$f(x) = x^2 - 5x + 4$ $[0, 6]$

f cont on $[0,6]$, poly. f diff on $(0,6)$, poly.

$f(0) = 4$

$f(6) = 10$

$$\frac{10 - 4}{6 - 0} = 1$$

$f'(x) = 2x - 5 = 1$

$x = 3$

$f(x) = x^3 - 4$ $\quad [1,2]$

f cont on $[1,2]$, poly. f diff on $(1,2)$, poly.

$f(1) = -3$

$f(2) = 4$

$$\frac{4-(-3)}{2-1} = 7$$

$f'(x) = 3x^2 = 7$

$x = \pm\sqrt{\frac{7}{3}}$

b/c $(1,2)$ must be $\boxed{\sqrt{\frac{7}{3}}}$

$f(x) = x^{\frac{1}{3}}$ $[1, 8]$

only not diff at $x=0$

f cont on $[1,8]$, cube root function f diff on $(1,8)$

$f(8) = 2$

$f(1) = 1$

$\frac{2-1}{8-1} = \frac{1}{7}$

$f'(x) = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}} = \frac{1}{7}$

f' undefined at $x=0$

$3x^{\frac{2}{3}} = 7$

$x^{\frac{2}{3}} = \frac{7}{3}$

$x = \left(\frac{7}{3}\right)^{\frac{3}{2}}$

$f(x) = x^{\frac{2}{3}} - 5x + 1$ $\quad [-1, 3]$

f cont on $[-1, 3]$, cube root function

$f'(x) = \frac{2}{3} x^{-\frac{1}{3}} - 5$

$= \frac{2}{3\sqrt[3]{x}} - 5$ undefined at $x = 0$

not diff. on $(-1, 3)$

MVT does not apply

$f(x) = e^{2x} - 1 \qquad [0,1]$

f cont $[0,1]$, exp. func. f diff on $(0,1)$, exp. function

$f(1) = e^2 - 1$

$f(0) = 0$

$$\frac{e^2 - 1 - 0}{1 - 0} = e^2 - 1$$

$$f'(x) = 2e^{2x} = e^2 - 1$$

$$e^{2x} = \frac{e^2 - 1}{2}$$

$$2x = \ln\left(\frac{e^2 - 1}{2}\right)$$

$$x = \frac{1}{2}\ln\left(\frac{e^2 - 1}{2}\right)$$