x(-\sin x) + \cos x\cos x - \cos x}{2x}$

$= \lim_{x\to 0} \frac{\cos^2 x - \sin^2 x - \cos x}{2x} = \lim_{x\to 0} \frac{\cos 2x - \cos x}{2x} = \frac{0}{0} \rightarrow \lim_{x\to 0} \frac{-2\sin 2x + \sin x}{2}$

$= \frac{0}{2} = 0$

4. $\lim_{x\to 0} \frac{\tan^3 2x}{x^3} = \lim_{x\to 0} \left(\frac{\tan 2x}{x}\right)^3 = \lim_{x\to 0} \left(\frac{2\tan 2x}{2x}\right)^3$

$= 8\lim_{x\to 0} \left(\frac{\tan 2x}{2x}\right)^3 = 8\left[\lim_{x\to 0} \frac{\tan 2x}{2x}\right]^3 = 8(1^3) = 8$

5. $\lim_{x\to 2^+} f(x) = 3(k)(2) - 5 = 6k-5$

$\lim_{x\to 2^-} f(x) = 4(2) - 5k = 8-5k$

$6k - 5 = 8 - 5k$

$11k = 13$

$\boxed{k = \frac{13}{11}}$

6. $3x^2 - 2xy + 3y = 1$

at $x = 2$, $3(2^2) - 2(2)y + 3y = 1$

$$12 - 4y + 3y = 1$$

$$-y = -11$$

$$y = 11$$

$$6x - 2x\frac{dy}{dx} + (-2y) + 3\frac{dy}{dx} = 0$$

$$6x - 2y = 2x\frac{dy}{dx} - 3\frac{dy}{dx}$$

$$\frac{dy}{dx} = \left.\frac{6x-2y}{2x-3}\right|_{(2,11)} = \frac{12-22}{4-3} = \boxed{-10}$$

7. $f(x) = x^2\sqrt{3x+1}$

$$f'(x) = \frac{x^2(3)}{2\sqrt{3x+1}} + 2x\sqrt{3x+1}$$

$$= \frac{3x^2}{2\sqrt{3x+1}} + \frac{2x\sqrt{3x+1}\,(2)\sqrt{3x+1}}{2\sqrt{3x+1}}$$

$$= \frac{3x^2 + 4x(3x+1)}{2\sqrt{3x+1}} = \boxed{\frac{15x^2 + 4x}{2\sqrt{3x+1}}}$$

8. $f(x) = x^4 - 2x^2 + 5x$

$f'(x) = 4x^3 - 4x + 5$

$f''(x) = 12x^2 - 4 \Big|_{x=3} = 12(3^2) - 4 = \boxed{104}$

9. $f(x) = (x^2 - 2x + 1)^5$

$f'(x) = 5(x^2 - 2x + 1)^4 (2x - 2)$

$= 5\left[(x-1)^2\right]^4 (2)(x-1)$

$= \boxed{10(x-1)^9}$

10. $f(x) = \dfrac{\ln x}{e^x}$ $\quad f'(x) = \dfrac{\frac{e^x}{x} - \ln x(e^x)}{(e^x)^2}\Bigg|_{x=1}$

$\dfrac{\frac{e^1}{1} - (\ln 1)(e^1)}{(e^1)^2} = \dfrac{e}{e^2} = \boxed{\dfrac{1}{e}}$

11. $f(x) = \sin 2x \quad f'(x) = 2\cos 2x = 0$ on $[0, 2\pi)$

$\cos 2x = 0$

$2x = \frac{\pi}{2} + \pi n$

$x = \frac{\pi}{4} + \frac{\pi n}{2}$

$n=0, \ x = \frac{\pi}{4}$ ✓ $\quad n=5, \ x = \frac{9\pi}{4}$ outside domain

$n=1, \ x = \frac{3\pi}{4}$ ✓

$n=2, \ x = \frac{5\pi}{4}$ ✓

$n=3, \ x = \frac{7\pi}{4}$ ✓

$\boxed{4}$

12. $f(x) = \ln(3x^2 - 6x)$

$$f'(x) = \frac{6x-6}{3x^2-6x}\Big|_{x=3} = \frac{12}{9} > 0$$

f increasing

$$f''(x) = \frac{(3x^2-6x)(6) - (6x-6)(6x-6)}{(3x^2-6x)^2}\Bigg|_{x=3}$$

$$= \frac{54 - 144}{81} < 0$$

f concave down

13. $f(x) = \frac{1}{4}x^4 - \frac{9}{2}x^2$

$f'(x) = x^3 - 9x$

$f''(x) = 3x^2 - 9 = 0$

$X = \pm\sqrt{3}$

$+ \quad | \quad - \quad | \quad +$ f''

$-\sqrt{3} \qquad \sqrt{3}$

concave up on $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

14. $f(x) = e^x(x^2 - 2x + 3)$

$f'(x) = e^x(2x-2) + e^x(x^2 - 2x + 3)$

$= e^x(x^2 - 2x + 3 + 2x - 2) = e^x(x^2+1) = 0$

NEVER

f does not have a local maximum

15. $f(x) = \sin^3(x^2)$

$f'(x) = 3\sin^2(x^2)\cos(x^2)\left[2x\right]$

$f'\left(\sqrt{\frac{\pi}{4}}\right) = 3\sin^2\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)\left[2\sqrt{\frac{\pi}{4}}\right)$

$= 3\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)(2)\frac{(\sqrt{\pi})}{2} = \frac{3\sqrt{2\pi}}{4}$

16. $\lim\limits_{x \to -2} \frac{2x^2+7x+6}{x+2} = \frac{0}{0} = \lim\limits_{x \to -2} \frac{4x+7}{1} = 4(-2)+7$

$= -1$

17. $f(x) = \frac{x^2+2}{x-1}$ not continuous at $x=1$ so MVT does not apply on $[0,2]$

18. $\lim\limits_{x \to -\infty} \frac{3x-2}{\sqrt{x^2-1}} = \lim\limits_{x \to -\infty} \frac{3x}{|x|} = \lim\limits_{x \to -\infty} \frac{3x}{-x} = \lim\limits_{x \to -\infty} \frac{3}{-1} : \boxed{-3}$

19. $V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = -10$ in^3/min $\quad r = 5$ in. $\quad \frac{dr}{dt} = ?$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-10 = 4\pi(5^2)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-10}{100\pi} = -\frac{1}{10\pi}$ in/min

20. $f(x) = \tan^{-1}(x^2)$

$$f'(x) = \frac{2x}{1+(x^2)^2}\Bigg|_{x=2} = \frac{4}{1+16} = \boxed{\frac{4}{17}}$$

21. $x^4 + 3xy^2 + 5y^3 = 9$

$$4x^3 + 3x\left(2y\frac{dy}{dx}\right) + 3y^2 + 15y^2\frac{dy}{dx} = 0$$

$$4x^3 + 3y^2 = (-6xy - 15y^2)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{4x^3+3y^2}{-6xy-15y^2}\Bigg|_{(1,1)} = \frac{4+3}{-6-15} = \frac{7}{-21} = -\frac{1}{3}$$

$$\boxed{y-1 = -\frac{1}{3}(x-1)}$$

22. $f(x) = \sqrt{x^2+5x+25}\,(x^2+x+1)^3$

$$f'(x) = \left(\sqrt{x^2+5x+25}\,(3)(x^2+x+1)^2(2x+1) + \frac{(2x+5)(x^2+x+1)^3}{2\sqrt{x^2+5x+25}}\right)\Bigg|_{x=0}$$

$$\sqrt{25}\,(3)(1^2)(1) + \frac{5(1)^3}{2(\sqrt{25})} = 15 + \frac{1}{2} = \boxed{\frac{31}{2}}$$

23. $f(x) = 4^x$

$$\boxed{f'(x) = 4^x \ln 4}$$

none of these on MC

24. If f and g are inverses and $(3,1)$ is on f and $f'(3) = 7$

$$g'(1) = \frac{1}{f'(3)} = \boxed{\frac{1}{7}}$$

25. $f(x) = x - 3\sqrt[3]{x} = x - 3x^{\frac{1}{3}}$

$$f'(x) = 1 - x^{-\frac{2}{3}} = 1 - \frac{1}{x^{\frac{2}{3}}} = \frac{x^{\frac{2}{3}} - 1}{x^{\frac{2}{3}}}$$

und at $x = 0$

$= 0$ if $x^{\frac{2}{3}} - 1 = 0$

$x = \pm 1$

$+ \quad - \quad - \quad +$

$-1 \quad 0 \quad 1$

$f(-1) = -1 - 3(-1) = 2$

$f(1) = 1 - 3 = -2$

inc: $(-\infty, -1] \cup [1, \infty)$ local max of 2 at $x = -1$

dec: $[-1, 1]$ local min of -2 at $x = 1$

$$f''(x) = \frac{2}{3} x^{-\frac{5}{3}}$$ und $x = 0$ $f'' \neq 0$

$- \quad +$

0

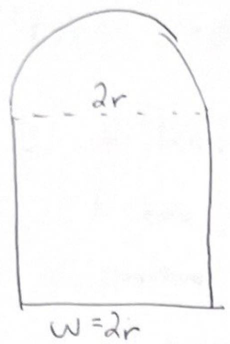
$f(0) = 0$

POI: $(0,0)$

conc up: $(0, \infty)$

conc down: $(-\infty, 0)$

26.



$A = \frac{1}{2}\pi r^2 + 2ry$

$P = 12 = \pi r + 2r + 2y$

$\frac{12 - \pi r - 2r}{2} = y$

$A = \frac{1}{2}\pi r^2 + 2r\left(\frac{12 - \pi r - 2r}{2}\right)$

$A = \frac{1}{2}\pi r^2 + 12r - \pi r^2 - 2r^2$

$A = -\frac{1}{2}\pi r^2 + 12r - 2r^2$

$A' = -\pi r + 12 - 4r = 0$ $\qquad A'' = -\pi - 4 < 0$, MAX

$12 = 4r + \pi r$

$12 = r(4+\pi)$

$r = \frac{12}{4+\pi}$ $\qquad$ width is asked for in question

$w = 2r = \boxed{\frac{24}{4+\pi}\text{ ft}}$

27. f and g are inverses $\qquad f(x) = 2x + \ln x$ $\qquad 2x + \ln x = 2$

TL to g at $x = 2$ $\qquad g: (2, 1)$ $\qquad x = 1$

$f: (1, 2)$

$g'(2) = \frac{1}{f'(1)} = \frac{1}{3}$

$f'(x) = 2 + \frac{1}{x}\Big|_{x=1} = 2 + 1 = 3$

$\boxed{y - 1 = \frac{1}{3}(x - 2)}$

28. $f'(x) = 3 - e^{\sin 3x}$ on $[0, 2\pi)$

I. f has four points of inflection on $[0, 2\pi)$

f has a point of inflection if f'' changes from positive to negative or vice versa. If f'' changes signs, f' goes from increasing to decreasing or vice versa. The graph of f' does this (has local max or min) 6 times on $[0, 2\pi)$ so NO

II. f is increasing on $[0, 2\pi)$

f is increasing on $[0, 2\pi)$ if $f'(x) > 0$ on $[0, 2\pi)$ which the graph shows that it is. YES

III. f has a local maximum at $x = \frac{\pi}{2}$

f has a local maximum at $x = \frac{\pi}{2}$ if f' changes from positive to negative at $x = \frac{\pi}{2}$, which the graph shows that it does not. NO

II only

29. speed is $|v(t)|$ so graph $s = |-t^3 + 2t^2 + 2^{-t}|$ for $t \geq 0$

and calculate intervals on increasing.

$$(.177, 1.256) \cup (2.057, \infty)$$

30. $y = e^x \qquad (k, e^k)$

$y' = e^x\big|_k = e^k$

$l - e^k = e^k(x-k)$

$l = e^k(x-k) + e^k$ contains $(0, \frac{1}{2})$

$\frac{1}{2} = e^k(0-k) + e^k$

$\frac{1}{2} = -ke^k + e^k$ → solve on calculator

$\boxed{k \approx .768}$

31. $f'(x)$ has a relative maximum if f' changes from positive to negative. On the interval $(-2.5, 2.5)$ $f'(x) = x^3 - 4\sin(x^2) + 1$ goes from positive to negative only at $\boxed{x = 0.542}$

32. $\frac{dV}{dt} = -3 \qquad \frac{dr}{dt} = -.25 \qquad r = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-3 = 4\pi r^2\left(-\frac{1}{4}\right)$

$r^2 = \frac{3}{\pi}$

$\boxed{r \approx .977}$

33. $f(x) = e^x + x$

$f(0) = e^0 + 0 = 1$

$f(-1) = e^{-1} - 1 = \frac{1}{e} - 1 < 0$

$e^x + x$ is a sum of two continuous functions and is therefore continuous on $(-1, 0)$, so by IVT, $e^x + x = 0$ on $(-1, 0)$

$$x_{n+1} = x_n - \frac{e^{x_n} + x_n}{e^{x_n} + 1}$$

$x_0 = -.5$

$x_1 = -.566311003$2

$x_2 = -.567143165$

$\boxed{x_3 = -.5671432904}$

$x_4 =$ same