A metal cylindrical container with an open top is to hold one cubic foot. If there is no waste in construction, find the dimensions that require the least amount of material.

$$V = 1\ ft^3 = \pi r^2 h \qquad h = \frac{1}{\pi r^2}$$

$$M = \pi r^2 + 2\pi r h$$

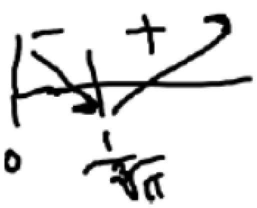
$$M = \pi r^2 + \frac{2\pi r}{\pi r^2} = \pi r^2 + 2r^{-1}$$

$$M' = 2\pi r - \frac{2}{r^2} = \frac{2\pi r^3 - 2}{r^2}$$

$M' = 0$ if $2\pi r^3 - 2 = 0$

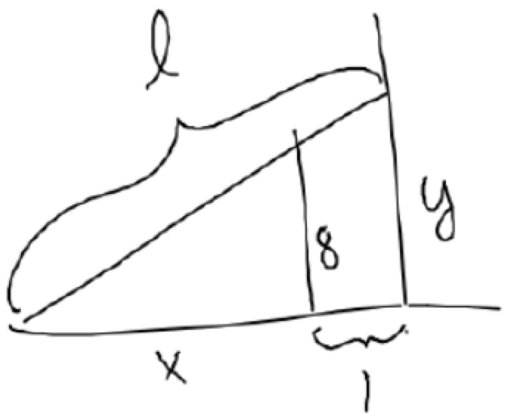
$$r^3 = \frac{1}{\pi}$$

$$\boxed{r = \frac{1}{\sqrt[3]{\pi}}\ ft.}$$



$$h = \frac{1}{\pi\left(\frac{1}{\sqrt[3]{\pi}}\right)^2} = \boxed{\frac{1}{\sqrt[3]{\pi}}\ ft = h}$$

An eight feet tall fence stands on level ground and runs parallel to a tall building. If the fence is 1 foot from the building, find the length of the shortest ladder that will extend from the ground over the fence to the wall on the building.



$$\frac{x+1}{y} = \frac{x}{8} \qquad y = \frac{8x+8}{x} = 8 + \frac{8}{x}$$

$$l^2 = (x+1)^2 + \left(8 + \frac{8}{x}\right)^2 = x^2 + 2x + 1 + 64 + \frac{128}{x} + \frac{64}{x^2}$$

$$l^2 = x^2 + 2x + 65 + \frac{128}{x} + \frac{64}{x^2}$$

l^2 at a min at same x value as l so...

$$(l^2)' = 2x + 2 - \frac{128}{x^2} - \frac{128}{x^3}$$

$$= 2x^4 + 2x^3 - 128x - 128 = 0$$

$$2x^3(x+1) - 128(x+1) = 0$$

$$(2x^3 - 128)(x+1) = 0$$

$$x = 4 \qquad x = -1$$

only $x=4$ makes sense

$$\ell^2 = (4+1)^2 + \left(8+\frac{8}{4}\right)^2 =$$

$$\ell = \sqrt{25+100} = \boxed{5\sqrt{5}\text{ feet}}$$

A hotel charges 80 dollars per day for a room gives special rates to organizations that reserve between 30 and 60 rooms. If more than 30 rooms are reserved, the charge per room is decreased by $1 times the number of rooms over 30. Under these conditions, how many rooms must be rented if the hotel is to receive the maximum income per day?

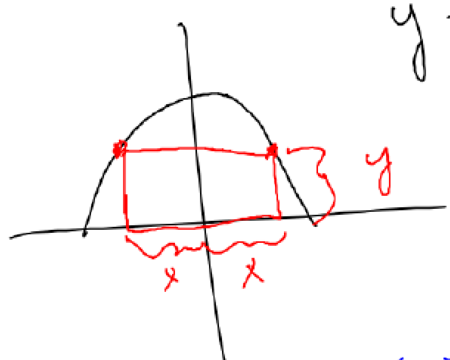
x = # of rooms > 30

$$I = (x+30)(80-x) = -x^2 + 50x + 240$$

$$I' = -2x + 50 = 0 \qquad x = 25$$

$\#$ of rooms $= x + 30 = 55$ rooms

Find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius a, if two vertices lie on the diameter.



$$y = \sqrt{a^2 - x^2}$$

$$l = 2x \qquad w = \sqrt{a^2 - x^2}$$

$$A = 2x\sqrt{a^2 - x^2}$$

$$A' = \frac{\cancel{2}x(-2x)}{\cancel{2}\sqrt{a^2 - x^2}} + \frac{2\sqrt{a^2 - x^2}^{\,2}}{\sqrt{a^2 - x^2}}$$

$$A' = \frac{-2x^2 + 2(a^2 - x^2)}{\sqrt{a^2 - x^2}} = \frac{-4x^2 + 2a^2}{\sqrt{a^2 - x^2}}$$

$$b = 2x = \boxed{\frac{2a}{\sqrt{2}}}$$

$$-2|x^2 + 2a^2 = 0$$

$$4x^2 = 2a^2$$

$$x^2 = \frac{1}{2}a^2$$

$$x = \pm\sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$$

$$\frac{a^3}{2\sqrt{2}}$$

$$h = \sqrt{a^2 - x^2} = \sqrt{a^2 - \frac{a^2}{2}}$$

$$= \sqrt{\frac{a^2}{2}}$$

$$= \frac{a}{\sqrt{2}}$$

$$A'' = \frac{\sqrt{a^2-x^2}(-8x) - \frac{(-4x^2+2a^2)(-2x)}{2\sqrt{a^2-x^2}}}{a^2-x^2} = \frac{\frac{-8a^2}{\sqrt{2}^2} + \left(\frac{-8x^3+4a^2x}{2\sqrt{a^2-x^2}}\right)}{a^2-x^2}$$

$$\frac{-4a^2 \frac{-\frac{2a^3}{\sqrt{2}} + \frac{2a^3}{\sqrt{2}}}{\frac{a}{\sqrt{2}}}}{\frac{a^2}{2}} \qquad \frac{-4a^2 \frac{-\sqrt{2}\,a^3 + \sqrt{2}\,a^3}{\frac{a}{\sqrt{2}}}}{\frac{a^2}{2}}$$

$$= \frac{-4a^2}{\frac{a^2}{2}} = \frac{-4a^2(2)}{a^2} = -8 < 0$$

<u>MAX</u>

A wholesaler sells running shoes at $20 per pair if less than 50 pairs are ordered. If 50 or more pairs are ordered (up to 600), the price per pair is reduced by 2 cents times the number ordered. What size order will produce the maximum amount of money for the wholesaler?

x = # of pairs

Amount = (# of pairs)(price per pair)

$$A = x(20 - .02x)$$

$$A = 20x - .02x^2$$

$$A' = 20 - .04x = 0$$

$\boxed{x = 500 \text{ pairs of shoes}}$

$$A'' = -.04 < 0, \text{ Max}$$

A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 feet, find the dimensions of the rectangle that will produce the largest area for the window.

$$A = xy + \frac{x^2\sqrt{3}}{4}$$

$$P = 12 = 3x + 2y$$

$$y = \frac{12-3x}{2}$$

$$A = x\left(\frac{12-3x}{2}\right) + \frac{x^2\sqrt{3}}{4}$$

$$A = 6x - \frac{3x^2}{2} + \frac{x^2\sqrt{3}}{4}$$

$$A' = 6 - 3x + \frac{x\sqrt{3}}{2} = 0$$

$$x = \frac{-6}{-3+\frac{\sqrt{3}}{2}}$$

$$A'' = -3 + \frac{\sqrt{3}}{2} < 0, \text{ MAX}$$

The owner of an apple orchard estimates that if 24 trees are planted per acre, then each mature tree will yield 600 apples per year. For each additional tree planted per acre, the number of apples produced by each tree decreases by 12 per year. How many trees should be planted per acre to obtain the most apples per year?

$x = \#$ of trees > 24

Apples = (Apples per tree)(# of trees)

$$A = (600 - 12x)(x + 24)$$

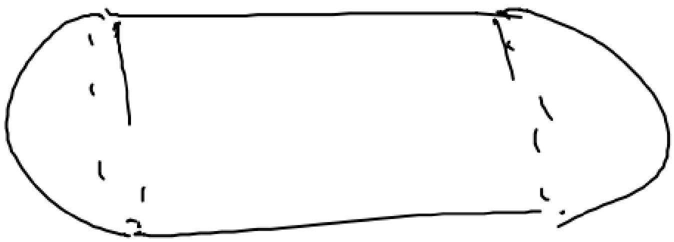
$$A = -12x^2 + 312x + 14400$$

$$A' = -24x + 312 = 0 \qquad x = 13$$

$$A'' = -24 < 0, \text{ Max}$$

$\#$ of trees $= 24 + 13 = \boxed{37 \text{ trees}}$

A steel storage tank for propane gas is to be constructed in the shape of a right circular cylinder with a hemisphere at each end. The construction cost per square foot for the end pieces is twice that for the cylindrical piece. If the desired capacity is 10π cubic feet, what dimensions will minimize the cost of construction?



$$V = 2\left(\frac{2}{3}\pi r^3\right) + \pi r^2 h$$

$$10\pi = \frac{4}{3}\pi r^3 + \pi r^2 h$$

$$h = \frac{10 - \frac{4}{3}r^3}{r^2} = \frac{10}{r^2} - \frac{4r}{3}$$

$$C = 2(2)(2\pi r^2) + 2\pi r h$$

$$C = 8\pi r^2 + 2\pi r\left(\frac{10}{r^2} - \frac{4r}{3}\right)$$

$$C = 8\pi r^2 + 20\pi r^{-1} - \frac{8\pi r^2}{3} = \frac{16\pi r^2}{3} + 20\pi r^{-1}$$

$$C' = \frac{32\pi r}{3} - \frac{20\pi}{r^2}$$

$$= \frac{32\pi r^3 - 60\pi}{3r^2} = 0 \text{ if } 32\pi r^3 - 60\pi = 0$$

undefined at $r=0$, makes no sense

$$r^3 = \frac{60}{32} = \frac{15}{8}$$

$$\boxed{r = \frac{\sqrt[3]{15}}{2} \text{ feet}}$$

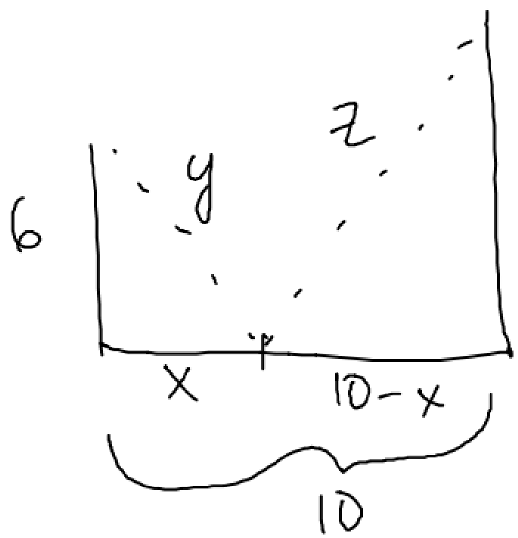
$-$ $+$

0 $\quad \frac{\sqrt[3]{15}}{2}$

$$h = \frac{10}{r^2} - \frac{4r}{3} \qquad h =$$

$$= \frac{10}{\left(\frac{\sqrt[3]{15}}{2}\right)^2} - \frac{4}{3}\left(\frac{\sqrt[3]{15}}{2}\right) = \boxed{2\sqrt[3]{15} \text{ feet}}$$

Two vertical poles of length 6 feet and 8 feet stand on level ground, with their bases ten feet apart. Approximate the minimal length of cable that can reach from the top of one pole to some point on the ground between the poles and then to the top of the other pole.



$$l = y + z = \sqrt{x^2+36} + \sqrt{(10-x)^2+64}$$

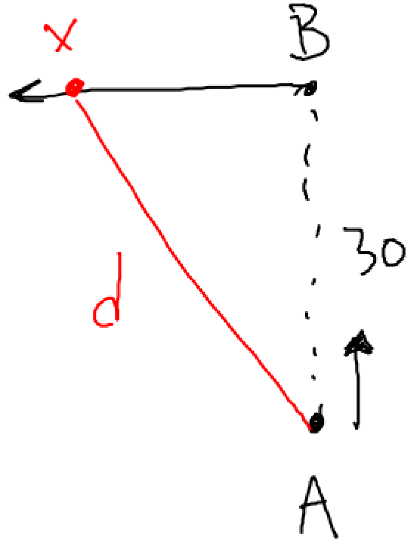
$$l = \sqrt{x^2+36} + \sqrt{x^2-20x+164}$$

$$l' = \frac{x}{\sqrt{x^2+36}} + \frac{x-10}{\sqrt{x^2-20x+164}}$$

$$\underset{\text{(calc)}}{l' = 0} \quad \text{if} \quad \boxed{x \approx 4.286 \text{ feet}}$$

from graph, clearly a minimum as derivative goes from negative to positive

At noon, ship A is 30 miles due south of ship B and is sailing north at a rate of 15 mph. If ship B is sailing west at a rate of 10 mph, find the time at which the distance d between the ships is minimal.



$\overline{AB} = 30 - 15t$

$\overline{BX} = 10t$

$$d^2 = (30-15t)^2 + (10t)^2$$

$$d^2 = 900 - 900t + 225t^2 + 100t^2$$

$$d^2 = 325t^2 - 900t + 900$$

since d is at min at same time as d^2, minimize d^2

$$(d^2)' = 650t - 900 = 0$$

− +

$\frac{18}{13}$

$$\boxed{t = \frac{18}{13} \text{ hrs after noon}} \approx \boxed{1:23:05}$$