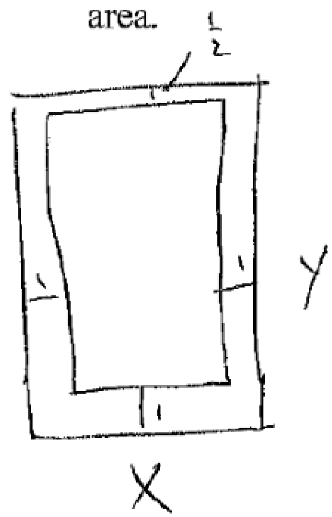
A page of a book is to have an area of 90 square inches, with one inch margins at the bottom and sides, and a one half inch margin at the top. Find the dimension of the page that will allow the largest printed area.



$$A_p = xy = 90$$

$$y = \frac{90}{x}$$

$$A_I = (x-2)\left(y - \frac{3}{2}\right)$$

$$A_I = (x-2)\left(\frac{90}{x} - \frac{3}{2}\right)$$

$$A_I = 90 - \frac{180}{x} - \frac{3}{2}x + 3$$

$$A_I = 93 - 180x^{-1} - \frac{3}{2}x$$

$$A_I' = \frac{180}{x^2} - \frac{3}{2} = \frac{360 - 3x^2}{2x^2}$$

$$= 0 \text{ if } 3x^2 = 360$$

$$x^2 = 120$$

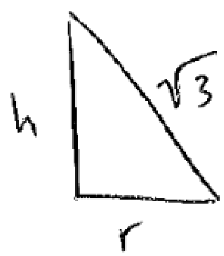
$$x = 2\sqrt{30}$$

$$A_I'' = \frac{-360}{x^3}\Big|_{x=2\sqrt{30}} < 0, \text{ max}$$

$$\boxed{\begin{array}{l} x = 2\sqrt{30} \\ y = \frac{45}{\sqrt{30}} \end{array}}$$

$$y = \frac{90}{2\sqrt{30}}$$

. A right triangle whose hypotenuse is $\sqrt{3}$ meters is revolved about one of its legs to generate a right circular cone. Find the radius, height, and volume of the cone of the greatest volume that can be made this way.



$$h^2 + r^2 = \sqrt{3}^2$$

$$h = \sqrt{3 - r^2}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi r^2 \sqrt{3 - r^2}$$

$$V' = \frac{1}{3}\pi r^2 \left(\frac{-2r}{2\sqrt{3-r^2}}\right) + \frac{2}{3}\pi r \sqrt{3 - r^2}$$

$$V' = \frac{-\pi r^3}{3\sqrt{3-r^2}} + \frac{2\pi r \sqrt{3-r^2}^2}{3\sqrt{3-r^2}}$$

$$V' = \frac{-\pi r^3 + 2\pi r(3 - r^2)}{3\sqrt{3-r^2}} = \frac{-\pi r^3 + 6\pi r - 2\pi r^3}{3\sqrt{3-r^2}}$$

$$-\pi r^3 - 2\pi r^3 + 6\pi r = 0 \rightarrow -3\pi r^3 = -6\pi r$$

$$\rightarrow r^2 = 2 \rightarrow \boxed{r = \sqrt{2}\ m}$$

$$h = \sqrt{3 - \sqrt{2}^2}$$

$$h = \sqrt{3 - 2}$$

$$\boxed{h = 1\ m}$$

$$V = \frac{1}{3}\pi (\sqrt{2})^2 (1)$$

$$\boxed{V = \frac{2\pi}{3} m^3}$$

. Sand falls from a conveyor belt at the rate of twenty cubic meters per minute onto the top of a cone-shaped pile. The height of the pile is always three-eighths of the base diameter. How fast is the height of the pile changing when it is 4 meters high? How fast is the radius changing when the pile is 4 meters high?

a) $\frac{dV}{dt} = 20 \; m^3/min$

$h = \frac{3}{8} d$

$h = \frac{3}{8}(2r)$

$h = \frac{3}{4} r$

$r = \frac{4}{3} h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{4}{3}h\right)^2 h$

$V = \frac{16\pi h^3}{27}$

$\frac{dV}{dt} = \frac{16\pi}{9} h^2 \frac{dh}{dt}$

$20 = \frac{16\pi}{9}(4^2)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{180}{256\pi}$

$\frac{dh}{dt} = \boxed{\frac{45}{64\pi} \; m/min}$

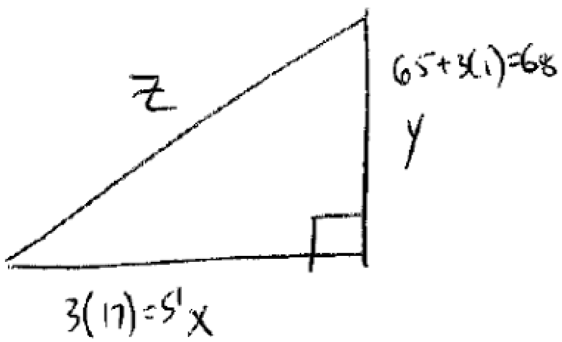
$h = \frac{3}{4} r$

$\frac{dh}{dt} = \frac{3}{4}\frac{dr}{dt}$

$\frac{45}{64\pi} = \frac{3}{4}\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{180}{192\pi} = \boxed{\frac{15}{16\pi} \; m/min}$

A balloon is rising vertically above a level, straight road at a constant rate of one foot per second. Just when the balloon is 65 feet above the ground, a bicycle moving at a constant rate of 17 feet per second passes under it. How fast is the distance between the bicycle and balloon increasing 3 seconds later?



$$z = \sqrt{51^2 + 68^2} = 85$$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

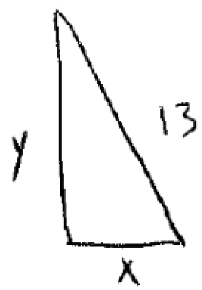
$$2(51)(17) + 2(68)(1) = 2(85)\frac{dz}{dt}$$

$$1734 + 136 = 170\frac{dz}{dt}$$

$$\frac{1870}{170} = \frac{dz}{dt} = \boxed{11 \text{ ft/s}}$$

A 13 foot ladder is leaning against the side of a house when its base starts to slide away. By the time the base is 12 feet from the house, the base is moving at a rate of 5 feet per second.

a. How fast is the top of the ladder sliding down the wall?



we know $\frac{dx}{dt} = 5$ we want $\frac{dy}{dt}$

when $x = 12$, $y = 5$

$$x^2 + y^2 = 13^2$$

$$y^2 = 169 - x^2$$

$$y = \sqrt{169 - x^2}$$

$$\frac{dy}{dt} = \frac{-2x}{2\sqrt{169-x^2}}\frac{dx}{dt} \quad \frac{-x}{\sqrt{169-x^2}}\frac{dx}{dt} = \frac{-12}{\sqrt{169-12^2}}(5) = \boxed{-12 \text{ ft/s}}$$

b. At what rate is the area of the triangle formed by the ladder, wall, and ground changing at that moment?

$$A = \frac{1}{2}xy$$

$$A = \frac{1}{2}xy$$

$$\frac{dA}{dt} = \frac{1}{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$$

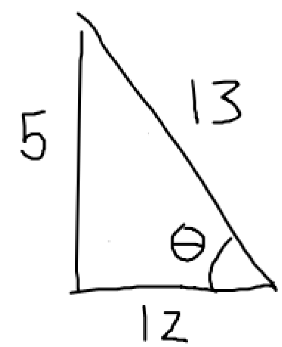
$$\frac{dA}{dt} = \frac{1}{2}\left[(12)(-12) + (5)(5)\right] = \boxed{\frac{-119}{2}\ ft^2/sec}$$

c. At what rate is the angle θ between the ladder and the ground changing at that moment?

$$\tan\theta = \frac{y}{x} \qquad \text{when } x = 12,\ y = 5,\ \tan\theta = \frac{5}{12},\ \theta = \tan^{-1}\left(\frac{5}{12}\right)$$

$$(\sec^2\theta)\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$$

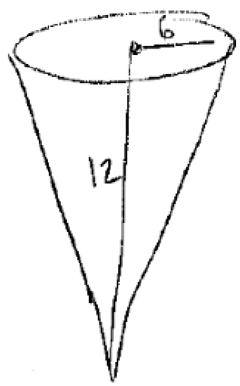
$$\left(\frac{13}{12}\right)^2\frac{d\theta}{dt} = \frac{(12)(-12) - (5)(5)}{(12)^2}$$



$$\sec\theta = \frac{13}{12}$$

$$\boxed{\frac{d\theta}{dt} = -1 \text{ rad/sec}}$$

A water tank has the shape of an inverted right circular cone of altitude 12 feet and base radius 6 feet. If water is begin pumped into the tank at a rate of 10 gallons per minute, approximate the rate at which the water level is rising when the water is 3 feet deep (1 gallon is approximately 0.1337 cubic feet).



$h = 2r$

$r = \frac{h}{2}$

$\frac{dV}{dt} = 10(.1337) = 1.337$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

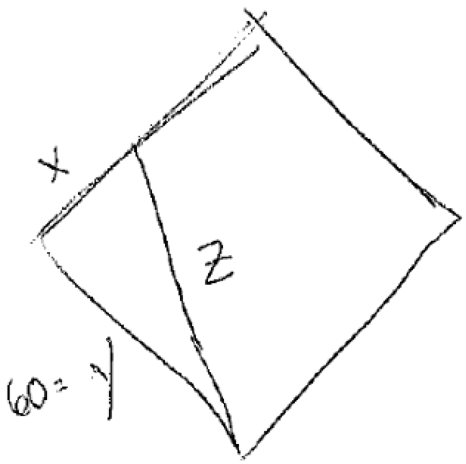
$\frac{1}{3}\pi \frac{h^2}{4} \cdot h$

$V = \frac{\pi h^3}{12}$

$\frac{dV}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt}$

$1.337 = \pi\left(\frac{3^2}{4}\right)\frac{dh}{dt} \rightarrow \boxed{\frac{dh}{dt} \approx .189 \text{ ft}/\text{min}}$

A softball diamond has the shape of a square with sides 60 feet long. If a player is running from second base to third at a speed of 24 feet per second, at what rate is her distance from home plate changing when she is 20 feet from third?



$$x^2 + y^2 = z^2$$

$x = 20$

$y = 60$

$z = \sqrt{20^2 + 60^2}$

$z = 20\sqrt{10}$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$(-24)(2)(20) + 2(60)(0) = 2(20\sqrt{10}) \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{-960}{40\sqrt{10}} = \boxed{\frac{-24}{\sqrt{10}} \text{ ft/s} \approx -7.589 \text{ ft/s}}$$