Gas is being pumped into a spherical balloon at a rate of 5 feet cubed per minute. Find the rate at which the radius is changing when the diameter is 18 inches.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 5 \text{ ft}^3/\text{min}$$

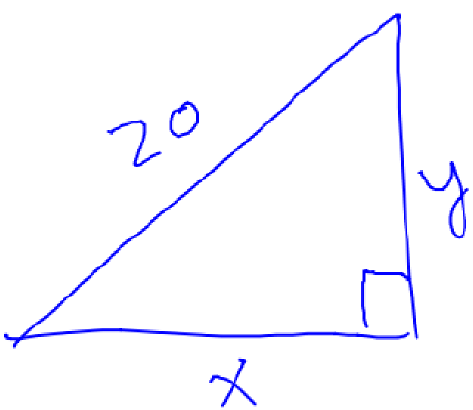
$$d = 18 \text{ in} \rightarrow r = 9 \text{ in} = \frac{3}{4} \text{ ft}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$5 = 4\pi\left(\frac{3}{4}\right)^2 \frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = \frac{20}{9\pi} \text{ ft}/\text{min}}$$

A 20 foot ladder leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of 3 feet per second, how fast is the ladder sliding down the building when the top of the ladder is 8 feet from the ground?



$$x^2 + y^2 = 400$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(4\sqrt{21})(3) + 2(8)\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{3\sqrt{21}}{2} \text{ ft/sec}$$

$\frac{dx}{dt} = 3$ $\quad \frac{dy}{dt} = ?$

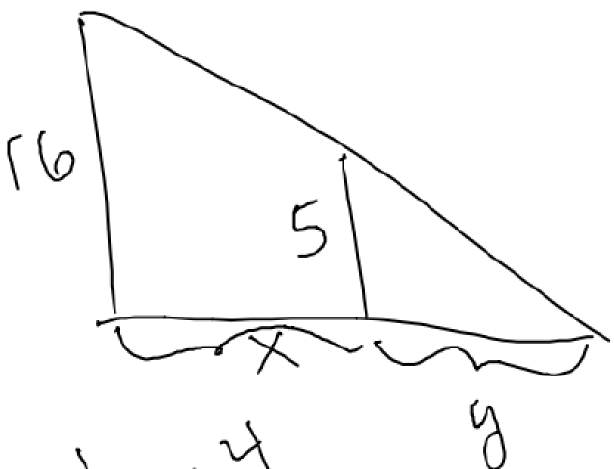
$y = 8$

$$x^2 + 64 = 400$$

$$x^2 = 336$$

$$x = 4\sqrt{21}$$

A light is at the top of a 16 foot pole. A 5 feet tall boy walks away from the pole at a rate of 4 feet per second. At what rate is the tip of his shadow moving when he is 18 feet from the pole? At what rate is the length of his shadow increasing?



$\frac{dx}{dt} = 4$

$\frac{dy}{dt} = ?$

$$\frac{x+y}{16} = \frac{y}{5}$$

$$\frac{1}{16}x + \frac{1}{16}y = \frac{1}{5}y$$

$$\frac{1}{16}x = \frac{11}{80}y$$

$$\frac{1}{16}\frac{dx}{dt} = \frac{11}{80}\frac{dy}{dt}$$

$$\frac{4}{16} = \frac{11}{80}\frac{dy}{dt}$$

length of shadow

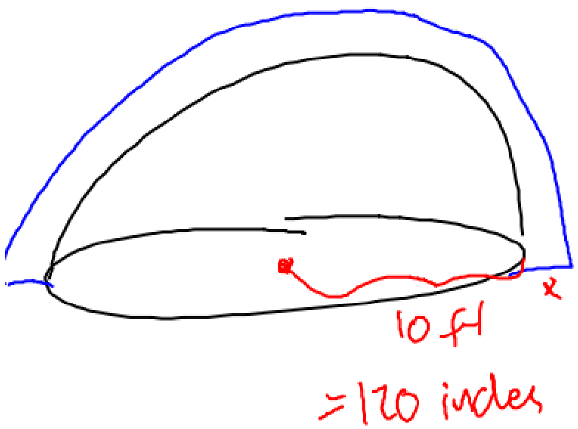
$$\boxed{\frac{dy}{dt} = \frac{20}{11} \text{ ft/sec}}$$

tip of shadow $= x+y$

$$\frac{dx}{dt} + \frac{dy}{dt} = 4 + \frac{20}{11}$$

$$\boxed{= \frac{64}{11} \text{ ft/sec}}$$

The top of a silo has the shape of a hemisphere of diameter 20 feet. If it is coated uniformly with a layer of ice and if the thickness is decreasing at a rate of ¼ inches per hour, how fast is the volume of the ice changing when the ice is 2 inches thick?



$$\frac{dx}{dt} = -\frac{1}{4}\ \text{in}/\text{hr}.$$

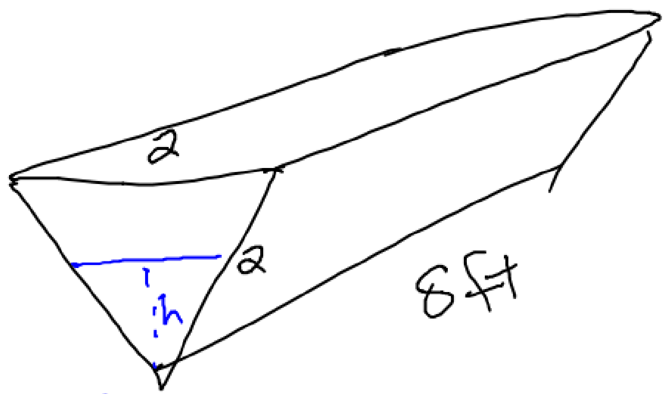
$$x = 2$$

$$V_{ice} = \frac{2}{3}\pi(x+120)^3 - \frac{2}{3}\pi(120^3)$$

$$\frac{dV_{ice}}{dt} = \left[2\pi(x+120)^2 - 0\right]\frac{dx}{dt}$$

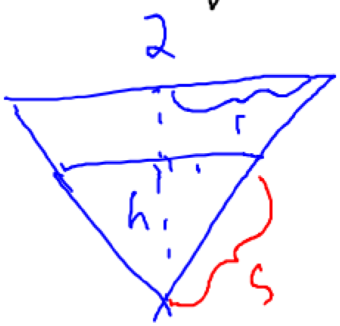
$$\frac{dV}{dt} = \left[2\pi(2+120)^2\right]\left(-\frac{1}{4}\right) = \boxed{-7442\pi\ \text{in}^3/\text{hr}}$$

The ends of a water trough 8 feet long are equilateral triangles whose sides are 2 feet long. If water is being pumped into the trough at a rate of 5 feet cubed per minute, find the rate at which the water level is rising when the depth of the water is 8 inches.



$h = 8 \text{ inches} = \frac{2}{3} \text{ ft}$

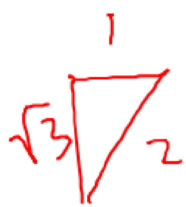
$$\frac{dV}{dt} = 5$$



$$A = \frac{s^2\sqrt{3}}{4}$$

$$V = \frac{8s^2\sqrt{3}}{4} = 2s^2\sqrt{3}$$

$$V = 2\left(\frac{2h}{\sqrt{3}}\right)^2\sqrt{3}$$



$$\frac{h}{s} = \frac{\sqrt{3}}{2} \rightarrow s = \frac{2h}{\sqrt{3}}$$

$$V = \frac{8h^2}{\sqrt{3}}$$

$$\frac{dV}{dt} = \frac{16h}{\sqrt{3}} \frac{dh}{dt}$$

$$5 = \frac{16\left(\frac{2}{3}\right)}{\sqrt{3}} \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{15\sqrt{3}}{32} \text{ ft}^3/\text{min}$$

The area of an equilateral triangle is decreasing at a rate of 4 cm squared per minute. Find the rate at which the length of a side is changing when the area of the triangle is 200 square centimeters.

$$A = \frac{s^2\sqrt{3}}{4}$$

$$\frac{dA}{dt} = -4$$

$$A = 200 = \frac{s^2\sqrt{3}}{4}$$

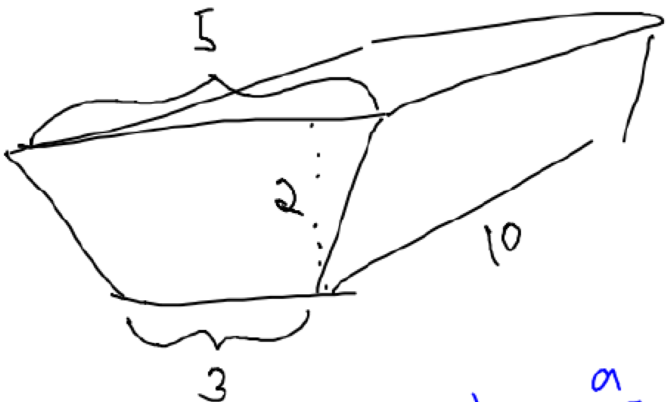
$$\frac{dA}{dt} = \frac{s\sqrt{3}}{2}\frac{ds}{dt}$$

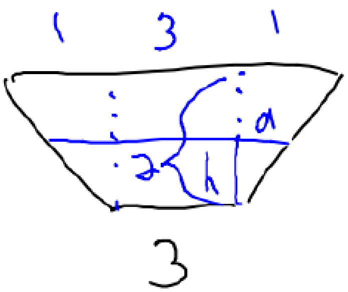
$$s = \sqrt{\frac{800}{\sqrt{3}}} = \frac{20\sqrt{2}}{\sqrt[4]{3}}$$

$$-4 = \frac{20\sqrt{2}}{\sqrt[4]{3}} \cdot \frac{\sqrt{3}}{2}\frac{ds}{dt} = \frac{10\sqrt{6}}{\sqrt[4]{3}}\frac{ds}{dt}$$

$$\frac{ds}{dt} = \frac{-2\sqrt[4]{3}}{5\sqrt{6}} \text{ cm/min}$$

The ends of a horizontal water trough 10 feet long are isosceles trapezoids with lower base 3 feet, upper base 5 feet, and altitude 2 feet. If the water level is rising at a rate of ¼ inches per minute when the depth of the water is 1 foot, how fast is water entering the trough?





$$\frac{1}{2} = \frac{a}{h}$$

$$a = \frac{h}{2}$$

$$V = 10\left(3h + 2\left[\frac{1}{2}(a)(h)\right]\right)$$

$$V = 10\left(3h + \frac{h^2}{2}\right)$$

$$V = 30h + 5h^2$$

$$\frac{dV}{dt} = 30\frac{dh}{dt} + 10h\frac{dh}{dt}$$

$$\frac{dV}{dt} = 30\left(\frac{1}{48}\right) + 10(1)\left(\frac{1}{48}\right) = \frac{40}{48}$$

$$= \frac{5}{6}\ ft^3/min$$

$$\frac{dh}{dt} = \frac{1}{4}\ in/min$$

$$= \frac{1}{48}\ ft/min$$

$$h = 1$$

As sand leaks out of a hole in a container, it forms a conical pile whose altitude is always the same as its radius. If the height of the pile is increasing at a rate of six inches per minute, find the rate at which he sand is leaking out when the altitude is ten inches.

$h = r$

$\frac{dh}{dt} = 6$

$h = 10$

$\frac{dV}{dt} = ?$

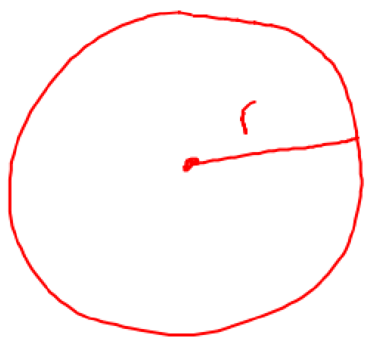
$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi h^3$$

$$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = \pi(10^2)(6) = \boxed{600\pi \text{ in}^3/\text{min}}$$

A stone is dropped into a lake, causing circular waves whose radii increase at a constant rate of .5 meters per second. At what rate is the circumference of a wave changing when its radius is 4 meters?



$$C = 2\pi r \qquad \frac{dr}{dt} = .5$$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt} \qquad r = 4$$

$$\frac{dc}{dt} = 2\pi(.5) = \boxed{\pi \text{ m/sec}}$$