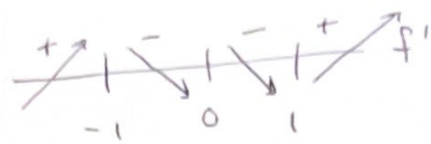
$f(x) = 3x^5 - 5x^3 + 3$

1. $f'(x) = 15x^4 - 15x^2 = 0$

$15x^2(x^2 - 1) = 0$

$15x^2(x+1)(x-1) = 0$

$x = 0 \quad x = -1 \quad x = 1$



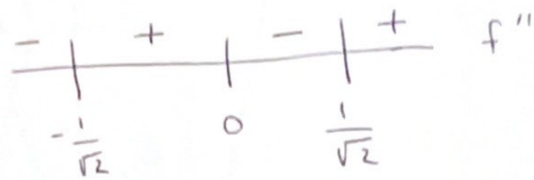
$f(-1) = 3(-1)^5 - 5(-1)^3 + 3 = -3 + 5 + 3 = 5$

$f(1) = 3 - 5 + 3 = 1$

$f''(x) = 60x^3 - 30x = 0$

$30x(2x^2 - 1) = 0$

$x = 0 \quad x = \pm \frac{1}{\sqrt{2}}$



$f(0) = 3 \quad f\left(-\frac{1}{\sqrt{2}}\right) = \frac{24 + 7\sqrt{2}}{8} \approx 4.2374369$

$f(x) = 0$ $\underline{\text{calc}}$ $\quad x \approx -1.419865 \quad$ x-int $(-1.419865, 0)$

$f\left(\frac{1}{\sqrt{2}}\right) = \frac{24 - 7\sqrt{2}}{8} \approx 1.7625631$

$f(0) = 3 \quad$ y-int: $(0, 3)$

no asymptotes

domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

conc up: $\left(-\frac{1}{\sqrt{2}}, 0\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right)$

conc down: $\left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(0, \frac{1}{\sqrt{2}}\right)$

POI: $(0, 3)$ $\left(-\frac{1}{\sqrt{2}}, \frac{24 + 7\sqrt{2}}{8}\right)$ $\left(\frac{1}{\sqrt{2}}, \frac{24 - 7\sqrt{2}}{8}\right)$

inc: $(-\infty, -1] \cup [1, \infty)$

dec: $[-1, 1]$

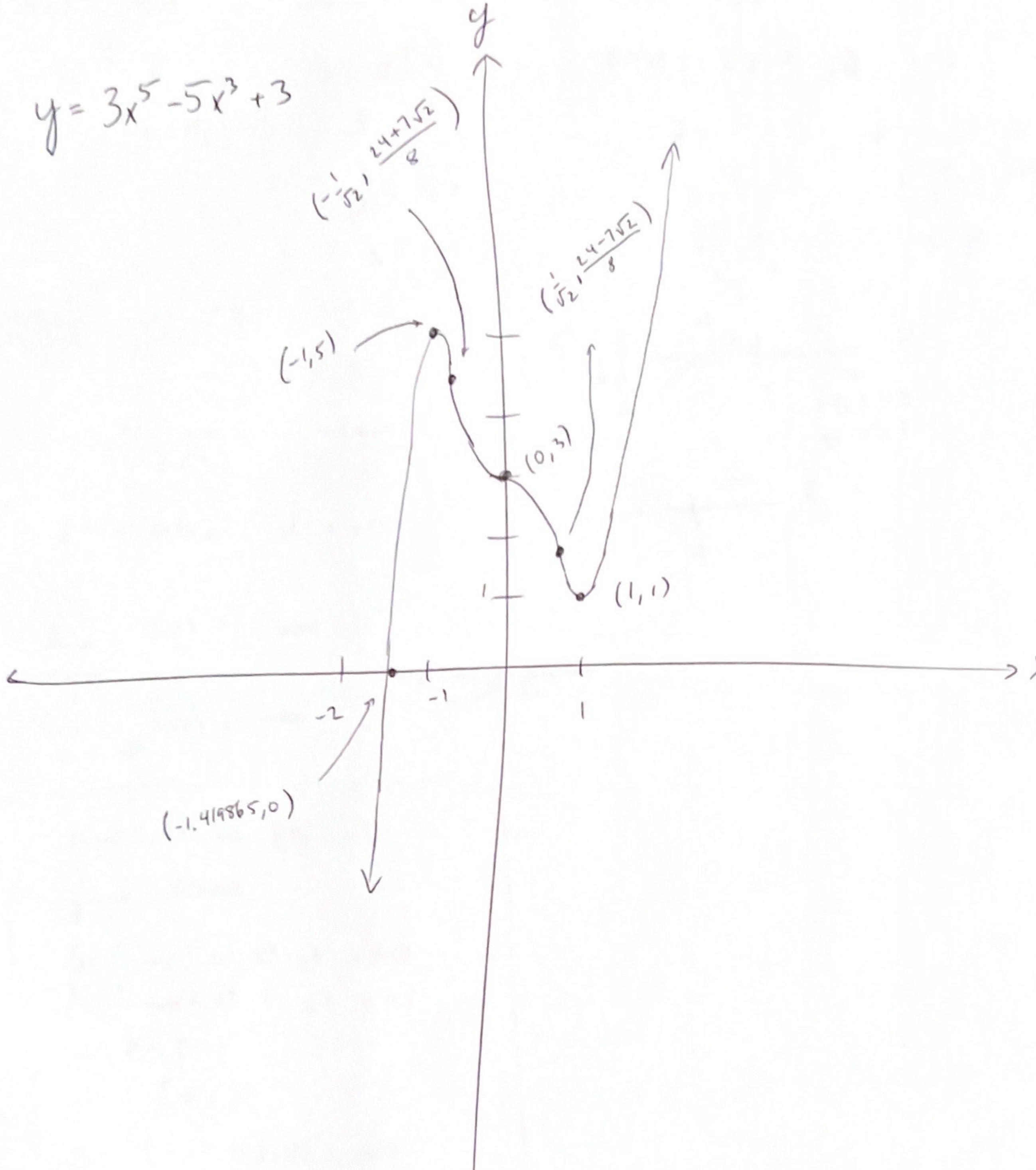
local max of 5 at $x = -1$

local min of 1 at $x = 1$

x-int: $(-1.419865, 0)$

y-int: $(0, 3)$

$y = 3x^5 - 5x^3 + 3$



y
x
$\left(-\frac{1}{\sqrt{2}}, \frac{24+7\sqrt{2}}{8}\right)$
$\left(\frac{1}{\sqrt{2}}, \frac{24-7\sqrt{2}}{8}\right)$
(-1,5)
(0,3)
(1,1)
1
-2
-1
1
(-1.419865,0)

2. $f(x) = \sqrt[3]{x^2}\,(3 - 2\sqrt[3]{x}) = 0$

$\sqrt[3]{x^2} = 0$

$x = 0$

$3 - 2\sqrt[3]{x} = 0$

$3 = 2\sqrt[3]{x}$

$\frac{3}{2} = \sqrt[3]{x}$

$x = \frac{27}{8}$

$f(x) = 3x^{2/3} - 2x$

$f'(x) = 2x^{-\frac{1}{3}} - 2 = 0$

$2x^{-\frac{1}{3}} = 2$

$x^{-\frac{1}{3}} = 1$

$\sqrt[3]{x} = 1$

$\underline{x = 1}$

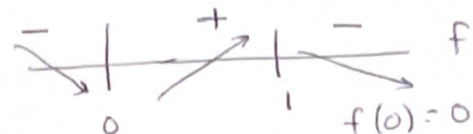
f' undefined if $\frac{2}{\sqrt[3]{x}}$ undefined

$\underline{x = 0}$

$f''(x) = -\frac{2}{3}x^{-\frac{4}{3}}$

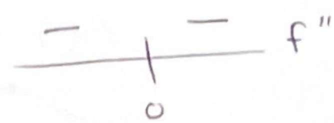
$= -\frac{2}{3x^{4/3}} = 0$ never

f'' undefined if $x = 0$



$f(0) = 0$

$f(1) = 1$



$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = \infty$

no asymptotes

x-int: $(0,0)$ $(\frac{27}{8}, 0)$

y-int: $(0,0)$

local min of 0 at $x = 0$

local max of 1 at $x = 1$

no POI

inc: $[0,1]$

dec: $(-\infty, 0] \cup [1, \infty)$

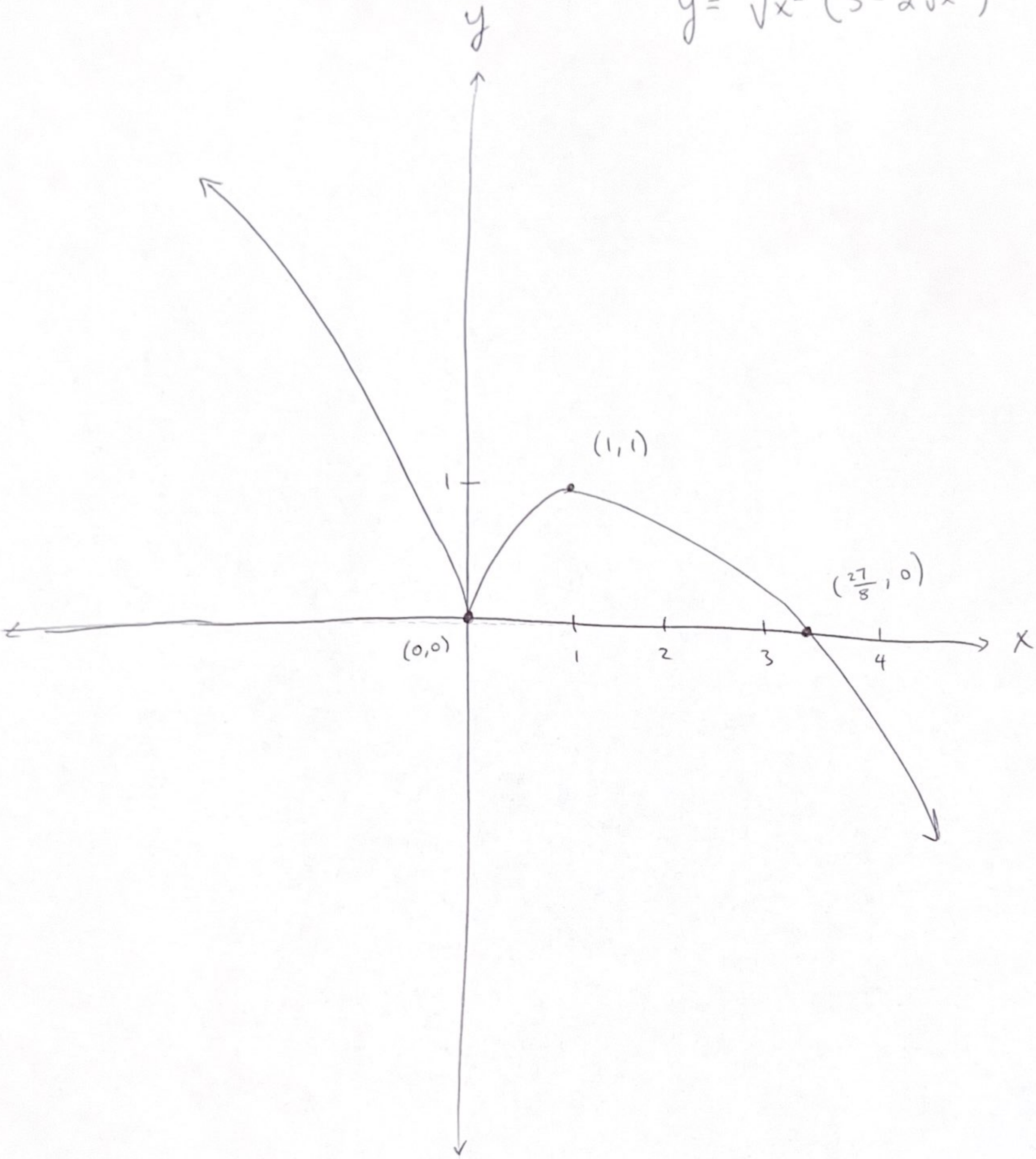
conc up never

conc down $(-\infty, \infty)$

domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

$$y = \sqrt[3]{x^2}\left(3 - 2\sqrt[3]{x}\right)$$



3. $f(x) = x\sqrt{9-x^2} = 0 \qquad x=0 \qquad 9-x^2=0$

$x = \pm 3$

$\lim_{x\to\infty} f(x)$ DNE

domain: $9-x^2 \geq 0$

$\lim_{x\to-\infty} f(x)$ DNE

$x^2 \leq 9$

$-3 \leq x \leq 3$

$f(0) = 0(9) = 0$

$$f'(x) = \frac{x(-2x)}{2\sqrt{9-x^2}} + \sqrt{9-x^2} = \frac{-x^2+9-x^2}{\sqrt{9-x^2}}$$

$$= \frac{9-2x^2}{\sqrt{9-x^2}} \quad \text{und at } x=\pm 3$$

$f(3)=0$

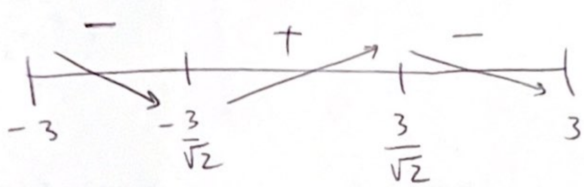
$f(-3)=0$

$f'=0 \quad 9-2x^2=0 \qquad x^2 = \frac{9}{2}$

$x = \pm\frac{3}{\sqrt{2}}$

$f\left(-\frac{3}{\sqrt{2}}\right) = -\frac{9}{2}$

$f\left(\frac{3}{\sqrt{2}}\right) = \frac{9}{2}$



$$f''(x) = \frac{\sqrt{9-x^2}\,(-4x) - \frac{(9-2x^2)(-\cancel{2}x)}{\cancel{2}\sqrt{9-x^2}}}{9-x^2} = \frac{(9-x^2)(-4x)+x(9-2x^2)}{\sqrt{(9-x^2)^3}}$$

und at $x = \pm 3$

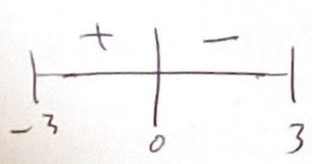
$f''=0$ if $-36x+4x^3+9x-2x^3=0$

$2x^3-27x=0$

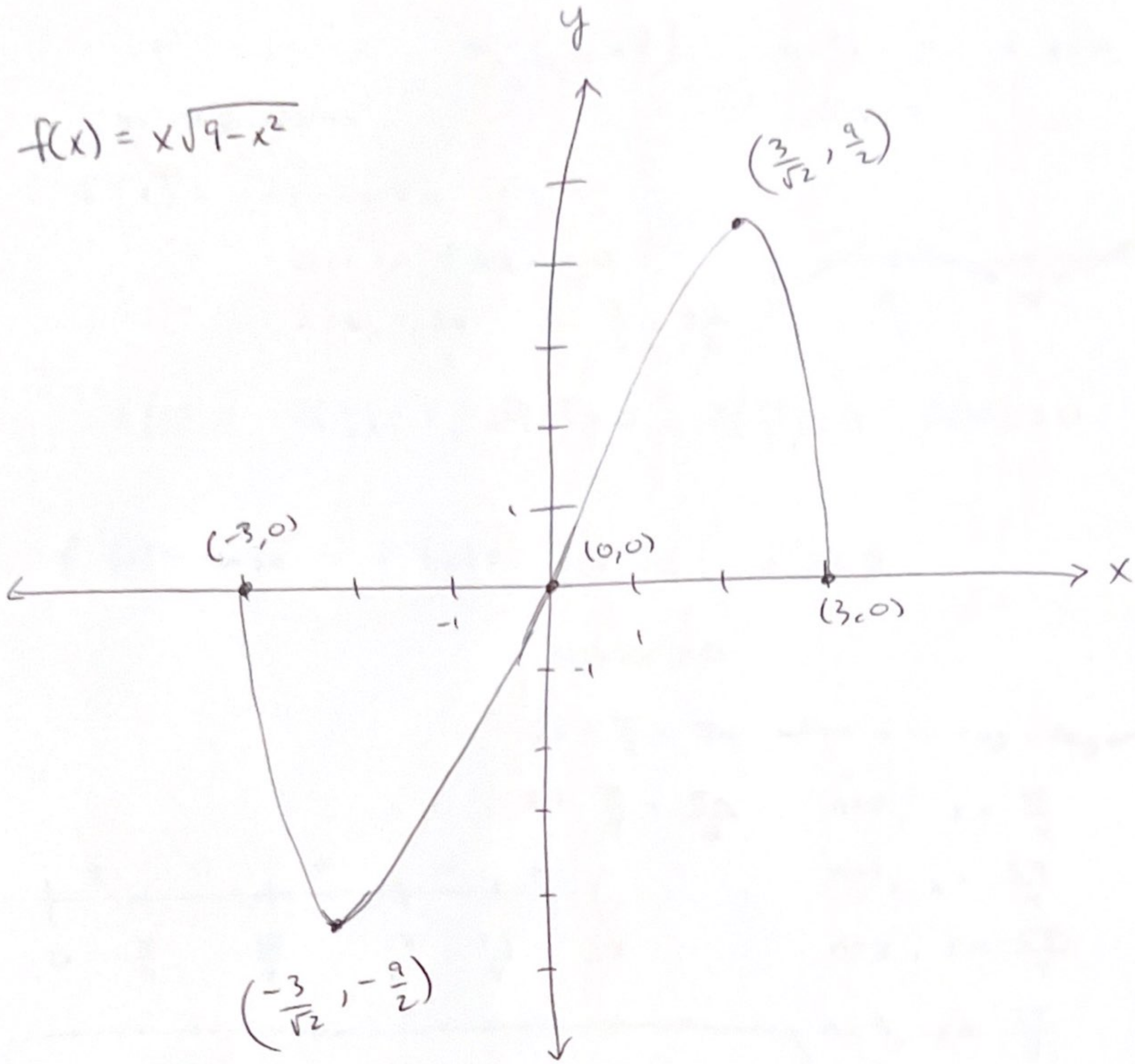
$x(2x^2-27)=0$

$x=0 \qquad x=\pm\sqrt{\frac{27}{2}}$

$\hookrightarrow$ not in domain



$f(x) = x\sqrt{9-x^2}$



x-int: $(-3,0), (0,0), (3,0)$ y-int: $(0,0)$

local max of 0 at $x=-3$ and $\frac{9}{2}$ at $x = \frac{3}{\sqrt{2}}$

local min of $-\frac{9}{2}$ at $x = -\frac{3}{\sqrt{2}}$ and 0 at $x=3$

POI: $(0,0)$

no asymptotes

inc: $\left[-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right]$ dec: $\left[-3, -\frac{3}{\sqrt{2}}\right] \cup \left[\frac{3}{\sqrt{2}}, 3\right]$

conc up: $(-3,0)$ conc down: $(0,3)$

Domain: $[-3,3]$ Range: $\left[-\frac{9}{2}, \frac{9}{2}\right]$

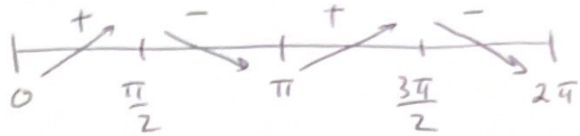
4. $f(x) = \sin^2 x$ on $[0, 2\pi]$ $\qquad \sin^2 x = 0$ if $x = 0, \pi, 2\pi$

no asymptotes $\qquad f(0) = 0$

$f'(x) = 2\sin x \cos x = 0$

$\sin x = 0 \qquad \cos x = 0$

$x = 0, \pi, 2\pi \qquad x = \frac{\pi}{2}, \frac{3\pi}{2}$



$f(0) = 0 \quad f(\frac{\pi}{2}) = 1 \quad f(\pi) = 0 \quad f(\frac{3\pi}{2}) = 1 \quad f(2\pi) = 0$

$f'(x) = \sin 2x \qquad f''(x) = 2\cos 2x \qquad f''(x) = 0$

$\cos 2x = 0$

$2x = \frac{\pi}{2} + \pi n$ where n is any integer

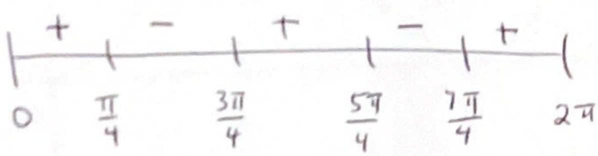
$x = \frac{\pi}{4} + \frac{\pi n}{2}$

$n = 0, \; x = \frac{\pi}{4}$

$n = 1, \; x = \frac{3\pi}{4}$

$n = 2, \; x = \frac{5\pi}{4}$

$n = 3, \; x = \frac{7\pi}{4}$



$f(\frac{\pi}{4}) = \frac{1}{2}$

$f(\frac{3\pi}{4}) = \frac{1}{2}$

$f(\frac{5\pi}{4}) = \frac{1}{2}$

$f(\frac{7\pi}{4}) = \frac{1}{2}$

x-int: $(0,0)$, $(\pi, 0)$, $(2\pi, 0)$ $\qquad$ y-int: $(0,0)$

local max of 1 at $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$

local min of 0 at $x = 0$, $x = \pi$, $x = 2\pi$

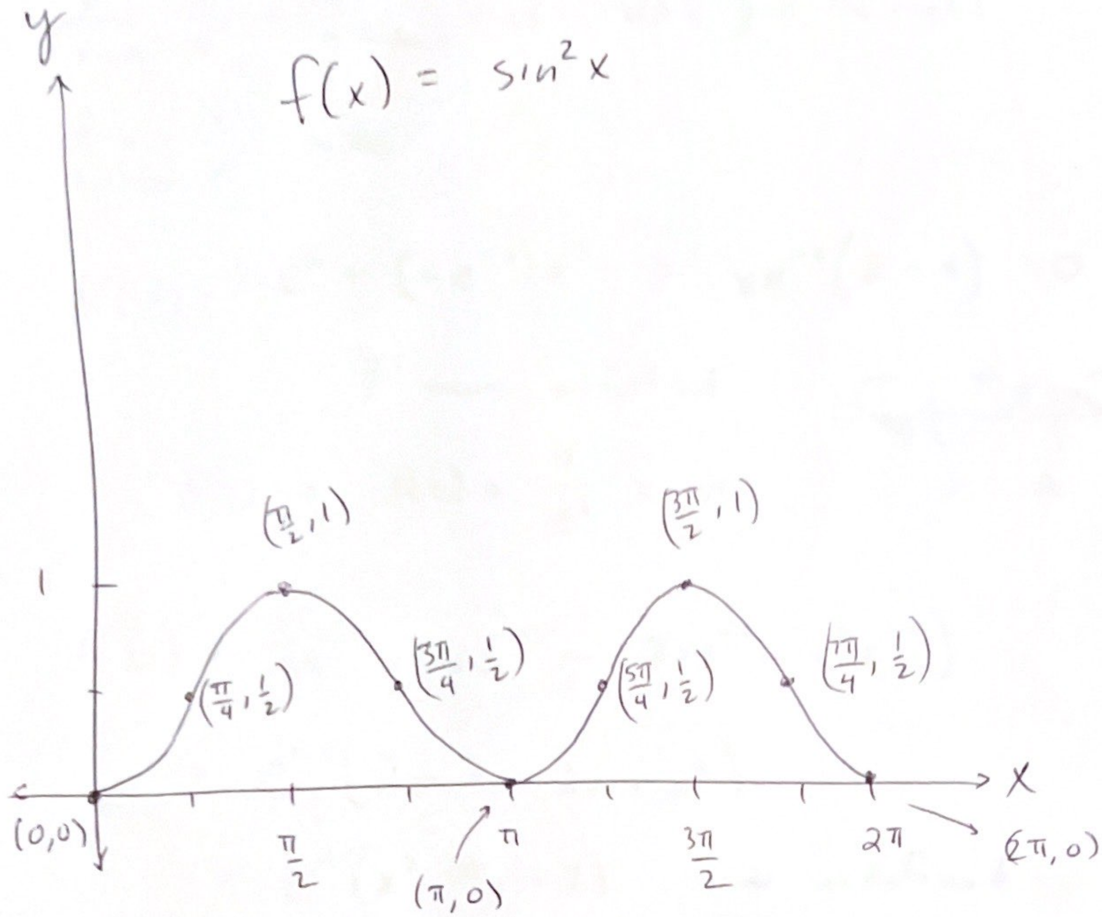
no asymptotes

POI: $(\frac{\pi}{4}, \frac{1}{2})$, $(\frac{3\pi}{4}, \frac{1}{2})$, $(\frac{5\pi}{4}, \frac{1}{2})$, $(\frac{7\pi}{4}, \frac{1}{2})$

inc: $[0, \frac{\pi}{2}] \cup [\pi, \frac{3\pi}{2}]$ $\qquad$ dec: $[\frac{\pi}{2}, \pi] \cup [\frac{3\pi}{2}, 2\pi]$

conc up: $(0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \frac{5\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$ $\qquad$ Domain: $[0, 2\pi]$

conc down: $(\frac{\pi}{4}, \frac{3\pi}{4}) \cup (\frac{5\pi}{4}, \frac{7\pi}{4})$ $\qquad$ Range: $[0, 1]$



y
$f(x) = \sin^2 x$
1
$\left(\frac{\pi}{2}, 1\right)$
$\left(\frac{3\pi}{2}, 1\right)$
$\left(\frac{\pi}{4}, \frac{1}{2}\right)$
$\left(\frac{3\pi}{4}, \frac{1}{2}\right)$
$\left(\frac{5\pi}{4}, \frac{1}{2}\right)$
$\left(\frac{7\pi}{4}, \frac{1}{2}\right)$
x
(0,0)
$\frac{\pi}{2}$
π
$(\pi, 0)$
$\frac{3\pi}{2}$
2π
$(2\pi, 0)$

5. $f(x) = x^2 e^{-x} = \dfrac{x^2}{e^x}$ $\quad f(0)=0 \quad$ $f(x)=0$ if $x=0$

$\lim\limits_{x\to\infty} \dfrac{x^2}{e^x} = \dfrac{\infty}{\infty \text{ faster}} = 0$ $\quad$ HA: $y=0$ to right

$\lim\limits_{x\to-\infty} \dfrac{x^2}{e^x} = \infty$

$f'(x) = 2xe^{-x} + (-e^{-x})x^2 = xe^{-x}(2-x) = 0$ if $x=0$, $x=2$

f' never undefined

f': $-$ | $+$ | $-$ at 0, 2

$f(0)=0 \quad f(2) = \dfrac{4}{e^2} \approx .541$

$f''(x) = 2e^{-x} - 2xe^{-x} - (2xe^{-x} - x^2e^{-x})$

$= e^{-x}(2 - 2x - 2x + x^2)$

$= e^{-x}(x^2 - 4x + 2)$ $\quad$ never undefined

$f'' = 0 \quad x^2 - 4x + 2 = 0$

$x^2 - 4x + 4 = 2$

$(x-2)^2 = 2$

$x - 2 = \pm\sqrt{2}$

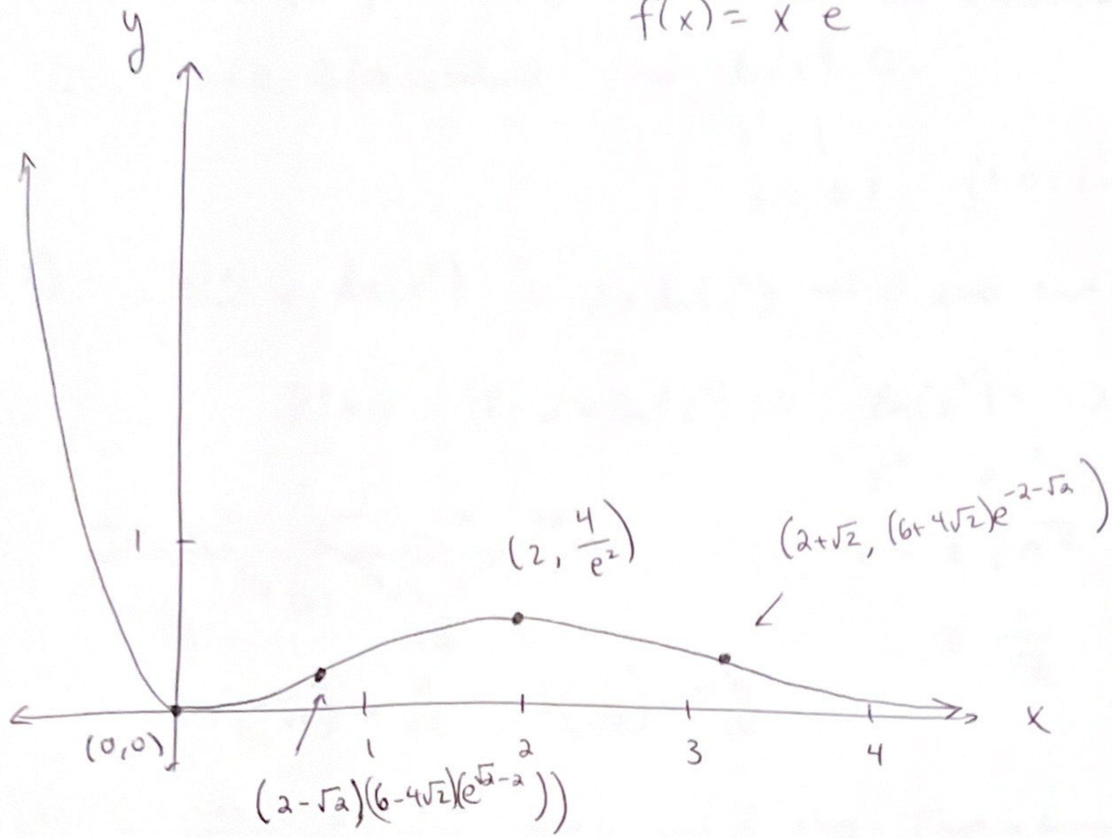
$x = 2 \pm \sqrt{2}$

$+$ | $-$ | $+$ at $2-\sqrt{2}$, $2+\sqrt{2}$

$f(2-\sqrt{2}) = (2-\sqrt{2})^2 e^{\sqrt{2}-2} = (6-4\sqrt{2})e^{\sqrt{2}-2} \approx .191$

$f(2+\sqrt{2}) = (2+\sqrt{2})^2 e^{-2-\sqrt{2}} = (6+4\sqrt{2})e^{-2-\sqrt{2}} \approx .384$

$f(x) = x^2 e^{-x}$



x-int: $(0,0)$ y-int: $(0,0)$

local max of $\frac{4}{e^2}$ at $x = 2$

local min of 0 at $x = 0$

POI: $\left(2-\sqrt{2}, (6-4\sqrt{2})e^{\sqrt{2}-2}\right)$ $\left(2+\sqrt{2}, (6+4\sqrt{2})e^{-2-\sqrt{2}}\right)$

HA: $y = 0$ to right

inc: $[0, 2]$ dec: $(-\infty, 0] \cup [2, \infty)$

conc up: $(-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, \infty)$

conc down: $(2-\sqrt{2}, 2+\sqrt{2})$

Dom: $(-\infty, \infty)$

Range: $[0, \infty)$

6. $f(x) = x\ln(x^2)$ $\quad = 0$ if $x = 0 \to$ not in domain

Dom: $x \neq 0$ $\ln(0)$ undefined $\quad$ or $\ln(x^2) = 0$

$x^2 = 1$

$x = \pm 1$ $\quad (1,0)\ (-1,0)$

$f'(x) = \dfrac{x(2x)}{x^2} + \ln(x^2) = 2 + \ln(x^2)$ und. if $x = 0$ (not in domain)

$f' = 0$ if $2 + \ln(x^2) = 0$ $\quad \ln(x^2) = -2$

$x^2 = e^{-2}$

$x = \pm\sqrt{e^{-2}}$

$= \pm \dfrac{1}{e}$

$+ \quad | \quad - \quad | \quad - \quad | \quad +$ $\quad f'$

$-\frac{1}{e} \qquad 0 \qquad \frac{1}{e}$

$f\left(-\frac{1}{e}\right) = \frac{2}{e}$ $\quad f\left(\frac{1}{e}\right) = -\frac{2}{e}$

$f''(x) = \dfrac{2x}{x^2} = \dfrac{2}{x}$ $\neq 0$ und. if $x = 0$ (not in domain)

$\lim_{x\to\infty} f(x) = \infty$ $\quad \lim_{x\to-\infty} f(x) = -\infty$

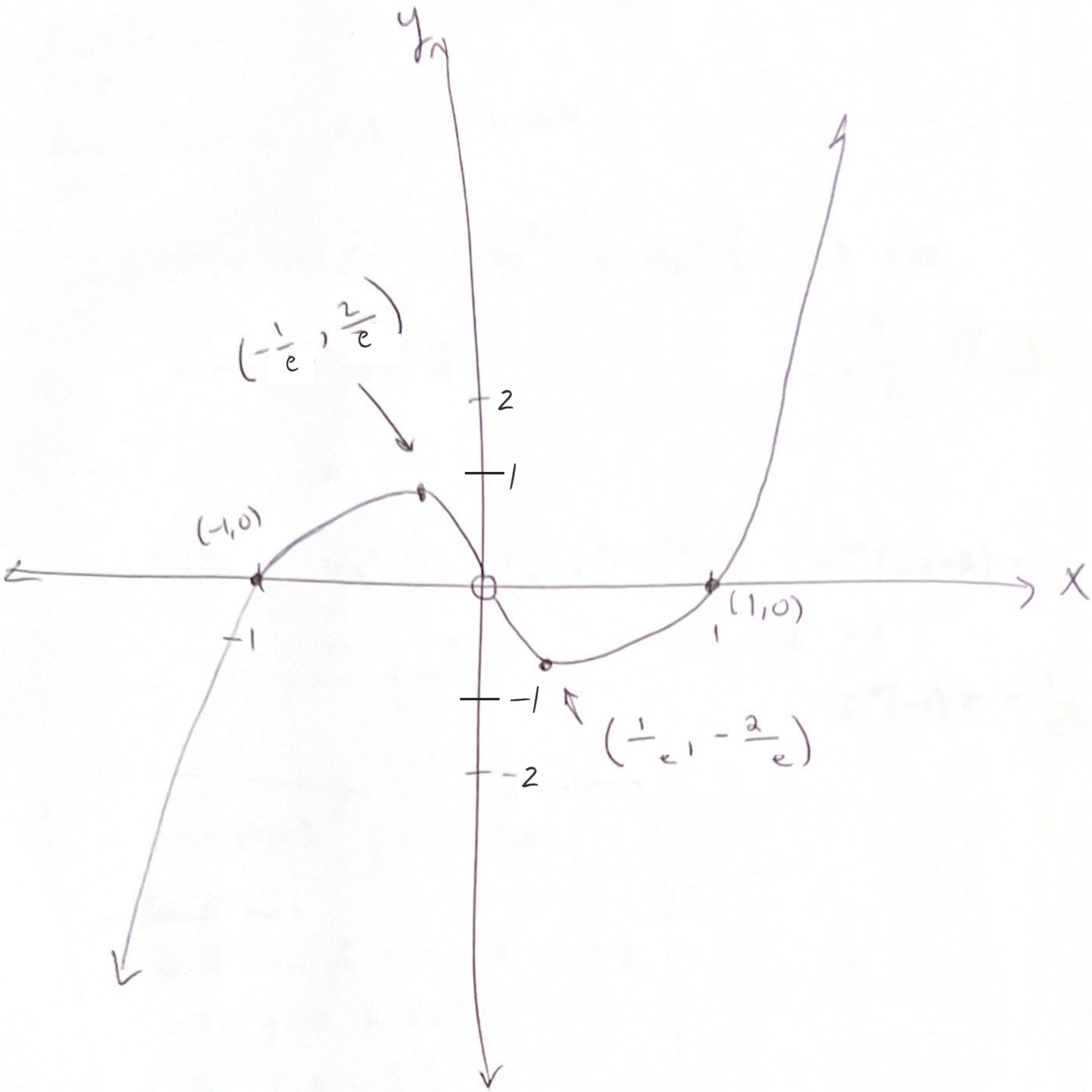
no asymptotes

$- \quad | \quad +$ $\quad f''$

0

$$\boxed{\begin{array}{l}
\text{x-int: } (1,0), (-1,0) \quad \text{no y-int} \\
\text{local max of } \frac{2}{e} \text{ at } x = -\frac{1}{e} \\
\text{local min of } -\frac{2}{e} \text{ at } x = \frac{1}{e} \\
\text{no POI} \\
\text{no asymptotes} \\
\text{inc: } \left(-\infty, -\frac{1}{e}\right] \cup \left[\frac{1}{e}, \infty\right) \\
\text{dec: } \left[-\frac{1}{e}, 0\right) \cup \left(0, \frac{1}{e}\right] \qquad \text{dom: } (-\infty, \infty) \\
\text{conc up: } (0, \infty) \qquad \text{range: } (-\infty, \infty) \\
\text{conc down: } (-\infty, 0)
\end{array}}$$

$f(x) = x \ln(x^2)$



y
x
$\left(-\frac{1}{e}, \frac{2}{e}\right)$
2
1
(-1,0)
-1
(1,0)
1
-1
$\left(\frac{1}{e}, -\frac{2}{e}\right)$
-2

7. $f(x) = 4xe^{2x}$ $f(x) = 0$ if $x = 0$ $(0,0)$

$f(0) = 0$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = 0$ HA: $y=0$ to left

$f'(x) = 4x(2e^{2x}) + 4e^{2x} = 4e^{2x}(2x+1) = 0$

$2x+1=0$

$x = -\frac{1}{2}$ $f(-\frac{1}{2}) = -\frac{2}{e}$

− | + f'

$-\frac{1}{2}$

$f''(x) = 4e^{2x}(2) + (2x+1)8e^{2x} = 8e^{2x}(2x+2) = 0$

$x = -1$

− | +

-1

$f(-1) = -\frac{4}{e^2}$

x-int: $(0,0)$ y-int: $(0,0)$

no local max

local min of $-\frac{2}{e}$ at $x = -\frac{1}{2}$

HA: $y = 0$ to left

POI: $(-1, -\frac{4}{e^2})$

inc: $[-\frac{1}{2}, \infty)$

dec: $(-\infty, -\frac{1}{2}]$

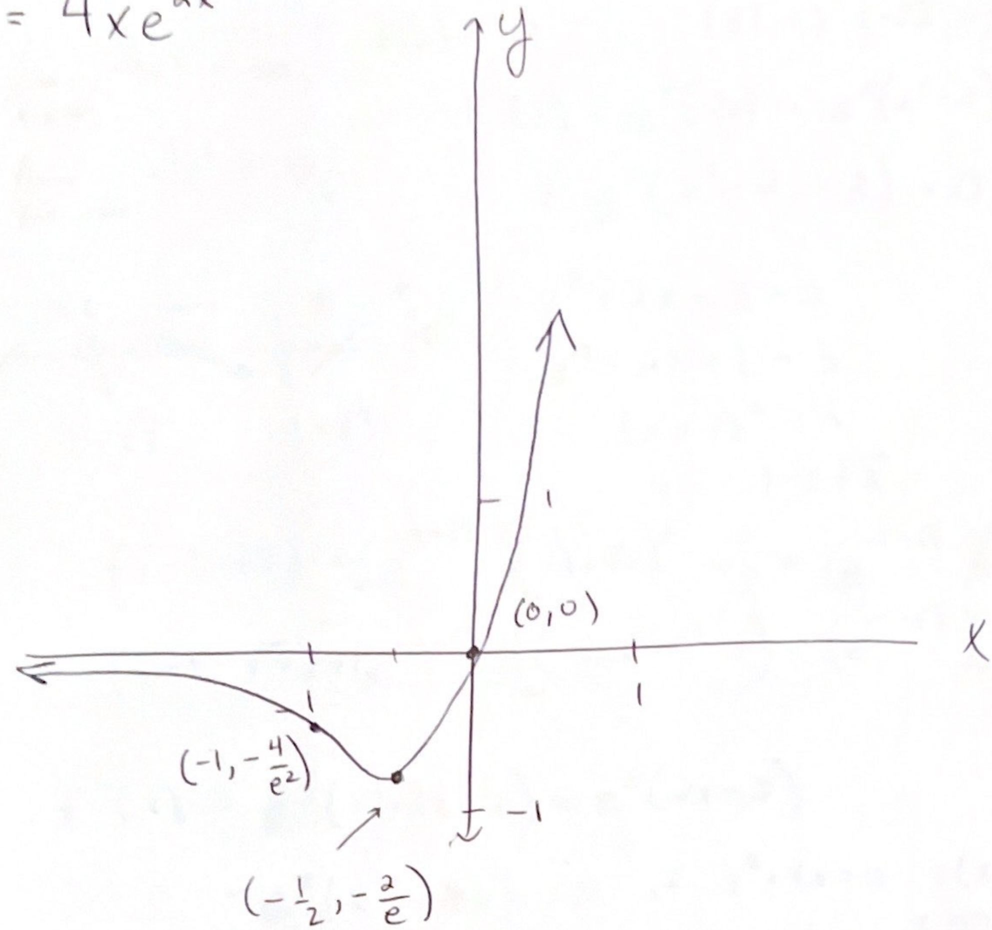
conc up: $(-1, \infty)$

conc down: $(-\infty, -1)$

domain: $(-\infty, \infty)$

range: $[-\frac{2}{e}, \infty)$

$f(x) = 4xe^{2x}$



8. $f(x) = e^x(x^2-2) = 0$ if $x^2-2=0$ $x = \pm\sqrt{2}$

$(\sqrt{2},0)$ $(-\sqrt{2},0)$

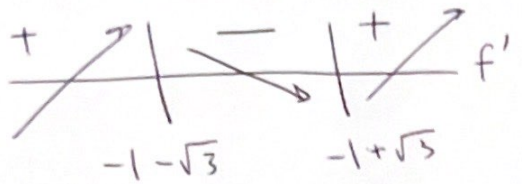
$f(0) = -2$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = 0$

$f'(x) = e^x(2x) + e^x(x^2-2)$

$= e^x(x^2+2x-2) = 0$



$x^2+2x-2=0$

$x^2+2x+1=3$

$(x+1)^2=3$

$x = -1\pm\sqrt{3}$

$f(-1+\sqrt{3}) = (e^{-1+\sqrt{3}})[(-1+\sqrt{3})^2-2] = (e^{-1+\sqrt{3}})(2-2\sqrt{3})$

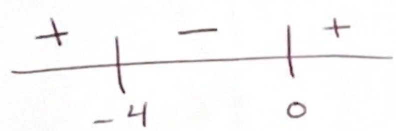
$f(-1-\sqrt{3}) = (e^{-1-\sqrt{3}})[(-1-\sqrt{3})^2-2] = (e^{-1-\sqrt{3}})(2+2\sqrt{3})$

$f''(x) = e^x(x^2+2x-2) + e^x(2x+2)$

$= e^x(x^2+4x) = 0$ if $x^2+4x=0$ $x(x+4)=0$

$x=0$ $x=-4$

$f(0) = -2$ $f(-4) = \frac{14}{e^4}$



x-int: $(\sqrt{2},0)$ $(-\sqrt{2},0)$ y-int: $(0,-2)$

local max of $(e^{-1-\sqrt{3}})(2+2\sqrt{3})$ at $x = -1-\sqrt{3}$

local min of $(e^{-1+\sqrt{3}})(2-2\sqrt{3})$ at $x = -1+\sqrt{3}$

inc: $(-\infty,-1-\sqrt{3}] \cup [-1+\sqrt{3},\infty)$ POI: $(0,-2)$ $(-4,\frac{14}{e^4})$

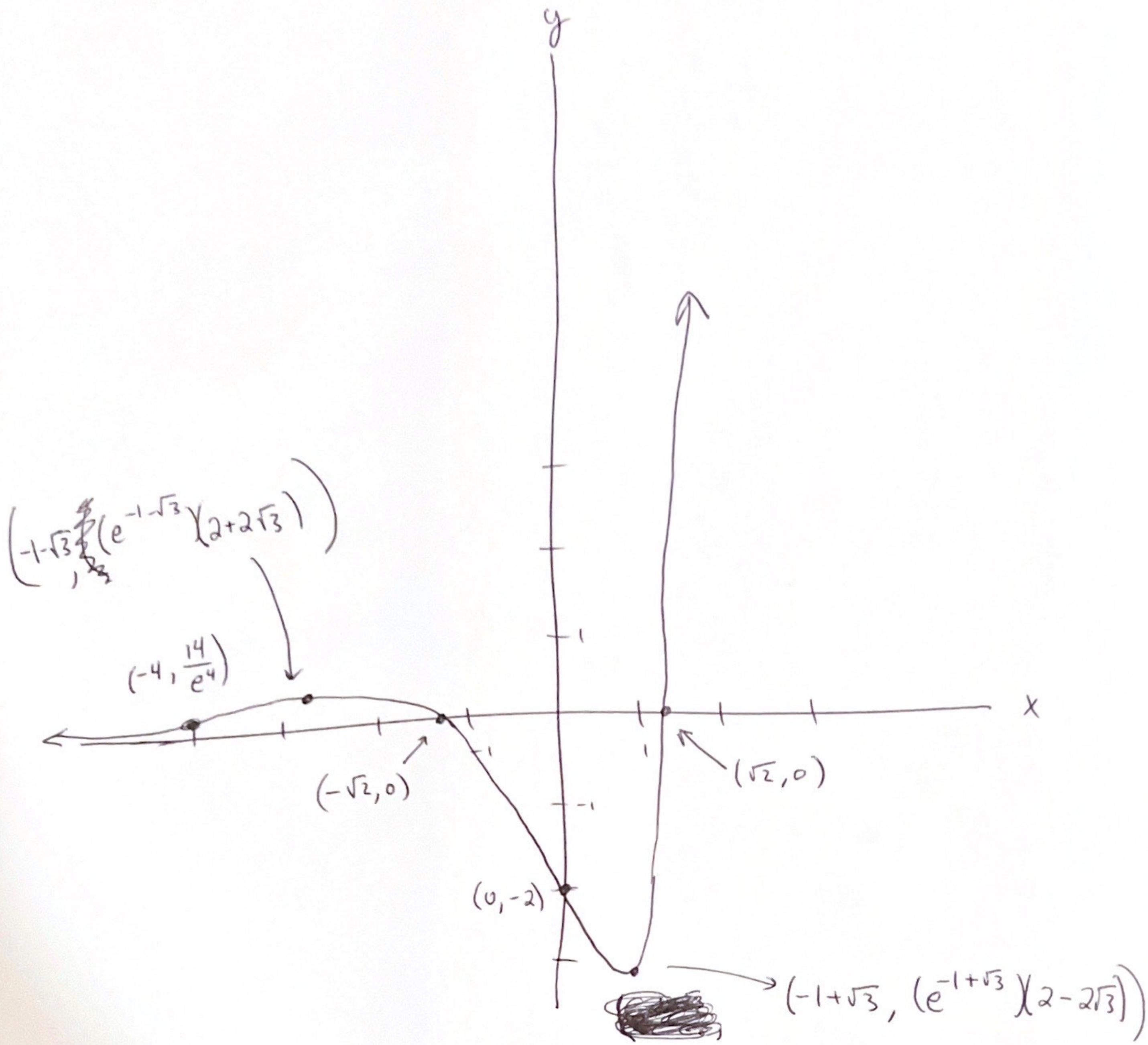
dec: $[-1-\sqrt{3},-1+\sqrt{3}]$

conc up: $(-\infty,-4)\cup(0,\infty)$

conc down: $(-4,0)$

dom: $(-\infty,\infty)$ range: $[(e^{-1+\sqrt{3}})(2-2\sqrt{3}),\infty)$

$f(x) = e^x(x^2-2)$



y
x
$\left(-1-\sqrt{3}, (e^{-1-\sqrt{3}})(2+2\sqrt{3})\right)$
$\left(-4, \frac{14}{e^4}\right)$
$(-\sqrt{2}, 0)$
$(\sqrt{2}, 0)$
$(0, -2)$
$\left(-1+\sqrt{3}, (e^{-1+\sqrt{3}})(2-2\sqrt{3})\right)$
1
-1

9. $f(x) = e^x(x^3-4) = 0 \quad x^3-4=0 \quad = \sqrt[3]{4} \quad$ x-int: $(\sqrt[3]{4}, 0)$

$f(0) = -4 \qquad \lim_{x\to\infty} f(x) = \infty \qquad \lim_{x\to-\infty} f(x) = 0$

$f'(x) = e^x(x^3-4) + e^x(3x^2) = e^x(x^3+3x^2-4)$

1	1	3	0	−4
		1	4	4
	1	4	4	0

$= e^x(x-1)(x^2+4x+4)$

$= e^x(x-1)(x+2)^2 = 0$

$x = 1 \quad x = -2$

f': $-$ (−2) $-$ (1) $+$

$f(1) = -3e$

$f''(x) = e^x(x^3+3x^2-4) + e^x(3x^2+6x)$

$= e^x(x^3+6x^2+6x-4)$

−2	1	6	6	−4
		−2	−8	4
	1	4	−2	0

f'': $-$ ($-2-\sqrt{6}$) $+$ (-2) $-$ ($-2+\sqrt{6}$) $+$

$= e^x(x+2)(x^2+4x-2) = 0$

$x+2=0 \qquad x^2+4x-2=0$

$x=-2 \qquad x^2+4x+4=6$

$(x+2)^2 = 6$

$x = -2 \pm \sqrt{6}$

$f(-2) = -\frac{12}{e^2} \approx -1.624$

$f(-2+\sqrt{6}) = (e^{-2+\sqrt{6}})((-2+\sqrt{6})^3-4)$

$f(-2-\sqrt{6}) = (e^{-2-\sqrt{6}})((-2-\sqrt{6})^3-4)$

≈ -6.128

≈ -1.076

x-int: $(\sqrt[3]{4}, 0)$

y-int: $(0, -4)$

no local max

local min of $-3e$ at $x=1$

POI: $\left(-2, -\frac{12}{e^2}\right)$, $\left(-2+\sqrt{6}, (e^{-2+\sqrt{6}})((-2+\sqrt{6})^3-4)\right)$

$\left((-2-\sqrt{6}), (e^{-2-\sqrt{6}})((-2-\sqrt{6})^3-4)\right)$

HA: $y=0$ to left

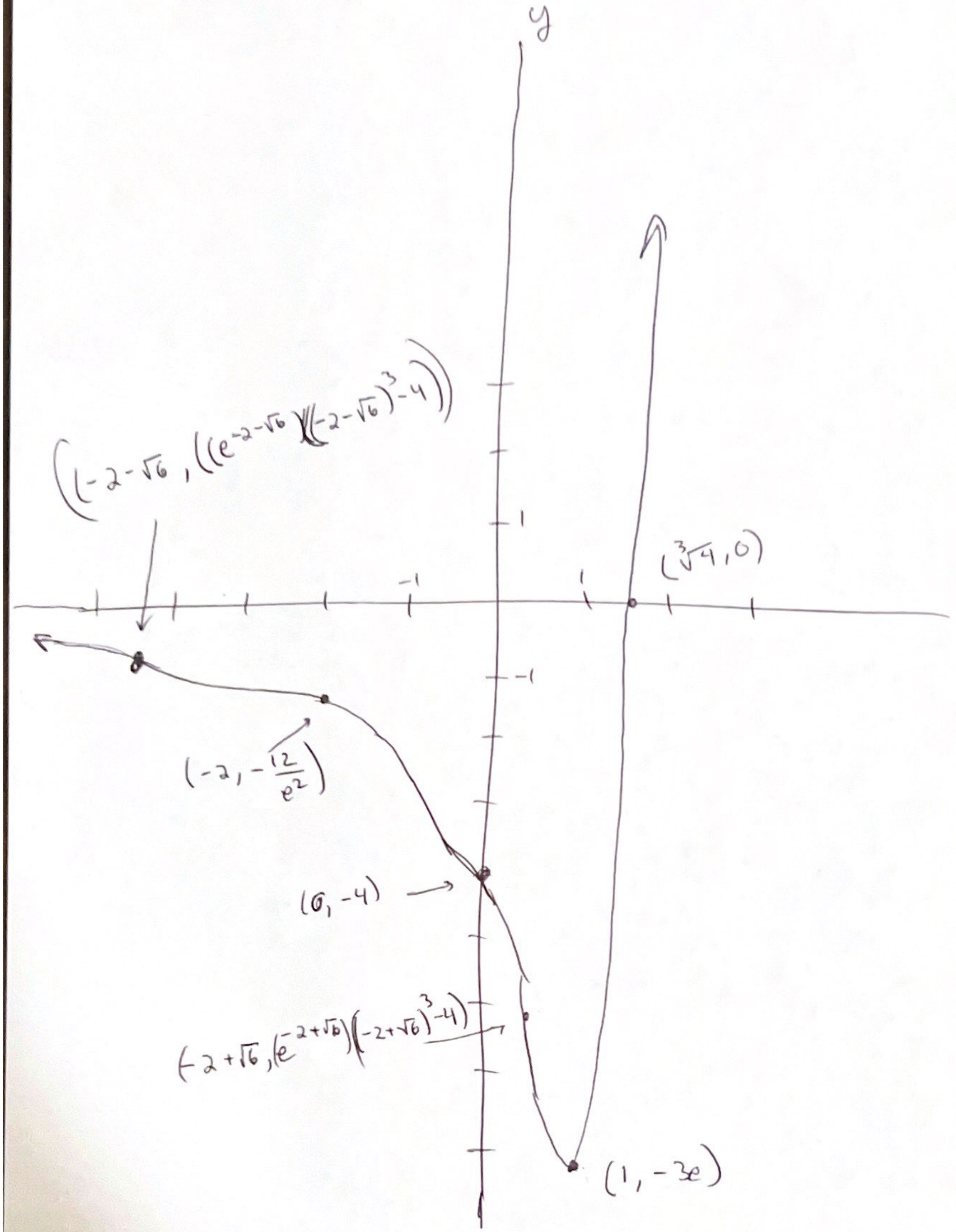
inc: $[1, \infty)$ dec: $(-\infty, 1]$

conc up: $(-2-\sqrt{6}, -2) \cup (-2+\sqrt{6}, \infty)$

conc down: $(-\infty, -2-\sqrt{6}) \cup (-2, -2+\sqrt{6})$

dom: $(-\infty, \infty)$ range: $[-3e, \infty)$

$f(x) = e^x(x^3-4)$



y
x
1
-1
1
-1
$(\sqrt[3]{4}, 0)$
$(-2-\sqrt{6}, ((e^{-2-\sqrt{6}})((-2-\sqrt{6})^3-4)))$
$(-2, -\frac{12}{e^2})$
$(0, -4)$
$(-2+\sqrt{6}, (e^{-2+\sqrt{6}})((-2+\sqrt{6})^3-4))$
$(1, -3e)$