1. A particle moves according to the position function $s(t) = t^2 - 10t + 12$ where t is measured in seconds and s in feet.

a. Find the velocity function, $v(t)$.

$s'(t) = \boxed{v(t) = 2t - 10}$

b. Find the acceleration function, $a(t)$.

$v'(t) = \boxed{a(t) = 2}$

c. When is the particle at rest?

$2t - 10 = 0$

$\boxed{t = 5 \text{ seconds}}$

d. What is the acceleration at $t = 1$ second?

$\boxed{2 \text{ ft/s}^2}$

e. What is the velocity at $t = 1$ second?

$2(1) - 10 = \boxed{-8 \text{ ft/s}}$

f. At what time is the velocity 3 feet per second?

$2t - 10 = 3$

$2t = 13$

$\boxed{t = \frac{13}{2} \text{ sec}}$ $= 6.5$ sec

2. A particle moves according to the position function $s(t) = t^3 - 9t^2 + 15t + 10$ where t is measured in seconds and s in feet.

a. Find the velocity function, $v(t)$.

$\boxed{v(t) = s'(t) = 3t^2 - 18t + 15}$

b. Find the acceleration function, $a(t)$.

$\boxed{v'(t) = a(t) = 6t - 18}$

c. When is the particle at rest? $v(t)=0=3t^2-18t+15$

$0=t^2-6t+5$

$(t-5)(t-1)=0$ $\boxed{t=1 \text{ sec} \quad t=5 \text{ sec}}$

d. What is the acceleration at $t = 4$ seconds?

$6(4)-18 = 24-18 = \boxed{6 \text{ ft/s}^2}$

e. What is the velocity at $t = 2$ seconds?

$3(2^2)-18(2)+15 = 12-36+15 = \boxed{-9 \text{ ft/s}}$

f. At what time is the velocity 3 feet per second?

$3t^2-18t+15=3$

$t^2-6t+5=1$

$t^2-6t+4=0$

$t=\dfrac{6\pm\sqrt{36-16}}{2}$

$t=\dfrac{6\pm\sqrt{20}}{2}=\dfrac{6\pm2\sqrt{5}}{2}$

$\boxed{t=3\pm\sqrt{5} \text{ sec} \approx \begin{matrix} .764 \text{ sec} \\ 5.236 \text{ sec} \end{matrix}}$

3. A bullet fired straight up from the moon's surface would reach a height of $s(t) = 832t - 2.6t^2$ feet after t seconds. On Earth, its height would be $s(t) = 832t - 16t^2$ feet after t seconds. How much longer does it take the bullet to hit the ground on the moon than on Earth? What is the velocity of the bullet on the moon and on Earth when it hits the ground?

$832t - 2.6t^2 = 0$

$\dfrac{-832 \pm \sqrt{832^2 - 0}}{-5.2}$

$t = 0$ or 320 sec

$832t - 16t^2 = 0$

$\dfrac{-832 \pm \sqrt{832^2 - 0}}{-32}$

$t = 0$ or 52 sec

$320 - 52$ sec $=$ 268 seconds longer

$s'(t) = v(t) = -5.2t + 832$

$-5.2(320) + 832$

moon: -832 ft/s

$s'(t) = v(t) = -32t + 832$

$-32(52) + 832$

Earth: -832 ft/s

4. A particle is moving along the x-axis so that its position at any time $t \geq 0$ is given by the equation $x(t) = \sin^{-1}\left(\frac{\sqrt{t}}{4}\right)$ in feet per second. Find the velocity at $t = 4$ seconds.

$$x'(t) = \frac{1}{\sqrt{1-\left(\frac{\sqrt{t}}{4}\right)^2}} \cdot \frac{1}{8\sqrt{t}} = \frac{1}{\sqrt{1-\frac{1}{4}}} \cdot \frac{1}{16} = \frac{1}{16\sqrt{\frac{3}{4}}} = \boxed{\frac{1}{8\sqrt{3}} \text{ ft/s}}$$

$$\boxed{\approx .072 \text{ ft/s}}$$

5. A weather balloon is released and rises vertically such that its distance $s(t)$ above the ground during the first 10 seconds of flight is given by $s(t) = 6 + 2t + t^2$, where $s(t)$ is in feet and t is in seconds.

a. Find the velocity of the balloon at $t = 1$, $t = 4$, and $t = 8$

$$v(t) = s'(t) = 2 + 2t$$

$$v(1) = 2 + 2(1) = \boxed{4 \text{ ft/s}}$$

$$v(4) = 2 + 2(4) = \boxed{10 \text{ ft/s}}$$

$$v(8) = 2 + 2(8) = \boxed{18 \text{ ft/s}}$$

b. Find the velocity of the balloon at the instant the balloon is 50 feet above the ground.

$s(t) = 50 = 6 + 2t + t^2 \rightarrow t^2 + 2t - 44 = 0$

$t = \frac{-2 \pm \sqrt{4 + 4(1)(44)}}{2} = \frac{-2 \pm \sqrt{180}}{2}$

$= \frac{-2 \pm 6\sqrt{5}}{2} = -1 \pm 3\sqrt{5}$, so $-1 + 3\sqrt{5}$

$v(-1+3\sqrt{5}) = 2 + 2(-1+3\sqrt{5})$

$= 2 - 2 + 6\sqrt{5}$

$= \boxed{6\sqrt{5} \text{ ft/s} \approx 13.416 \text{ ft/s}}$

c. Find the acceleration of the balloon at any given time, $t = a$.

$a(t) = v'(t) = \boxed{2 \text{ ft/s}^2}$

6. A ball rolls down an inclined plane such that the distance in centimeters that rolls in t seconds is given by $s(t) = 2t^3 + 3t^2 + 4$ for $0 \leq t \leq 3$.

a. Find the velocity of the ball at $t = 2$ seconds.

$$v(t) = s'(t) = 6t^2 + 6t$$

$$v(2) = 6(2^2) + 6(2) = 24 + 12 = \boxed{36 \text{ cm}/s}$$

b. At what time is the velocity of the ball 30 cm/sec?

$$6t^2 + 6t = 30 \rightarrow t^2 + t = 5 \rightarrow t^2 + t - 5 = 0$$

$$t = \frac{-1 \pm \sqrt{1 - 4(1)(-5)}}{2} \rightarrow t = \frac{-1 \pm \sqrt{21}}{2}$$

$$\boxed{t = \frac{-1 + \sqrt{21}}{2} \text{ sec} \approx 1.791 \text{ sec.}}$$

c. What is the ball's acceleration at $t = 1$ second? What is its acceleration at $t = 2$ seconds?

$a(t) = v'(t) = 12t + 6$

$a(1) = 12(1) + 6 = \boxed{18\ cm/s^2}$

$a(2) = 12(2) + 6 = \boxed{30\ cm/s^2}$

7. A penny is dropped in a vacuum from a height of 1920 feet according to the equation $s(t) = -16t^2 + 1920$.

a. Find the instantaneous velocity of the penny at any value $t = a$.

$s'(t) = v(t) = -32t\big|_{t=a} = \boxed{-32a\ ft/sec}$

b. Find the instantaneous velocity at 2 seconds.

$-32t\big|_{t=2} = \boxed{-64\ ft/sec}$

c. Find the instantaneous velocity at 5 seconds.

$$-32t\Big|_{t=5} = \boxed{-160 \text{ ft/sec}}$$

d. Find the velocity when the penny hits the ground.

$$-16t^2+1920=0$$

$$-16t^2=-1920$$

$$t^2=120$$

$$t=\sqrt{120}=2\sqrt{30} \text{ sec.}$$

$$-32(2\sqrt{30}) = \boxed{-64\sqrt{30} \text{ ft/s} \approx -350.54 \text{ ft/s}}$$

8. A toy rocket is projected directly upward with a velocity of 192 feet per second from a launching pad that is 9 feet above ground level. The distance $s(t)$ in feet from the ground to the rocket after t seconds is: $s(t) = -16t^2 + 192t + 9$. What is the maximum altitude of the rocket above the ground?

max altitude at $v=0$

$v(t) = s'(t) = -32t + 192 = 0$

$t = 6$ sec

$s(6) = -16(6^2) + 192(6) + 9$

$= 585$ feet

9. A weather balloon is released and rises vertically such that its distance $s(t)$ above ground during the first 14 seconds of flight is given by: $s(t) = 10 + 2t + t^2$, where $s(t)$ is in feet and t is in seconds. Find the velocity of the balloon at the instant that it is 130 feet above the ground.

$$s(t) = 10 + 2t + t^2 = 130$$

$$t^2 + 2t - 120 = 0$$

$$v(t) = s'(t) = 2 + 2t \Big|_{t=10} \qquad (t+12)(t-10) = 0$$

$$= 2 + 2(10) \qquad \underline{t = -12} \quad \underline{t = 10}$$

NO

$$= \boxed{22 \text{ ft/sec}}$$