1. $f(x) = (2x-1)^5(x+1)$ $\quad f(1) = 2$

$f'(x) = (2x-1)^5(1) + 5(2x-1)^4(2)(x+1)$

$f'(1) = 1 + 20 = 21$

$y - 2 = 21(x-1)$

$y - 2 = 21x - 21$

$\boxed{y = 21x - 19}$

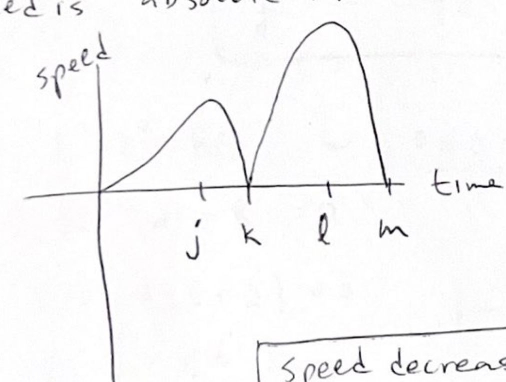
2. $x^2 - 3xy = 10$ $\quad (1, -3)$

$2x - 3x\frac{dy}{dx} - 3y = 0$

$\frac{dy}{dx} = \left.\frac{3y - 2x}{-3x}\right|_{(1,-3)} = \frac{3(-3) - 2(1)}{-3(1)} = \frac{-11}{-3} = \frac{11}{3}$

$\boxed{y + 3 = \frac{11}{3}(x-1)}$

3. speed is absolute value of velocity so graph of speed is



speed decreasing $\quad j \le t \le k$ and $l \le t \le m$

4. $x(t) = (t-a)(t-b)$

$v(t) = (t-a)(1) + (t-b)(1) = 2t - a - b = 0$

$$\boxed{t = \frac{a+b}{2}}$$

5. $x(t) = \sin t - \cos t$

$v(t) = \cos t + \sin t = 0 \qquad \cos t = -\sin t$

$t = \frac{3\pi}{4}$ first time for $t \geq 0$

$a(t) = -\sin t + \cos t$

$a\left(\frac{3\pi}{4}\right) = -\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = \frac{-2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$

6. $x^4 + 3xy^2 + 5y^3 = 9 \qquad (1,1)$

$4x^3 + 6xy\frac{dy}{dx} + 3y^2 + 15y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \left.\frac{-4x^3 - 3y^2}{6xy + 15y^2}\right|_{(1,1)} = \frac{-4-3}{6+15} = \frac{-7}{21} = -\frac{1}{3}$

$$\boxed{y - 1 = -\frac{1}{3}(x-1)}$$

7. $s(t) = t^3 + t^2 - 8t$ for $\underline{0 \leq t \leq 10}$, moves right if $v > 0$

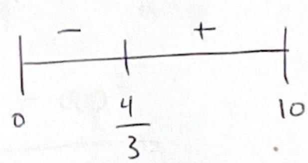
$v(t) = 3t^2 + 2t - 8 = 0$

$(3t-4)(t+2) = 0$

$t = \frac{4}{3} \qquad t = -2$

↓

not in domain



$$\boxed{\left(\frac{4}{3}, 10\right]}$$

8. $y = 3e^{-2x}$ at $x = \ln 2$, find normal line

$y(\ln 2) = 3e^{-2\ln 2} = 3e^{\ln\frac{1}{4}} = \frac{3}{4}$ $\quad (\ln 2, \frac{3}{4})$

$y' = 3e^{-2x}(-2) = -6e^{-2x}\Big|_{x=\ln 2} = -6e^{-2\ln 2} = -\frac{6}{4} = -\frac{3}{2}$

$= m_{TL}$

$m_{NL} = \frac{2}{3}$ $\quad y - \frac{3}{4} = \frac{2}{3}(x - \ln 2)$

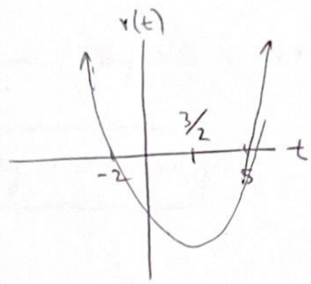
$$\boxed{y = \frac{2}{3}(x - \ln 2) + \frac{3}{4}}$$

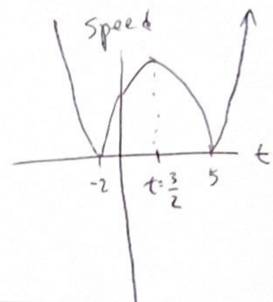
9. $v(t) = t^2 - 3t - 10 = 0$

$(t-5)(t+2)$

$v'(t) = 2t - 3 = 0$

$t = \frac{3}{2}$





$\boxed{\text{Speed increases} \quad 0 < t < \frac{3}{2} \text{ and } t > 5}$

10. $y = ax^2$ $\quad y(p) = ap^2$ $\quad (p, ap^2)$

$y' = 2ax\Big|_{x=p} = 2ap$ $\quad y - ap^2 = 2ap(x - p)$

y-int, $x = 0$ $\longrightarrow$ $y - ap^2 = 2ap(-p)$

$y - ap^2 = -2ap^2$

$y = -ap^2$

$$\boxed{(0, -ap^2)}$$

11. $y = 4x^3 - 7x^2$ at $x = 3$ $\quad y(3) = 4(3^3) - 7(3^2) = 108 - 63 = 45$

$(3, 45)$

$y' = 12x^2 - 14x \Big|_{x=3} = 12(3^2) - 14(3) = 108 - 42 = 66$

$$\boxed{y - 45 = 66(x-3)}$$

12. $9x^2 + 16y^2 = 52 \quad (2, -1)$

$18x + 32y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-18x}{32y} = \frac{-9x}{16y}\Big|_{(2,-1)} = \frac{-18}{-16} = \frac{9}{8}$

$y + 1 = \frac{9}{8}(x-2) \rightarrow 8y + 8 = 9x - 18$

$\rightarrow \boxed{9x - 8y - 26 = 0}$

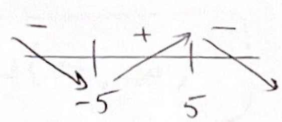
13. $s = t^3 - 6t^2 + 9t$

$v(t) = 3t^2 - 12t + 9$

$a(t) = 6t - 12$

$a(4) = 6(4) - 12 = \boxed{12}$

14. $f(x) = -x^3 + 75x$



$f'(x) = -3x^2 + 75 = 0$

$x^2 = 25$

$x = \pm 5$

$\boxed{\text{decreasing}: (-\infty, -5] \cup [5, \infty)}$

15. $\lim\limits_{x \to \frac{\pi}{4}} \frac{2\tan 2x}{1 + 3\sec 2x} = \frac{\infty}{\infty} \rightarrow \lim\limits_{x \to \frac{\pi}{4}} \frac{4\sec^2 2x}{6\sec 2x \tan 2x} = \lim\limits_{x \to \frac{\pi}{4}} \frac{4\sec 2x}{6\tan 2x}$

$= \lim\limits_{x \to \frac{\pi}{4}} \frac{\frac{2}{\cos 2x}}{\frac{3\sin 2x}{\cos 2x}} = \lim\limits_{x \to \frac{\pi}{4}} \frac{2}{3\sin 2x} = \frac{2}{3\sin \frac{\pi}{2}} = \boxed{\frac{2}{3}}$

16. $f(x) = \dfrac{x}{\ln x}$ $\quad f'(x) = \dfrac{\ln x - x\left(\frac{1}{x}\right)}{(\ln x)^2} = \dfrac{\ln x - 1}{(\ln x)^2} = 0$ if

$\ln x - 1 = 0$

$\ln x = 1$

$x = e$

f' undefined if $x = 1$, not defined on $f(x)$ so not a C.V.

$-$ $+$

1 e

$f(e) = \dfrac{e}{\ln e} = e$

local min of e at $x = e$

(e, e)

17. $g(x) = xe^{kx}$ $\quad x = 2$

$g'(x) = e^{kx} + kxe^{kx} = e^{kx}(1 + kx) = 0$

$1 + kx = 0$

at $x = 2$, $1 + 2k = 0$, $\boxed{k = -\frac{1}{2}}$

18. $h(x) = \frac{1}{4}x^4 - 3x^2$ $\quad h'(x) = x^3 - 6x$ $\quad h''(x) = 3x^2 - 6 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$+$ $-$ $+$ h''

$-\sqrt{2}$ $\sqrt{2}$

conc up: $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

19. $V = \pi r^2 h = 250\pi$ $\quad SA = 2\pi r^2 + 2\pi r h$

$h = \dfrac{250\pi}{\pi r^2} = \dfrac{250}{r^2}$ $\quad SA = 2\pi r^2 + 500\pi r^{-1}$

$SA' = 4\pi r - \dfrac{500\pi}{r^2} = \dfrac{4\pi r^3 - 500\pi}{r^2} = 0$

$-$ $+$

0 5

$\sim$ und. $r = 0$, makes no sense

if $4\pi r^3 - 500\pi = 0$

$r^3 = 125$

$r = 5$

$h = \dfrac{250}{5^2} = \boxed{10 \text{ feet}}$

20. $V = \dfrac{a^3}{6\sqrt{2}} \rightarrow dV = \dfrac{3a^2\,da}{6\sqrt{2}} \rightarrow dV = \dfrac{a^2\,da}{2\sqrt{2}}$ $a = 10$, $da = .1$

$$dV = \frac{10^2(.1)}{2\sqrt{2}} = \frac{10}{2\sqrt{2}} = \boxed{\frac{5}{\sqrt{2}} \text{ cubic inches}}$$

21. $x(t) = 6t^3 - 2t$

$v(t) = 18t^2 - 2$

$a(t) = 36t$ $a(6) = 36(6) = \boxed{216}$

22.

x	2	3	4
$f(x)$	1	7	12
$f'(x)$	3	0	-3
$f''(x)$	0	1	-2

I. $f(x)$ has a local maximum at $x = 2$ NO $f'(2) \neq 0$

II. $f(x)$ has a local maximum at $x = 3$

NO $f'(3) = 0$ but $f''(3) = 1$

$1 > 0$, local minimum by 2nd derivative test

III. $(2,1)$ is a possible point of inflection

YES $f''(2) = 0$ and $f(2) = 1$

IV. There must be a point of inflection on the interval $(3,4)$

YES $f''(3) = 1$ and $f''(4) = -2$, because f'' is a continuous function, The Intermediate Value Theorem guarantees $f''(x) = 0$ on $(3,4)$ because $-2 < 0 < 1$

The sign change must occur on $(3,4)$.

V. $f(x)$ is increasing at $x = 2$ YES $f'(2) = 3 > 0$

VI. $f(x)$ is decreasing at $x = 4$ YES $f'(4) = -3 < 0$

$$\boxed{\text{III, IV, V, VI}}$$

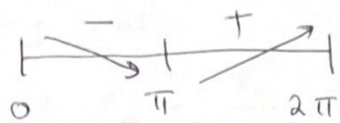
23. $f(x) = \sin(\cos x)$ $\quad f'(x) = \cos(\cos x)[-\sin x] = 0 \quad [0, 2\pi]$

if $\cos(\cos x) = 0$ $\qquad -\sin x = 0$

$\checkmark$

$\cos\frac{\pi}{2} = 0$ so, $\cos x$ must $= \frac{\pi}{2}$ or $\frac{3\pi}{2}$ $\qquad x = 0, \pi, 2\pi$

$\cos\frac{3\pi}{2} = 0$



$\cos x \neq \frac{\pi}{2} > 1$

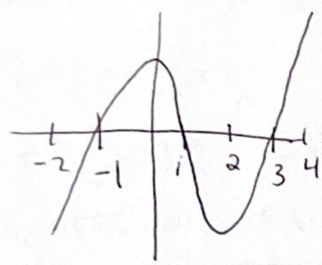
$\cos x \neq \frac{3\pi}{2} > 1$

range of $\cos x$ is $[-1, 1]$

$\boxed{\text{increasing on } [\pi, 2\pi]}$

24. $\lim\limits_{x \to 4} \dfrac{\sin\frac{\pi}{2}x}{x^3 - 64} = \frac{0}{0} = \lim\limits_{x \to 4} \dfrac{\frac{\pi}{2}\cos\frac{\pi}{2}x}{3x^2} = \dfrac{\frac{\pi}{2}\cos 2\pi}{48} = \boxed{\dfrac{\pi}{96}}$

25. $f''(x) \rightarrow$



f is concave down if $f''(x) < 0$

$\boxed{-2 < x < -1 \text{ and } 1 < x < 3}$

26. $f'(x) = x - 4e^{-\sin 2x}$ Points of Inflection on f occur when f' has local maxima or minima

Use graphing calculator to graph $f'(x)$ on $(0, 2\pi)$.

It has $\boxed{4}$ turning points

27. $f'(x) = \sqrt{x^4 + 1} + x^3 - 3x$ f has a local maximum if f' changes from positive to negative

Graph f' and calculate the appropriate x-int $\boxed{x \approx .350}$

28. $x^3 - 2x + 1 = x + 5$ if $x^3 - 3x - 4 = 0$

$2^3 - 3(2) - 4 = -2$ $3^3 - 3(3) - 4 = 14$

by IVT, zero must exist on $(2,3)$

choose 2.5 as starting point

$x_0 = 2.5$

$x_1 = 2.238095238$

$x_2 = 2.196814629$

$x_3 = 2.19582391$

$\boxed{x_4 = 2.195823345}$

$x_5 = 2.195823345$

$$x_{n+1} = x_n - \frac{x_n^3 - 3x_n - 4}{3x_n^2 - 3}$$

29. $R: 3 \to 3.004$ $r: 2 \to 2.004$ $R = 3$ $dR = .004$

$r = 2$ $dr = .004$

$A = \pi^2(R^2 - r^2)$

$dA = \pi^2(2R\,dR - 2r\,dr)$

$dA = \pi^2(6(.004) - 4(.004))$

$\boxed{dA = .0789568352}$

30. $\sqrt[4]{16.003}$ $f(x) = \sqrt[4]{x}$ $a = 16$ $x = 16.003$

$L(x) = \sqrt[4]{a} + f'(a)[x - a]$

$= 2 + \frac{1}{4a^{\frac{3}{4}}}(16.003 - 16)$

$= 2 + \frac{.003}{4(8)} = \boxed{2.00009375}$