1. $f(x) = 2x^{\frac{1}{3}}$ on $[-8, 1]$

$f'(x) = \frac{2}{3}x^{-\frac{2}{3}} = \frac{2}{3x^{\frac{2}{3}}} = 0$ never

f' undefined if $3x^{\frac{2}{3}} = 0 \rightarrow x = 0$

endpoints $f(-8) = -4$

$f(1) = 2$

CV: $f(0) = 0$

$\boxed{\begin{array}{l}\text{Absolute Maximum of } 2 \text{ at } x=1 \\ \text{Absolute Minimum of } -4 \text{ at } x=-8\end{array}}$

2. $h(x) = x\sqrt{x} - x = x^{\frac{3}{2}} - x$ on $[0, 4]$

$h'(x) = \frac{3}{2}\sqrt{x} - 1$ never undefined on domain given

$h' = 0$ if $\frac{3}{2}\sqrt{x} - 1 = 0 \rightarrow \sqrt{x} = \frac{2}{3}$

$x = \frac{4}{9}$

endpoints $h(0) = 0$

$h(4) = 4$

CV: $h\left(\frac{4}{9}\right) = \frac{4}{9}\left(\frac{2}{3}\right) - \frac{4}{9} = -\frac{4}{27}$

$\boxed{\begin{array}{l}\text{Absolute maximum of } 4 \text{ at } x=4 \\ \text{Absolute minimum of } -\frac{4}{27} \text{ at } x=\frac{4}{9}\end{array}}$

3. $g(x) = \frac{x-1}{\sqrt{x}}$ on $[1, 9]$

$g(x) = \sqrt{x} - x^{-\frac{1}{2}}$

$g'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{2x^{\frac{3}{2}}}$

$= \frac{x+1}{2x^{\frac{3}{2}}}$

endpoints $g(1) = 0$

$g(9) = \frac{8}{3}$

$\boxed{\begin{array}{l}\text{Absolute maximum of } \frac{8}{3} \text{ at } x=9 \\ \text{Absolute minimum of } 0 \text{ at } x=1\end{array}}$

$g'(x) = 0$ if $x+1 = 0 \rightarrow x = -1$

g' undefined if $2x^{\frac{3}{2}} = 0 \rightarrow x = 0$

not on given domain

4. $f(x) = x^3 - 3x^2 + 4$ on $[-1, 4]$

$f'(x) = 3x^2 - 6x$ never undefined

$f'(x) = 0$ if $3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$

CV: $f(0) = 4$, $f(2) = 0$

endpoints: $f(-1) = 0$, $f(4) = 20$

$$\boxed{\begin{array}{l}\text{Absolute maximum of } 20 \text{ at } x = 4 \\ \text{Absolute minimum of } 0 \text{ at } x = -1 \text{ and } x = 2\end{array}}$$

5. $f(x) = x\sqrt{4-x^2}$ on $[-\sqrt{2}, 2]$

$$f'(x) = \frac{x(-2x)}{2\sqrt{4-x^2}} + \sqrt{4-x^2} = \frac{-2x^2 + (4-x^2)(2)}{2\sqrt{4-x^2}}$$

$$= \frac{8-4x^2}{2\sqrt{4-x^2}}$$ never undefined on domain

$f' = 0$ if $8 - 4x^2 = 0 \quad x = \pm\sqrt{2}$

endpoint $\rightarrow f(-\sqrt{2}) = -2$

CV $\rightarrow f(\sqrt{2}) = 2$

endpoint $\rightarrow f(2) = 0$

$$\boxed{\begin{array}{l}\text{Absolute maximum of } 2 \text{ at } x = \sqrt{2} \\ \text{Absolute minimum of } -2 \text{ at } x = -\sqrt{2}\end{array}}$$

6. $g(x) = e^{-x}\sin x$ on $[-\pi, \pi]$

$g'(x) = e^{-x}\cos x - e^{-x}\sin x$

$= e^{-x}(\cos x - \sin x)$ never undefined

$g'(x) = 0$ if $e^{-x} = 0$ OR $\cos x - \sin x = 0$

never $\quad \sin x = \cos x$

$x = -\frac{3\pi}{4}$, $x = \frac{\pi}{4}$ on $[-\pi, \pi]$

CV

$g(-\frac{3\pi}{4}) = e^{\frac{3\pi}{4}}(-\frac{\sqrt{2}}{2}) < 0$

$g(\frac{\pi}{4}) = e^{-\frac{\pi}{4}}(\frac{\sqrt{2}}{2}) > 0$

endpoints

$g(-\pi) = 0$

$g(\pi) = 0$

$\boxed{\begin{array}{l}\text{Absolute maximum of } \frac{\sqrt{2}e^{-\frac{\pi}{4}}}{2} \text{ at } x = \frac{\pi}{4} \\ \text{Absolute minimum of } -\frac{\sqrt{2}e^{\frac{3\pi}{4}}}{2} \text{ at } x = -\frac{3\pi}{4}\end{array}}$

7. $f(x) = x^2\sqrt{8-x^3}$ on $[-2, 2]$

$$f'(x) = \frac{x^2(-3x^2)}{2\sqrt{8-x^3}} + 2x\sqrt{8-x^3} = \frac{-3x^4 + 4x(8-x^3)}{2\sqrt{8-x^3}}$$

$$= \frac{-7x^4 + 32x}{2\sqrt{8-x^3}}$$

never undefined on given domain

$f'(x) = 0$ if $-7x^4 + 32x = 0$

$x(-7x^3 + 32) = 0$

$x = 0 \quad x = \sqrt[3]{\frac{32}{7}}$

endpoints

$f(-2) = 16$

$f(2) = 0$

CV

$f(0) = 0$

$f(\sqrt[3]{\frac{32}{7}}) = \sqrt{\frac{24}{7}}\left(\frac{32}{7}\right)^{\frac{2}{3}} < 16$

> 0

$\boxed{\begin{array}{l}\text{Absolute maximum of 16 at } x = -2 \\ \text{Absolute minimum of 0 at } x = 2 \text{ and } x = 0\end{array}}$

8. $h(x) = \sin x + \cos x$ on $[0, \pi]$

$h'(x) = \cos x - \sin x$ never undefined

$\cos x - \sin x = 0$

$\sin x = \cos x$

$x = \frac{\pi}{4}$

endpoints $f(0) = 1$, $f(\pi) = -1$

CV: $f\left(\frac{\pi}{4}\right) = \sqrt{2}$

$$\boxed{\begin{array}{l}\text{Absolute Max of } \sqrt{2} \text{ at } x = \frac{\pi}{4} \\ \text{Absolute Min of } -1 \text{ at } x = \pi\end{array}}$$

9. $g(x) = -\frac{1}{2}\cos 2x - \cos x$ on $[-\pi, \pi]$

$g'(x) = \sin 2x + \sin x$ → never undefined

$= 2\sin x\cos x + \sin x = 0$

$\sin x(2\cos x + 1) = 0$

$x = 0$

$x = -\pi$

$x = \pi$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}$

$x = -\frac{2\pi}{3}$

endpoints: $f(-\pi) = \frac{1}{2}$, $f(\pi) = \frac{1}{2}$

$f(0) = -\frac{3}{2}$

CV: $f\left(\frac{2\pi}{3}\right) = \frac{3}{4}$, $f\left(-\frac{2\pi}{3}\right) = \frac{3}{4}$

$$\boxed{\begin{array}{l}\text{Absolute Maximum of } \frac{3}{4} \text{ at } x = \frac{2\pi}{3} \text{ and } x = -\frac{2\pi}{3} \\ \text{Absolute Minimum of } -\frac{3}{2} \text{ at } x = 0\end{array}}$$

10. $h(x) = x(5-x)^{\frac{2}{3}}$ on $[0,6]$

$$h'(x) = x\left(\tfrac{2}{3}\right)(5-x)^{-\frac{1}{3}}(-1) + (5-x)^{\frac{2}{3}}$$

$$= -\frac{2x}{3(5-x)^{\frac{1}{3}}} + \frac{(5-x)^{\frac{2}{3}}(3)(5-x)^{\frac{1}{3}}}{3(5-x)^{\frac{1}{3}}}$$

$$= \frac{-2x + 3(5-x)}{3(5-x)^{\frac{1}{3}}} = \frac{-5x+15}{3(5-x)^{\frac{1}{3}}}$$ $=0$ at $x=3$

undefined at $x=5$

end points $\left\{ \begin{array}{l} f(0) = 0 \\ f(6) = 6 \end{array} \right.$

CV $\left\{ \begin{array}{l} f(3) = 3\left(2^{\frac{2}{3}}\right) < 6 \\ f(5) = 0 \end{array} \right.$

Absolute maximum of 6 at $x=6$

Absolute minimum of 0 at $x=0$ and $x=5$