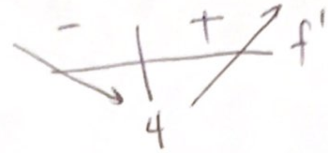
1. $f(x) = x^2 - 8x + 16$ Domain: $(-\infty, \infty)$

$f'(x) = 2x - 8$ f' never undefined

$f' = 0$ if $2x - 8 = 0$

$x = 4$



$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = \infty$

$f(4) = 0$

$f''(x) = 2$ f'' is never undefined

$f'' \neq 0$

$2 > 0$

increasing: $[4, \infty)$

decreasing: $(-\infty, 4]$

concave up: $(-\infty, \infty)$

never concave down

local (and absolute) minimum of 0 at $x = 4$

no local/absolute maximum

no points of inflection

2. $f(x) = \frac{1}{3}x^3 - 2x^2$ Domain: $(-\infty, \infty)$

$f(0) = 0$

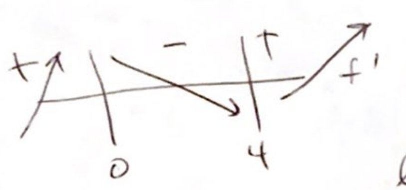
$f(4) = -\frac{32}{3}$

$f'(x) = x^2 - 4x$ f' never undefined

$f' = 0$ if $x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ $x = 4$



$\lim_{x \to \infty} f(x) = \infty$

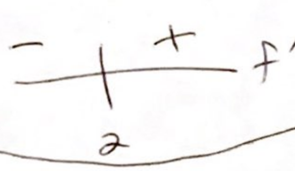
$\lim_{x \to -\infty} f(x) = -\infty$

$f''(x) = 2x - 4$ f'' never undefined

$f(2) = -\frac{16}{3}$

$f'' = 0$ if $2x - 4 = 0$

$x = 2$



increasing: $(-\infty, 0] \cup [4, \infty)$

decreasing: $[0, 4]$

concave up: $(2, \infty)$

concave down $(-\infty, 2)$

local minimum of $-\frac{32}{3}$ at $x = 4$

local maximum of 0 at $x = 0$

no absolute max/min

POI: $(2, \frac{-16}{3})$

3. $\ln(x^2-1) = f(x)$

$$x^2-1>0$$
$$x^2>1$$
$$x>1 \text{ or } x<-1$$

Domain: $(-\infty,-1)\cup(1,\infty)$

$f'(x) = \dfrac{2x}{x^2-1}$ $\quad$ f' undefined if $x^2-1=0$

$x=\pm 1 \rightarrow$ not in domain of f

$f'=0$ if $2x=0$

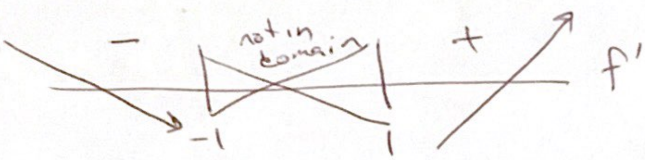
$x=0 \rightarrow$ not in domain of f

$\lim\limits_{x\to\infty} f(x) = \infty$

$\lim\limits_{x\to-\infty} f(x) = \infty$

$\lim\limits_{x\to1^+} f(x) = -\infty$

$\lim\limits_{x\to-1^-} f(x) = -\infty$



$$f''(x) = \frac{(x^2-1)(2)-(2x)(2x)}{(x^2-1)^2} = \frac{2x^2-2-4x^2}{(x^2-1)^2}$$

$$= \frac{-2x^2-2}{(x^2-1)^2}$$

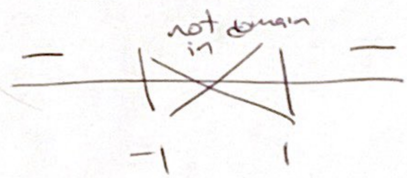
f'' undefined if $(x^2-1)^2=0$

$x=\pm1 \rightarrow$ not in domain of f

$f''=0$ if $-2x^2-2=0$

$x^2=-1$

no real solutions



increasing: $(1,\infty)$

decreasing: $(-\infty,-1)$

concave down: $(-\infty,-1)\cup(1,\infty)$

never concave up

no local or absolute max/min

no POI.

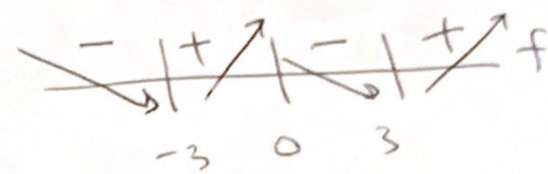
4. $f(x) = x^4 - 18x^2$ domain: $(-\infty, \infty)$

$f'(x) = 4x^3 - 36x$ f' never undefined

$f' = 0$ if $4x^3 - 36x = 0$

$4x(x^2 - 9) = 0$

$x = 0 \quad x = \pm 3$



$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = \infty$

$f(-3) = -81$

$f(0) = 0$

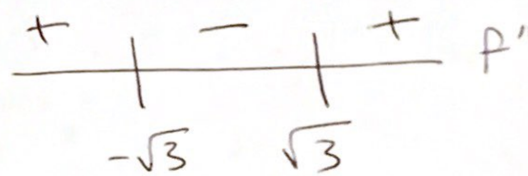
$f(3) = -81$

$f''(x) = 12x^2 - 36$ f'' never undefined

$f'' = 0$ if $12x^2 - 36 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$



$f(-\sqrt{3}) = -45$

$f(\sqrt{3}) = -45$

increasing: $[-3, 0] \cup [3, \infty)$

decreasing: $(-\infty, -3] \cup [0, 3]$

concave up: $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

concave down: $(-\sqrt{3}, \sqrt{3})$

POIs: $(-\sqrt{3}, -45), (\sqrt{3}, -45)$

local max $= 0$ at $x = 0$

local (and absolute) min of -81 at $x = 3$ and $x = -3$

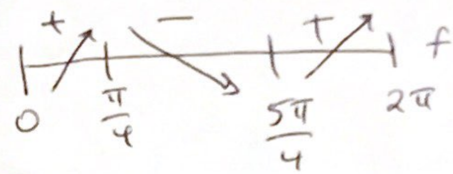
5. $f(x) = \sin x + \cos x$ for $[0, 2\pi]$

$f'(x) = \cos x - \sin x$ $\quad$ f' never undefined

$f' = 0$ if $\cos x - \sin x = 0$

$\cos x = \sin x$

$x = \frac{\pi}{4}$ $\quad$ $x = \frac{5\pi}{4}$



$f(0) = 1$ $\quad$ $f\left(\frac{5\pi}{4}\right) = -\sqrt{2}$

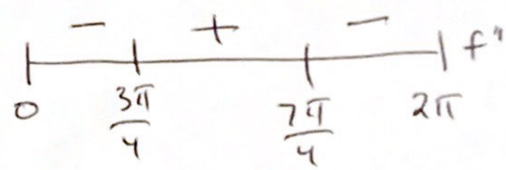
$f\left(\frac{\pi}{4}\right) = \sqrt{2}$ $\quad$ $f(2\pi) = 1$

$f''(x) = -\sin x - \cos x$ $\quad$ f'' never undefined

$f'' = 0$ if $-\sin x - \cos x = 0$

$\sin x = -\cos x$

$x = \frac{3\pi}{4}$ $\quad$ $x = \frac{7\pi}{4}$



$f\left(\frac{3\pi}{4}\right) = 0$

$f\left(\frac{7\pi}{4}\right) = 0$

increasing: $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{5\pi}{4}, 2\pi\right]$

decreasing: $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

concave up: $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

concave down: $\left(0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$

POIs: $\left(\frac{3\pi}{4}, 0\right), \left(\frac{7\pi}{4}, 0\right)$

local min of 1 at $x = 0$

local (and absolute) min of $-\sqrt{2}$ at $x = \frac{5\pi}{4}$

local max of 1 at $x = 2\pi$

local (and absolute) max of $\sqrt{2}$ at $x = \frac{\pi}{4}$