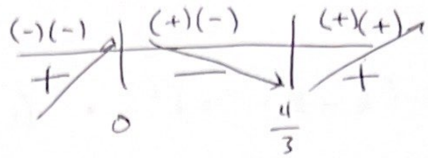
1. $f(x) = x^3 - 2x^2 + 3$



$f'(x) = 3x^2 - 4x = 0$

f' never undefined $x(3x-4) = 0$

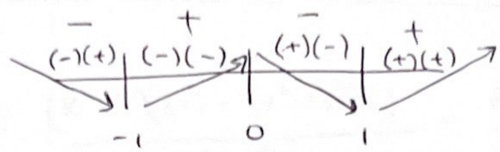
$x = 0 \quad x = \frac{4}{3}$

$f(0) = 3$

$f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2 + 3 = \frac{49}{27}$

$$\boxed{\begin{array}{ll} \text{local max of } 3 \text{ at } x = 0 & \text{inc: } (-\infty, 0] \cup [\frac{4}{3}, \infty) \\ \text{local min of } \frac{49}{27} \text{ at } x = \frac{4}{3} & \text{dec: } [0, \frac{4}{3}] \end{array}}$$

2. $f(x) = x^4 - 2x^2 + 8$



$f'(x) = 4x^3 - 4x = 0$

f' never undefined $4x(x^2 - 1) = 0$

$x = 0 \quad x = \pm 1$

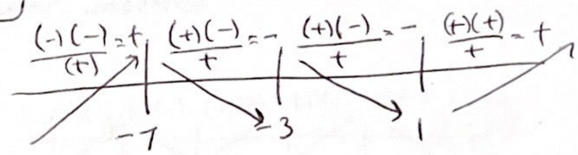
$f(-1) = (-1)^4 - 2(-1)^2 + 8 = 7$

$f(0) = 8$

$f(1) = 1^4 - 2(1)^2 + 8 = 7$

$$\boxed{\begin{array}{l} \text{local min of } 7 \text{ at } x = -1 \text{ and at } x = 1 \\ \text{local max of } 8 \text{ at } x = 0 \\ \text{inc: } [-1, 0] \cup [1, \infty) \\ \text{dec: } (-\infty, -1] \cup [0, 1] \end{array}}$$

3. $f(x) = \dfrac{x^2 - 2x + 1}{x + 3}$ $x \neq -3$, VA



$f'(x) = \dfrac{(x+3)(2x-2) - (x^2 - 2x + 1)(1)}{(x+3)^2}$

$= \dfrac{2x^2 + 6x - 2x - 6 - x^2 + 2x - 1}{(x+3)^2}$

$= \dfrac{x^2 + 6x - 7}{(x+3)^2} = 0$ if $x^2 + 6x - 7 = 0$

$(x+7)(x-1) = 0$

$x = -7 \quad x = 1$

$f(-7) = \dfrac{49 + 14 + 1}{-4} = -16$

$f(1) = \dfrac{1 - 2 + 1}{4} = 0$

$$\boxed{\begin{array}{ll} \text{local max of } -16 \text{ at } x = -7 & \text{inc: } (-\infty, -7] \cup [1, \infty) \\ \text{local min of } 0 \text{ at } x = 1 & \text{dec: } [-7, -3) \cup (-3, 1] \end{array}}$$

f' undefined if $x = -3$ but no max/min because $x = -3$ is not in domain of $f(x)$

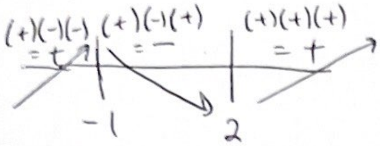
4. $f(x) = e^x(x^2-3x+1)$

$f'(x) = e^x(2x-3) + e^x(x^2-3x+1) = e^x(x^2-x-2) = e^x(x-2)(x+1)$

f' never undefined $\quad f'=0$ if $e^x = 0 \quad x-2=0 \quad x+1=0$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \downarrow \quad\quad\; x=2 \quad\quad x=-1$

never



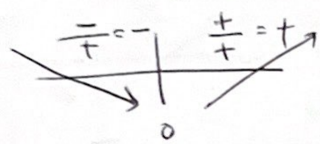
$f(-1) = e^{-1}(1+3+1) = \frac{5}{e}$

$f(2) = e^2(4-6+1) = -e^2$

local max of $\frac{5}{e}$ at $x=-1$ $\quad$ inc: $(-\infty,-1] \cup [2,\infty)$

local min of $-e^2$ at $x=2$ $\quad$ dec: $[-1,2]$

5. $f(x) = \frac{x^2-1}{x^2+2}$ $\quad f'(x) = \frac{(x^2+2)(2x)-(x^2-1)(2x)}{(x^2+2)^2} = \frac{2x^3+4x-2x^3+2x}{(x^2+2)^2}$



$= \frac{6x}{(x^2+2)^2}$ $\quad f'$ never undefined

$f'=0$ if $6x=0$, $x=0$

$f(0) = -\frac{1}{2}$

local min of $-\frac{1}{2}$ at $x=0$ $\quad$ inc: $[0,\infty)$ $\quad$ dec: $(-\infty,0]$

6. $f(x) = \sin^3 x \cos x$ on $(0,2\pi)$

$f'(x) = \sin^3 x(-\sin x) + 3\sin^2 x \cos x \cos x = 3\sin^2 x\cos^2 x - \sin^4 x$

$= \sin^2 x(3\cos^2 x - \sin^2 x)$ $\quad f'$ never undefined

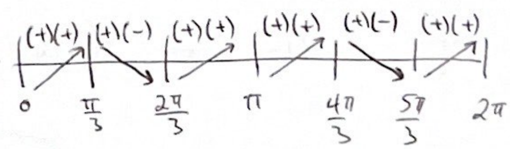
$f'=0$ if $\sin^2 x = 0$ $\quad\quad 3\cos^2 x - \sin^2 x = 0$

$x = \pi$ $\quad\quad 3\cos^2 x = \sin^2 x$

$3 = \tan^2 x$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$



$f(\frac{\pi}{3}) = \left(\frac{\sqrt{3}}{2}\right)^3\left(\frac{1}{2}\right) = \frac{3\sqrt{3}}{16}$

$f(\frac{2\pi}{3}) = \left(\frac{\sqrt{3}}{2}\right)^3\left(-\frac{1}{2}\right) = -\frac{3\sqrt{3}}{16}$

$f(\frac{4\pi}{3}) = \left(-\frac{\sqrt{3}}{2}\right)^3\left(-\frac{1}{2}\right) = \frac{3\sqrt{3}}{16}$

$f(\frac{5\pi}{3}) = \left(-\frac{\sqrt{3}}{2}\right)^3\left(\frac{1}{2}\right) = -\frac{3\sqrt{3}}{16}$

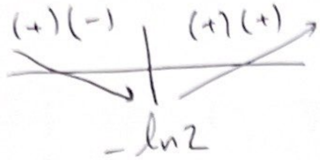
local max of $\frac{3\sqrt{3}}{16}$ at $x=\frac{\pi}{3}$ and $x=\frac{4\pi}{3}$

local min of $-\frac{3\sqrt{3}}{16}$ at $x=\frac{2\pi}{3}$ and $x=\frac{5\pi}{3}$

inc: $(0,\frac{\pi}{3}] \cup [\frac{2\pi}{3},\frac{4\pi}{3}] \cup [\frac{5\pi}{3},2\pi)$

dec: $[\frac{\pi}{3},\frac{2\pi}{3}] \cup [\frac{4\pi}{3},\frac{5\pi}{3}]$

7. $f(x) = e^{2x} - e^{x}$ $\quad f'(x) = 2e^{2x} - e^{x} = 2e^{x}(e^{x}) - e^{x}$



$= e^{x}(2e^{x} - 1) = 0$ if $e^{x} = 0 \rightarrow$ never

$2e^{x} - 1 = 0 \rightarrow e^{x} = \frac{1}{2}$

f' never undefined $\rightarrow x = \ln\frac{1}{2} = \ln 1 - \ln 2$

$= -\ln 2$

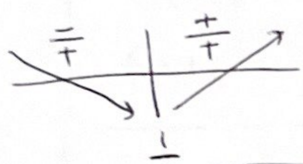
$f(-\ln 2) = e^{-2\ln 2} - e^{-\ln 2} = e^{\ln\frac{1}{4}} - e^{\ln\frac{1}{2}}$

$= \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

$$\boxed{\begin{array}{l} \text{local min of } -\frac{1}{4} \text{ at } x = -\ln 2 \\ \text{dec: } (-\infty, -\ln 2] \quad \text{inc: } [-\ln 2, \infty) \end{array}}$$

8. $f(x) = \ln(x^2 - x + 2)$ $\quad f'(x) = \dfrac{2x-1}{x^2 - x + 2}$ $\quad f' = 0$ if $2x - 1 = 0$

$x = \frac{1}{2}$



f' undefined if $x^2 - x + 2 = 0$ $\quad f(\frac{1}{2}) = \ln(\frac{1}{4} - \frac{1}{2} + 2) = \ln\frac{7}{4}$

$b^2 - 4ac = 1 - 4(1)(2) = -7 < 0$, no real sol., so never

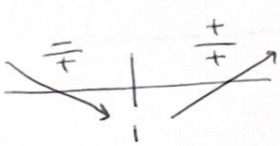
$$\boxed{\text{local min of } \ln\left(\tfrac{7}{4}\right) \text{ at } x = \tfrac{1}{2} \quad \text{inc: } [\tfrac{1}{2}, \infty) \quad \text{dec: } (-\infty, \tfrac{1}{2}]}$$

9. $f(x) = \tan^{-1}(x^2 - 2x)$ $\quad f'(x) = \dfrac{2x - 2}{1 + (x^2 - 2x)^2} = \dfrac{2(x-1)}{x^4 - 4x^3 + 4x^2 + 1}$

$f' = 0$ if $x - 1 = 0$

$x = 1$

f' never undefined because $1 + (x^2 - 2x)^2 \neq 0$



$f(1) = \tan^{-1}(1^2 - 2(1)) = \tan^{-1}(-1) = -\frac{\pi}{4}$

$$\boxed{\begin{array}{c} \text{local min of } -\frac{\pi}{4} \text{ at } x = 1 \\ \text{inc: } [1, \infty) \quad \text{dec: } (-\infty, 1] \end{array}}$$

10. $f(x) = \sqrt[3]{x^3 - 3x + 2}$ $\qquad f'(x) = \dfrac{3x^2 - 3}{3(x^3 - 3x + 2)^{\frac{2}{3}}}$

$f' = 0$ if $3x^2 - 3 = 0$

$3(x+1)(x-1) = 0$

$x = \pm 1$

f' undefined if

$x^3 - 3x + 2 = 0$

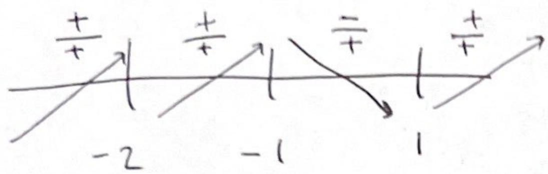
$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$(x-1)(x^2 + x - 2) = 0$

$(x-1)(x-1)(x+2) = 0$

$(x-1)^2(x+2) = 0$

$x = 1 \quad x = -2$



$f(-1) = \sqrt[3]{-1+3+2} = \sqrt[3]{4}$

$f(1) = \sqrt[3]{1-3+2} = 0$

local max of $\sqrt[3]{4}$ at $x = -1$

local min of 0 at $x = 1$

inc: $(-\infty, -1] \cup [1, \infty)$

dec: $[-1, 1]$