

$$1. \quad x^3 + 3x = -1 \rightarrow x^3 + 3x + 1 = 0 \quad f'(x) = 3x^2 + 3$$

$$f(-1) = -3$$

$$f(0) = 1$$

continuous, polynomial
by IVT, root on $(-1, 0)$

$$x_0 = -.5$$

$$x_1 = -.3$$

$$x_2 = -.32$$

$$x_3 = -.322185355$$

$$x_4 = -.3221853546$$

$$x_5 = \text{same}$$

$$x_{n+1} = x_n - \frac{x_n^3 + 3x_n + 1}{3x_n^2 + 3}$$

$$2. \quad \sin x + x^2 = 3 \text{ on } [1, 2]$$

$$\sin x + x^2 - 3 = 0$$

$$x_0 = 1.5$$

$$x_1 = 1.419402095$$

$$x_2 = 1.418310293$$

$$x_3 = 1.418310092$$

$$x_4 = \text{same}$$

$$f'(x) = \cos x + 2x$$

$$x_{n+1} = x_n - \frac{\sin(x_n) + x_n^2 - 3}{\cos(x_n) + 2x_n}$$

$$3. \quad \tan x = e$$

$$\tan x - e = 0$$

$$f'(x) = \sec^2 x$$

$$x_{n+1} = x_n - \frac{\tan(x_n) - e}{\sec^2(x_n)}$$

$$x_0 = 1.5$$

$$x_1 = 1.443041603$$

⋮

$$x_7 = x_8 = 1.218282905$$

4. $x^4 + x^3 - 5x^2 - 7 = 0$ on $[2, 3]$ $f'(x) = 4x^3 + 3x^2 - 10x$

$$x_0 = 2.5$$

$$x_1 = 2.207$$

⋮

$$x_5 = x_6 = \boxed{2.111518833}$$

$$x_{n+1} = x_n - \frac{x_n^4 + x_n^3 - 5x_n^2 - 7}{4x_n^3 + 3x_n^2 - 10x_n}$$

5. $\sin x - \sin 2x = 2 + \cos x$ on $[2, 3]$ $f'(x) = \cos x - 2\cos 2x + \sin x$

$$\sin x - \sin 2x - \cos x - 2 = 0$$

$$x_{n+1} = x_n - \frac{\sin(x_n) - \sin(2x_n) - \cos(x_n) - 2}{\cos(x_n) - 2\cos(2x_n) + \sin(x_n)}$$

$$x_0 = 2.5$$

$$x_1 = 2.965638922$$

⋮

$$x_5 = x_6 = \boxed{2.755833558}$$

6. $\sqrt[5]{2} \rightarrow x^5 - 2 = 0$ $f'(x) = 5x^4$

$f(1) = -1 \rightarrow$ continuous, by IVT, root on $(1, 2)$
 $f(2) = 30$

$$x_0 = 1.5$$

$$x_1 = 1.279012346$$

⋮

$$x_5 = x_6 = \boxed{1.148698355}$$

$$x_{n+1} = x_n - \frac{x_n^5 - 2}{5x_n^4}$$

7. $\sqrt[4]{8}$

$$x^4 - 8 = 0$$

$$f'(x) = 4x^3$$

$$f(1) = -7$$

$$f(2) = 8$$

continuous, by IVT root on (1, 2)

$$x_0 = 1.5$$

$$x_1 = 1.717592593$$

⋮

$$x_4 = x_5 = \boxed{1.681792831}$$

$$x_{n+1} = x_n - \frac{x_n^4 - 8}{4x_n^3}$$

8. $\sqrt[3]{7}$

$$x^3 - 7 = 0$$

$$f'(x) = 3x^2$$

$$f(1) = -6$$

$$f(2) = 1$$

continuous, by IVT root on (1, 2)

$$x_0 = 1.5$$

$$x_1 = 2.037037$$

⋮

$$x_4 = x_5 = \boxed{1.912931183}$$

$$x_{n+1} = x_n - \frac{x_n^3 - 7}{3x_n^2}$$