1. $f(x) = \frac{x^3}{3} + \frac{5}{2}x^2 + 6x - 1$

$f'(x) = x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$x = -2 \quad x = -3$

$f''(x) = 2x + 5$

$f''(-2) = 1 > 0$, min

$\boxed{\text{local min at } x = -2}$

$f''(-3) = -1 < 0$, max

$\boxed{\text{local max at } x = -3}$

2. $f(x) = (x-1)^{\frac{2}{3}}(x+4)$

$f'(x) = (x-1)^{\frac{2}{3}} + \frac{2}{3}(x-1)^{-\frac{1}{3}}(x+4)$

$= \frac{(x-1)^{\frac{2}{3}}(3)(x-1)^{\frac{1}{3}}}{3(x-1)^{\frac{1}{3}}} + \frac{2(x+4)}{3(x-1)^{\frac{1}{3}}}$

$= \frac{3(x-1) + 2x + 8}{3(x-1)^{\frac{1}{3}}} = \frac{5x+5}{3(x-1)^{\frac{1}{3}}}$

$f' = 0 = 5x + 5$

$x = -1$

f' und. $x = 1$

don't include 2nd deriv. test only works for $f' = 0$

$f'' = \frac{3(x-1)^{\frac{1}{3}}(5) - (5x+5)\left(\frac{1}{3}\right)(3)(x-1)^{-\frac{2}{3}}}{9(x-1)^{\frac{2}{3}}}$

$f''(-1) = \frac{15(-2)^{\frac{1}{3}} - 0}{9(-2)^{\frac{2}{3}}} = \frac{-}{+} < 0$, max

$\boxed{\text{local max at } x = -1}$

3. $f(x) = x^4 - 5x^2$

$f'(x) = 4x^3 - 10x = 0$

$2x(2x^2 - 5) = 0$

$x = 0 \qquad x = \pm\sqrt{\frac{5}{2}}$

$f''(x) = 12x^2 - 10$

$f''\left(-\sqrt{\frac{5}{2}}\right) = 20 > 0$, min

$f''\left(\sqrt{\frac{5}{2}}\right) = 20 > 0$, min

$f''(0) = -10 < 0$, max

$\boxed{\text{local max at } x = 0 \qquad \text{local min at } x = -\sqrt{\tfrac{5}{2}} \text{ and } \sqrt{\tfrac{5}{2}}}$

4. $f(x) = x^5 - 3x^3$

$f'(x) = 5x^4 - 9x^2 = 0$

$x^2(5x^2 - 9) = 0$

$x = 0 \qquad x = \pm\frac{3}{\sqrt{5}}$

$f''(x) = 20x^3 - 18x$

$f''(0) = 0$

2nd deriv. test fails

$f''\left(\frac{3}{\sqrt{5}}\right) > 0$, min

$f''\left(-\frac{3}{\sqrt{5}}\right) < 0$, max

$\boxed{\begin{array}{l}\text{local min at } x = \frac{3}{\sqrt{5}} \\ \text{local max at } x = -\frac{3}{\sqrt{5}}\end{array}}$

5. $f(x) = \ln(x^3 - 3x)$ $\rightarrow$ domain: $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

$f'(x) = \dfrac{3x^2 - 3}{x^3 - 3x} = 0$ if $x = \pm 1$

$x = 1$ not in domain

$x = -1$ in domain

$$f''(x) = \frac{(x^3 - 3x)(6x) - (3x^2 - 3)(3x^2 - 3)}{(x^3 - 3x)^2}$$

$f''(-1) = \dfrac{(2)(-6) - 0}{4} < 0$, max $\qquad \boxed{\text{local max at } x = -1}$