1. $x^2y + x^2 = y^2 - 2y$

$x^2\frac{dy}{dx} + 2xy + 2x = 2y\frac{dy}{dx} - 2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2xy + 2x}{2y - 2 - x^2}$

$\frac{dy}{dx} = 0$ if $2xy + 2x = 0$ and $2y - 2 - x^2 \neq 0$

$2x(y+1) = 0$

$x = 0$ $\quad$ $y = -1$

into original

$0 + 0 = y^2 - 2y$

$y(y-2) = 0$

$y = 0$ $\quad$ $y = 2$

$(0,0)$ $\quad$ $(0,2)$

check in denominator of $\frac{dy}{dx}$

$0 - 2 - 0 \neq 0$

$y = 0$

check in denominator of $\frac{dy}{dx}$

$4 - 2 - 0 \neq 0$

$y = 2$

into original

$-x^2 + x^2 = 1 + 2$

$0 = 3$

NO!

horizontal asymptote $y = -1$

2. $2y - x^2y^2 = y^3$

$$2\frac{dy}{dx} - 2x^2y\frac{dy}{dx} - 2xy^2 = 3y^2\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2xy^2}{2 - 2x^2y - 3y^2}$$

$\frac{dy}{dx} = 0$ if $2xy^2 = 0$ and $2 - 2x^2y - 3y^2 \neq 0$

$2xy^2 = 0$ → $x = 0$ or $y = 0$

$2 - 2x^2y - 3y^2 \neq 0$

$x = 0$: check in original

$2y - 0 = y^3$

$y^3 - 2y = 0$

$y(y^2 - 2) = 0$

$y = 0$ or $y^2 = 2$

$y = \pm\sqrt{2}$

$y = 0$ already taken care of on above right

$(0, \sqrt{2})$ $(0, -\sqrt{2})$

Check in denominator of $\frac{dy}{dx}$

$2 - 2(0^2)(\sqrt{2}) - 3(\sqrt{2})^2$

$= 2 - 0 - 6 \neq 0$

$y = \sqrt{2}$

$y = 0$: check in original

$0 - 0 = 0$

yes → the line $y = 0$ is part of the graph so $y = 0$ is HTL for every x-value when $y = 0$

for $y = 0$, regardless of value of x

→ Check in denominator of $\frac{dy}{dx}$

$2 - 2(0^2)(-\sqrt{2}) - 3(-\sqrt{2})^2$

$= 2 - 0 - 6 \neq 0$

$y = -\sqrt{2}$

3. $y^3 - y^2 = xy$

$3y^2 \frac{dy}{dx} - 2y \frac{dy}{dx} = x \frac{dy}{dx} + y$

$\frac{dy}{dx} = \frac{y}{3y^2 - 2y - x}$

$\frac{dy}{dx}$ undefined if $3y^2 - 2y - x = 0$ with $y \neq 0$

$3y^2 - 2y - x = 0 \longrightarrow x = 3y^2 - 2y$

↓ plug into original

$y^3 - y^2 = (3y^2 - 2y)y$

$y^3 - y^2 = 3y^3 - 2y^2$

$2y^3 - y^2 = 0$

$y^2(2y - 1) = 0$

↓ $y = 0$ NO! makes numerator of $\frac{dy}{dx} = 0$

↘ $2y - 1 = 0$

$y = \frac{1}{2}$

↓ plug into original

$\frac{1}{8} - \frac{1}{4} = \frac{1}{2}x$

$-\frac{1}{8} = \frac{1}{2}x$

$\boxed{x = -\frac{1}{4}}$

$\left(-\frac{1}{4}, \frac{1}{2}\right)$ does not make numerator of $\frac{dy}{dx} = 0$

4. $x + y - x^2 + y^2 = xy$

$$1 + \frac{dy}{dx} - 2x + 2y\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\frac{dy}{dx} = \frac{y + 2x - 1}{1 + 2y - x}$$

$\frac{dy}{dx}$ undefined if $1 + 2y - x = 0$ but $y + 2x - 1 \neq 0$

$$x = 1 + 2y$$

↓

plug into original

$$1 + 2y + y - (1+2y)^2 + y^2 = y(1+2y)$$

$$1 + 3y - 1 - 4y - 4y^2 + y^2 = y + 2y^2$$

$$5y^2 + 2y = 0$$

$$y(5y+2) = 0$$

$y = 0$ → into original

$$x - x^2 = 0$$

$$x(1-x) = 0$$

$x = 0$ → $(0,0)$ → Check in numerator of $\frac{dy}{dx}$ → $0 + 0 - 1 \neq 0$ → Check in denominator → $1 + 0 - 0 \neq 0$ → so, disregard

$x = 1$ → $(1,0)$ → Check in numerator of $\frac{dy}{dx}$ → $0 + 2 - 1 \neq 0$ → Check in denominator of $\frac{dy}{dx}$ → $1 + 0 - 1 = 0$ → so, yes

$\boxed{x = 1}$

$y = -\frac{2}{5}$ → into original

$$x - \frac{2}{5} - x^2 + \frac{4}{25} = -\frac{2}{5}x$$

$$25x - 10 - 25x^2 + 4 = -10x$$

$$25x^2 - 35x + 6 = 0$$

$$(5x-1)(5x-6) = 0$$

$\boxed{x = \frac{1}{5}}$ → $\left(\frac{1}{5}, -\frac{2}{5}\right)$ → Check in numerator → $-\frac{2}{5} + \frac{2}{5} - 1 \neq 0$ → Check in denominator → $1 + \left(-\frac{4}{5}\right) - \frac{1}{5} = 0$ → so, yes

$x = \frac{6}{5}$ NO → $\left(\frac{6}{5}, -\frac{2}{5}\right)$ → Check in numerator → $-\frac{2}{5} + \frac{12}{5} - 1 \neq 0$ → Check in denominator → $1 + \left(-\frac{4}{5}\right) - \frac{6}{5} \neq 0$ → so, disregard

5. $x^2 - 2y + y^2 - 1 = x^3$

$2x - 2\frac{dy}{dx} + 2y\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{3x^2 - 2x}{2y - 2}$

HTL

$3x^2 - 2x = 0$

$x(3x - 2) = 0$

$x = 0 \quad x = \frac{2}{3}$

VTL

$2y - 2 = 0$

$y = 1$

plug into original

$-2y + y^2 - 1 = 0$

$y^2 - 2y + 1 = 2$

$(y-1)^2 = 2$

$y - 1 = \pm\sqrt{2}$

$y = 1 \pm \sqrt{2}$

HTL

$y = 1 + \sqrt{2}$

$y = 1 - \sqrt{2}$

do not make denominator $= 0$ of $\frac{dy}{dx}$ ←

plug into original

$\frac{4}{9} - 2y + y^2 - 1 = \frac{8}{27}$

$12 - 54y + 27y^2 - 27 = 8$

$27y^2 - 54y - 23 = 0$

$y = \frac{54 \pm \sqrt{(-54)^2 - 4(27)(-23)}}{2(27)}$

$y = \frac{54 \pm \sqrt{5400}}{54}$

$y = \frac{54 \pm 30\sqrt{6}}{54}$

do not make denominator $= 0$ of $\frac{dy}{dx}$ ←

HTL

$y = 1 + \frac{5\sqrt{6}}{9}$

$y = 1 - \frac{5\sqrt{6}}{9}$

plug into original

$x^2 - 2 + 1 - 1 = x^3$

$x^3 - x^2 + 2 = 0$

-1	1	-1	0	2
		-1	2	-2
	1	-2	2	0

$(x+1)(x^2 - 2x + 2) = 0$

VTL $x = -1$ → no real solutions

↓

does not make numerator of $\frac{dy}{dx} = 0$

6. $4x + y^2 = 4x^3$

$4 + 2y\frac{dy}{dx} = 12x^2$

$\frac{dy}{dx} = \frac{12x^2 - 4}{2y} = \frac{6x^2 - 2}{y}$

HTL

$6x^2 - 2 = 0$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

VTL

$y = 0$

plug into original

$4x = 4x^3$

$4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

VTL

$x = 0 \quad x = 1 \quad x = -1$

↓

do not make numerator of $\frac{dy}{dx} = 0$

↓

in original, $y = 0$

$x = \sqrt{\frac{1}{3}}$

↓

into original

$4\sqrt{\frac{1}{3}} + y^2 = 4\left(\sqrt{\frac{1}{3}}\right)^3$

$y^2 = \frac{4}{3\sqrt{3}} - \frac{4}{\sqrt{3}}$

$y^2 = \frac{-8}{3\sqrt{3}}$

no real sol.

$x = -\sqrt{\frac{1}{3}}$

↓

into original

$-4\sqrt{\frac{1}{3}} + y^2 = 4\left(-\sqrt{\frac{1}{3}}\right)^3$

$y^2 = \frac{4}{\sqrt{3}} - \frac{4}{3\sqrt{3}}$

$y^2 = \frac{8}{3\sqrt{3}}$

HTL

$y = \sqrt{\frac{8}{3\sqrt{3}}} \qquad y = -\sqrt{\frac{8}{3\sqrt{3}}}$

↓ ↓

both do not make denominator of $\frac{dy}{dx} = 0$