1. $\lim_{x\to 3} \frac{2x-6}{x^2-9} = \frac{0}{0} \qquad \lim_{x\to 3} \frac{2}{2x} = \frac{2}{6} = \boxed{\frac{1}{3}}$

2. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \frac{0}{0} \qquad \lim_{x\to 3} \frac{\frac{1}{2\sqrt{x+1}}}{1} = \frac{1}{2\sqrt{4}} = \boxed{\frac{1}{4}}$

3. $\lim_{x\to \infty} \frac{5x^2-3x+1}{3x^2-5} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{10x-3}{6x} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{10}{6} = \boxed{\frac{5}{3}}$

4. $\lim_{x\to 2} \frac{x^3-x-2}{x-2} = \frac{4}{0} \quad \boxed{\text{DNE}}$

5. $\lim_{x\to 0} \frac{\sqrt{4-x^2}-2}{x} = \frac{0}{0} = \lim_{x\to 0} \frac{\frac{(-2x)}{2\sqrt{4-x^2}}}{1} = \frac{0}{4} = \boxed{0}$

6. $\lim_{x\to 0} \frac{e^x-(1-x)}{x} = \frac{0}{0} = \lim_{x\to 0} \frac{e^x+1}{1} = \boxed{2}$

7. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \frac{0}{0} = \lim_{x\to 0} \frac{2\cos 2x}{3\cos 3x} = \frac{2\cos 0}{3\cos 0} = \boxed{\frac{2}{3}}$

8. $\lim_{x\to 0} \frac{\sin^{-1} x}{x} = \frac{0}{0} = \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = \boxed{1}$

9. $\lim_{x\to \infty} \frac{3x^2-2x+1}{2x^2+3} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{6x-2}{4x} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{6}{4} = \boxed{\frac{3}{2}}$

10. $\lim_{x\to \infty} \frac{x^2+2x+1}{x-1} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{2x+2}{1} = \boxed{\infty}$

11. $\lim_{x\to \infty} \frac{x}{\sqrt{x^2+1}} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{1}{\frac{2x}{2\sqrt{x^2+1}}} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{2x}{2\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} =$ infinite loop! so $\lim_{x\to\infty} \frac{x}{|x|} = \lim_{x\to\infty} \frac{x}{x}$

$= \lim_{x\to\infty} 1 = \boxed{1}$

12. $\lim\limits_{x\to\infty} \dfrac{\ln x}{x} = \dfrac{\infty}{\infty} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x}}{1} = \dfrac{1}{\infty} = \boxed{0}$

13. $\lim\limits_{x\to\infty} \dfrac{(\ln x)^3}{x} = \dfrac{\infty}{\infty} = \lim\limits_{x\to\infty} \dfrac{\frac{3(\ln x)^2}{x}}{1} = \dfrac{\infty}{\infty}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{6\ln x}{x}}{1} = \dfrac{\infty}{\infty} = \lim\limits_{x\to\infty} \dfrac{6}{x} = \boxed{0}$

14. $\lim\limits_{x\to\infty} \dfrac{x^2}{e^{5x}} = \dfrac{\infty}{\infty} = \lim\limits_{x\to\infty} \dfrac{2x}{5e^{5x}} = \dfrac{\infty}{\infty} = \lim\limits_{x\to\infty} \dfrac{2}{25e^{5x}} = \dfrac{2}{\infty}$

$= \boxed{0}$

15. $\lim\limits_{x\to 0} \dfrac{e^{2x}-1}{e^x} = \dfrac{1-1}{1} = \dfrac{0}{1} = \boxed{0}$ no L'Hopital's Rule

16. $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{e^x - x - 1} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{-\sin x}{e^x - 1} = \dfrac{0}{0}$

$= \lim\limits_{x\to 0} \dfrac{-\cos x}{e^x} = \dfrac{-\cos 0}{e^0} = \dfrac{-1}{1} = \boxed{-1}$

17. $\lim\limits_{x\to 0} \dfrac{\ln(\cos(\sin x))}{x^2} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{\frac{-\sin(\sin x)\cos x}{\cos(\sin x)}}{2x} = \lim\limits_{x\to 0} \dfrac{-\tan(\sin x)\cos x}{2x}$

$= \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{-\tan(\sin x)(-\sin x) + (-\sec^2(\sin x))(\cos x)(\cos x)}{2}$

$= \dfrac{0 + (-1)(1)(1)}{2} = \boxed{-\dfrac{1}{2}}$

18. $\lim_{x \to 0} \dfrac{1-\cos 2x}{1-\cos 7x} = \dfrac{0}{0} = \lim_{x \to 0} \dfrac{2\sin 2x}{7\sin 7x} = \dfrac{0}{0}$

$= \lim_{x \to 0} \dfrac{4\cos 2x}{49\cos 7x} = \boxed{\dfrac{4}{49}}$

19. $\lim_{x \to 0} \dfrac{e^{2+x} - \sin x - e^2}{x} = \dfrac{0}{0} = \lim_{x \to 0} \dfrac{e^{2+x} - \cos x}{1}$

$= \boxed{e^2 - 1}$

20. $\lim_{x \to 3} \dfrac{\sqrt{3x} - 3}{\sqrt{2x-4} - \sqrt{2}} = \dfrac{0}{0} = \lim_{x \to 3} \dfrac{\dfrac{1(3)}{2\sqrt{3x}}}{\dfrac{1(2)}{2\sqrt{2x-4}}} = \dfrac{\dfrac{3}{6}}{\dfrac{2}{2\sqrt{2}}} = \boxed{\dfrac{\sqrt{2}}{2}}$