1. Find horizontal and vertical tangent lines of $x \ln x = y^3 - xy$

$$x\left(\frac{1}{x}\right) + \ln x = 3y^2 \frac{dy}{dx} - x\frac{dy}{dx} - y$$

$$1 + \ln x + y = \frac{dy}{dx}(3y^2 - x)$$

$$\frac{dy}{dx} = \frac{1 + \ln x + y}{3y^2 - x}$$

HTL: $1 + \ln x + y = 0$

$y = -1 - \ln x$

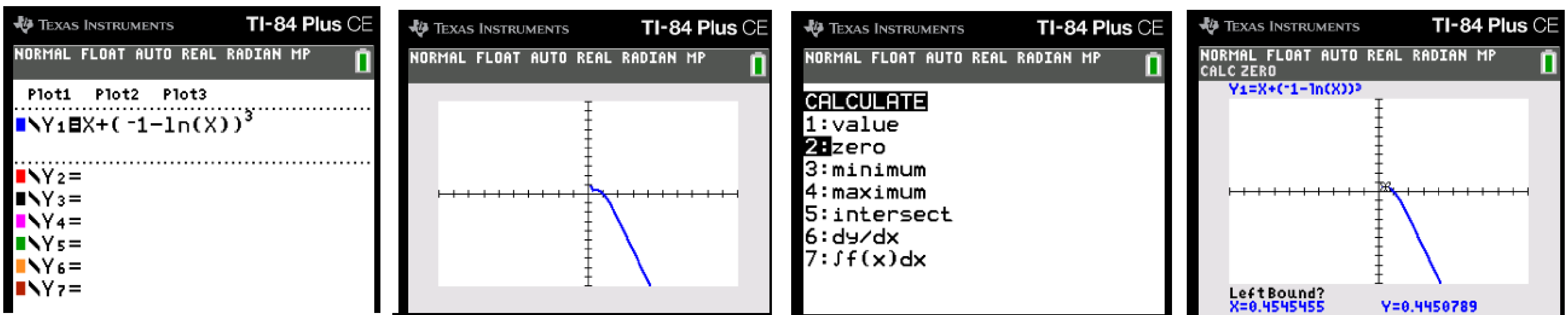
VTL: $3y^2 - x = 0$

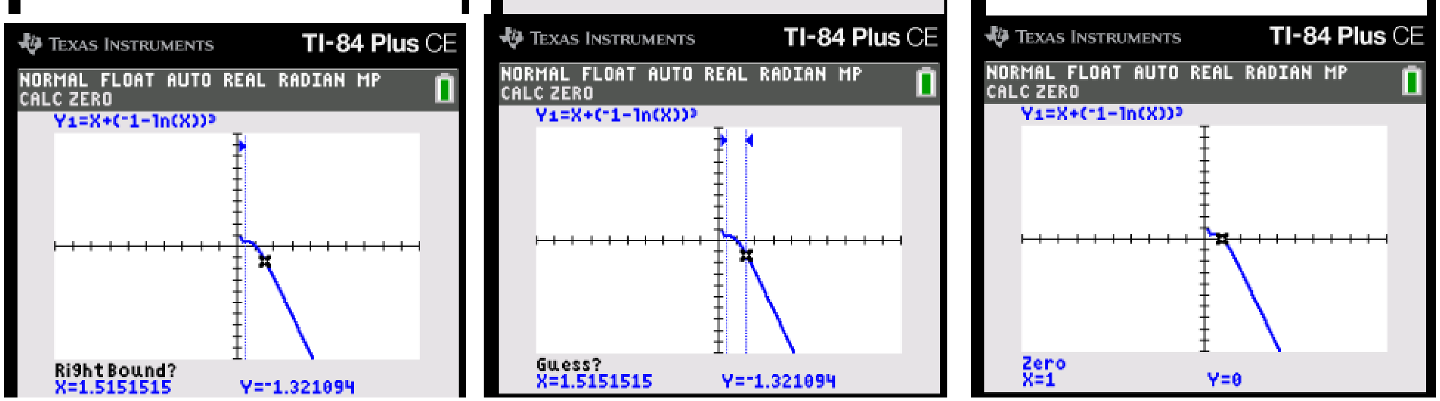
$x = 3y^2$

HTL: $x\ln x = (-1-\ln x)^3 - x(-1-\ln x)$

$$(-1-\ln x)^3 + x + x\ln x - x\ln x = 0$$

$$x + (-1-\ln x)^3 = 0$$





possible horizontal tangent line at $x = 1$

if $x=1$, $\quad 1(\ln 1) = y^3 - y$

$$y(y^2-1)=0$$

$$y=0 \quad y=\pm 1$$

$$\boxed{\begin{array}{c}\text{HTL}\\ y=-1\end{array}}$$

$1+\ln x+y\Big|_{(1,0)} = 1$ NO

$1+\ln x+y\Big|_{(1,-1)} = 1+0-1=0$ yes

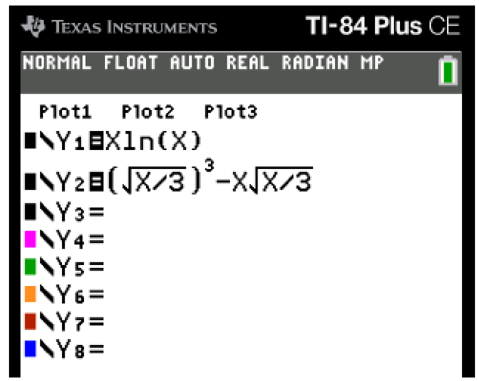
$1+\ln x+y\Big|_{(1,1)} = 1+0+1=2$ NO

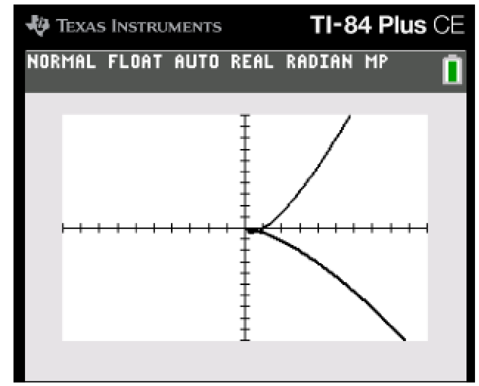
VTL

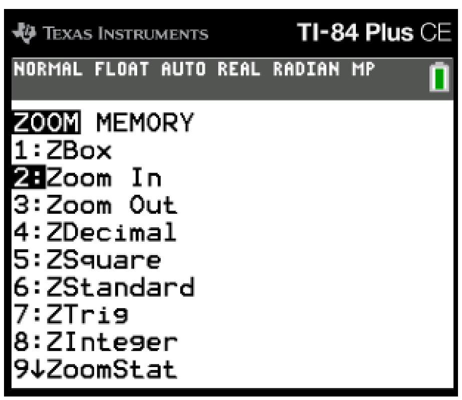
$$x = 3y^2 \longrightarrow y = \pm\sqrt{\frac{x}{3}}$$

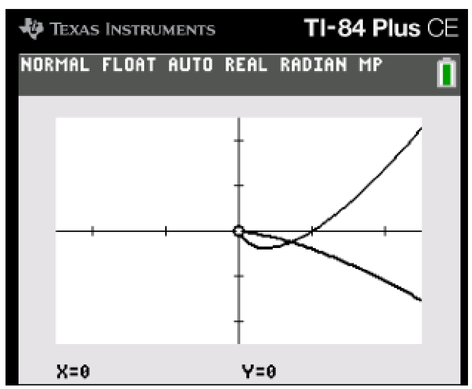
$$y = \sqrt{\frac{x}{3}}$$

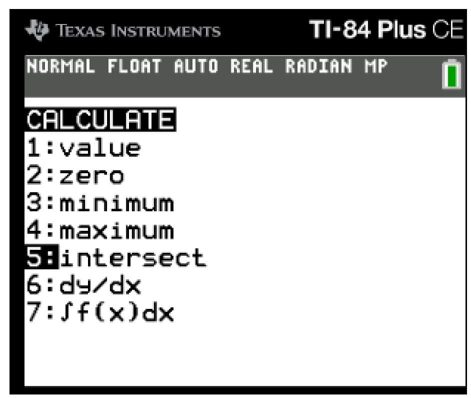
$$x\ln x = \left(\sqrt{\frac{x}{3}}\right)^3 - x\sqrt{\frac{x}{3}}$$

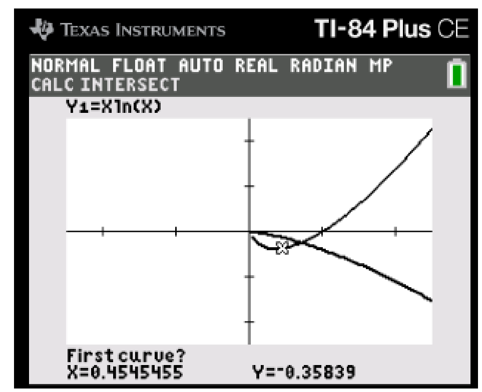


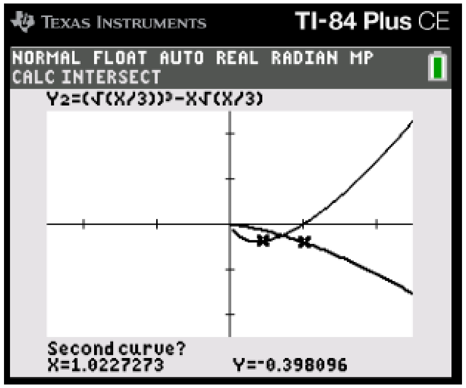


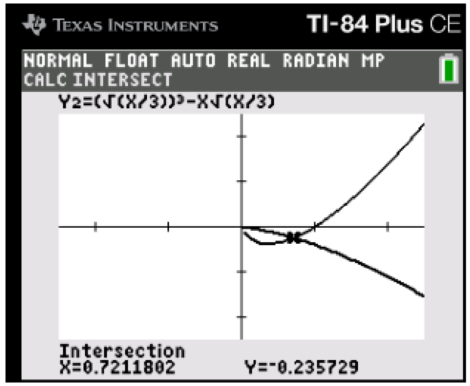






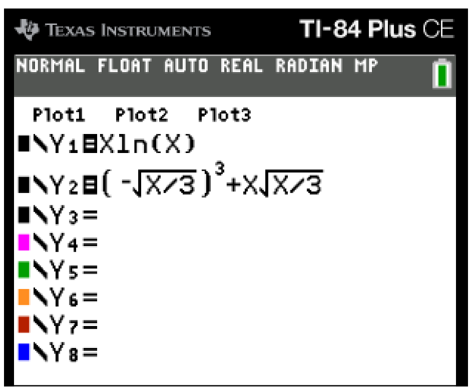


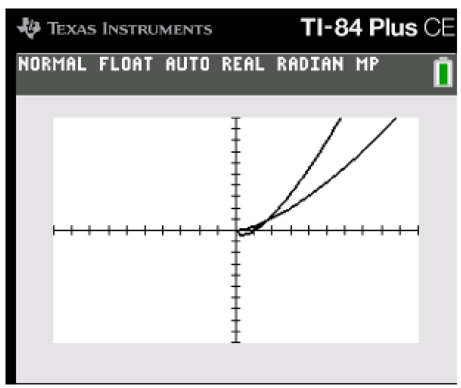


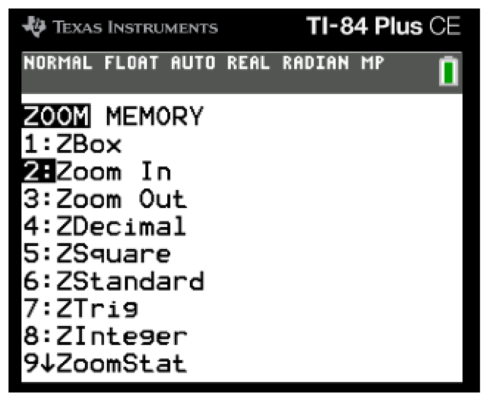


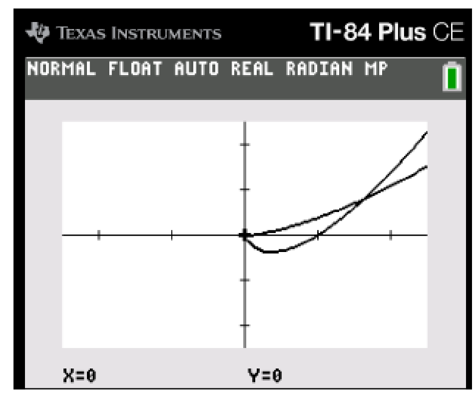
$\boxed{x = .721}$ VTL

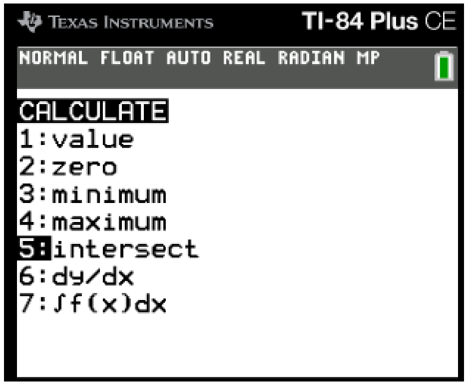
$$y = -\sqrt{\frac{x}{3}} \rightarrow x\ln x = \left(-\sqrt{\frac{x}{3}}\right)^3 + x\sqrt{\frac{x}{3}}$$

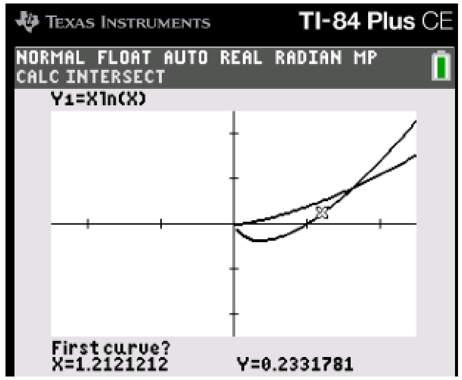


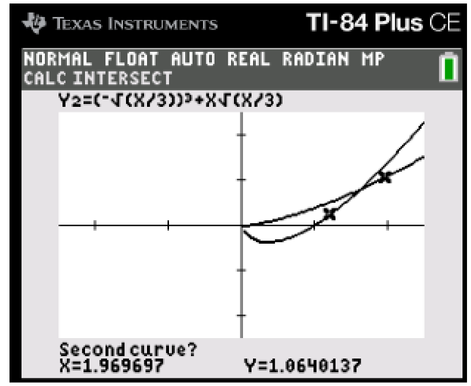


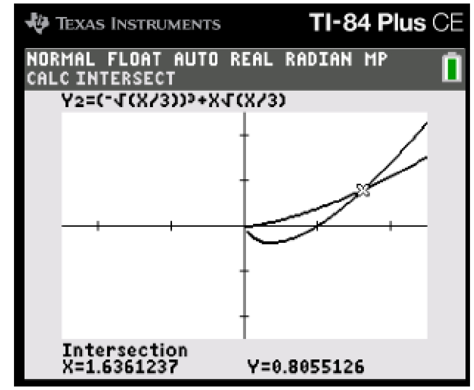












VTL $\boxed{X = 1.636}$

or $x = 3y^2$

yields $3y^2 \ln(3y^2) = y^3 - 3y^3$

$\to 3y^2 \ln(3y^2) + 2y^3 = 0$

$$y \approx -7.81671$$

$$x \ln x = (-7.81671)^3 + 7.81671 x$$

VTL $x \approx 183.30284$

2. Find vertical tangent line to $x^2 \ln xy = xy^2$

$$\frac{x^2\left(x\frac{dy}{dx}+y\right)}{xy} + 2x\ln(xy) = 2xy\frac{dy}{dx} + y^2$$

$$\frac{x^2}{y}\left(\frac{dy}{dx}\right) + x + 2x\ln(xy) = 2xy\frac{dy}{dx} + y^2$$

$$x^2\frac{dy}{dx} + xy + 2xy\ln(xy) = 2xy^2\frac{dy}{dx} + y^3$$

$$\frac{dy}{dx}(x^2 - 2xy^2) = y^3 - xy - 2xy\ln(xy)$$

$$\frac{dy}{dx} = \frac{y^3 - xy - 2xy\ln(xy)}{x^2 - 2xy^2}$$

VTL

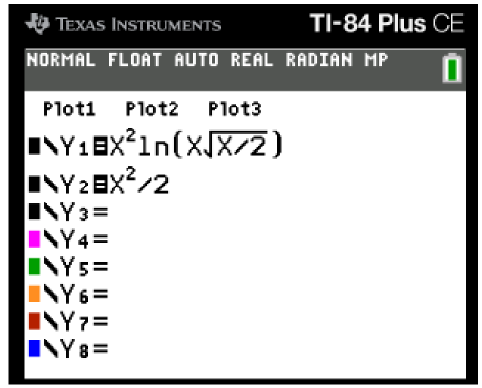
$$x^2 - 2xy^2 = 0$$

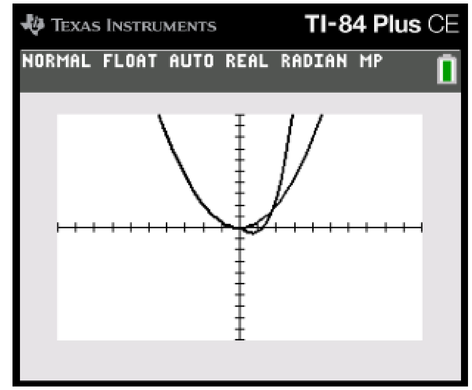
$$y^2 = \frac{-x^2}{-2x}$$

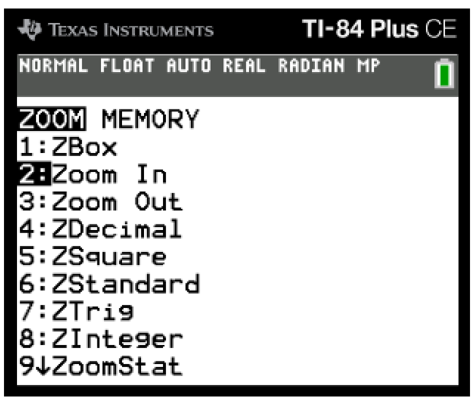
$$y = \pm\sqrt{\frac{x}{2}}$$

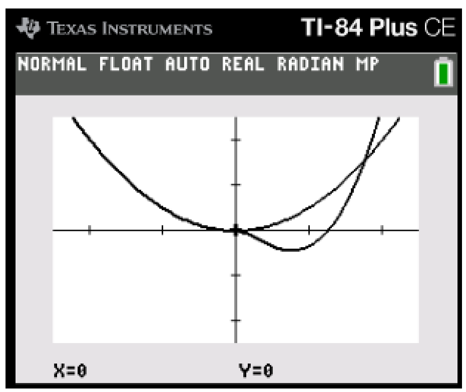
$$y = \sqrt{\frac{x}{2}}$$

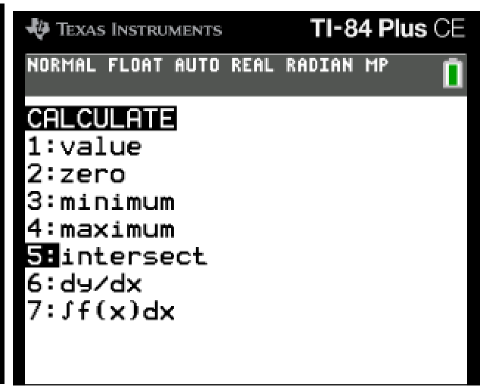
$$x^2\ln\left(x\sqrt{\frac{x}{2}}\right) = \frac{x^2}{2}$$

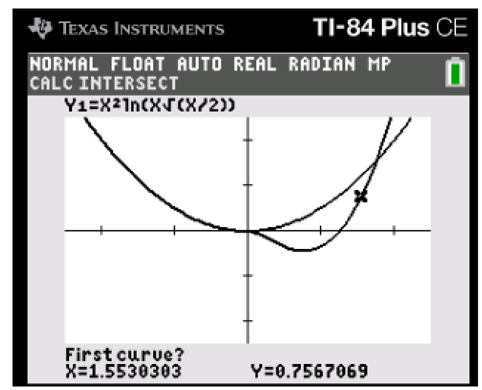


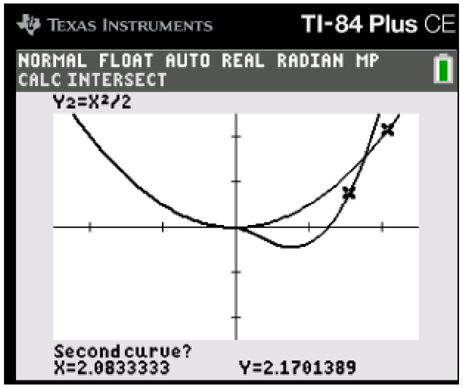


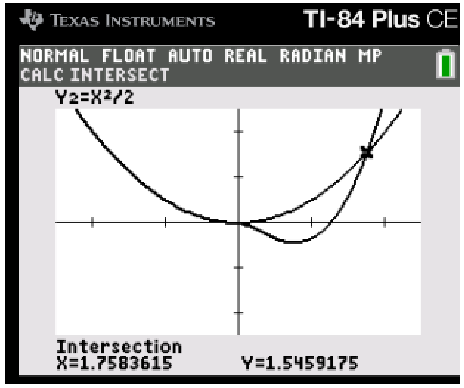












$\boxed{x = 1.758}$ ☆

$y = -\sqrt{\frac{x}{2}}$

$$x^2 \ln\left(-x\sqrt{\frac{x}{2}}\right) = \frac{x^2}{2}$$

↳ there are no values in the domain

NO SOLUTION

3. Find vertical tangent line of $x \ln y^2 = x^2y^2$

$$\frac{x(2y)}{y^2}\left(\frac{dy}{dx}\right) + \ln(y^2) = 2x^2y\frac{dy}{dx} + 2xy^2$$

$$\frac{2xy}{y^2}\frac{dy}{dx} - 2x^2y\frac{dy}{dx} = 2xy^2 - \ln(y^2)$$

$$2x\frac{dy}{dx} - 2x^2y^2\frac{dy}{dx} = 2xy^3 - y\ln(y^2)$$

$$\frac{dy}{dx} = \frac{2xy^3 - y\ln(y^2)}{2x - 2x^2y^2}$$

$$2x - 2x^2y^2 = 0$$

$$2x(1 - xy^2) = 0$$

$$\boxed{x = 0} \quad y^2 = \frac{1}{x}$$

$$y^2 = \frac{1}{x}$$

$$x \ln \frac{1}{x} = \frac{x^2}{x}$$

$$x \ln \frac{1}{x} = x$$

$$e^{\ln \frac{1}{x}} = e^1$$

$$\frac{1}{x} = e$$

$$\boxed{x = \frac{1}{e}} \approx .368$$

4. Find horizontal tangent lines of $x^2y - y^2 = 3xy^3$

$$x^2\frac{dy}{dx} + 2xy - 2y\frac{dy}{dx} = 9xy^2\frac{dy}{dx} + 3y^3$$

$$\left(x^2 - 2y - 9xy^2\right)\frac{dy}{dx} = 3y^3 - 2xy$$

$$\frac{dy}{dx} = \frac{3y^3 - 2xy}{x^2 - 2y - 9xy^2}$$

$$3y^3 - 2xy = 0$$

$$y\left(3y^2 - 2x\right) = 0$$

$$\boxed{y = 0}$$

$$3y^2 - 2x = 0$$

$$2x = 3y^2$$

$$x = \frac{3y^2}{2}$$

$x = \frac{3y^2}{2}$

$$\frac{9y^5}{4} - y^2 = \frac{9y^5}{2}$$

$$\frac{9y^5}{4} + y^2 = 0$$

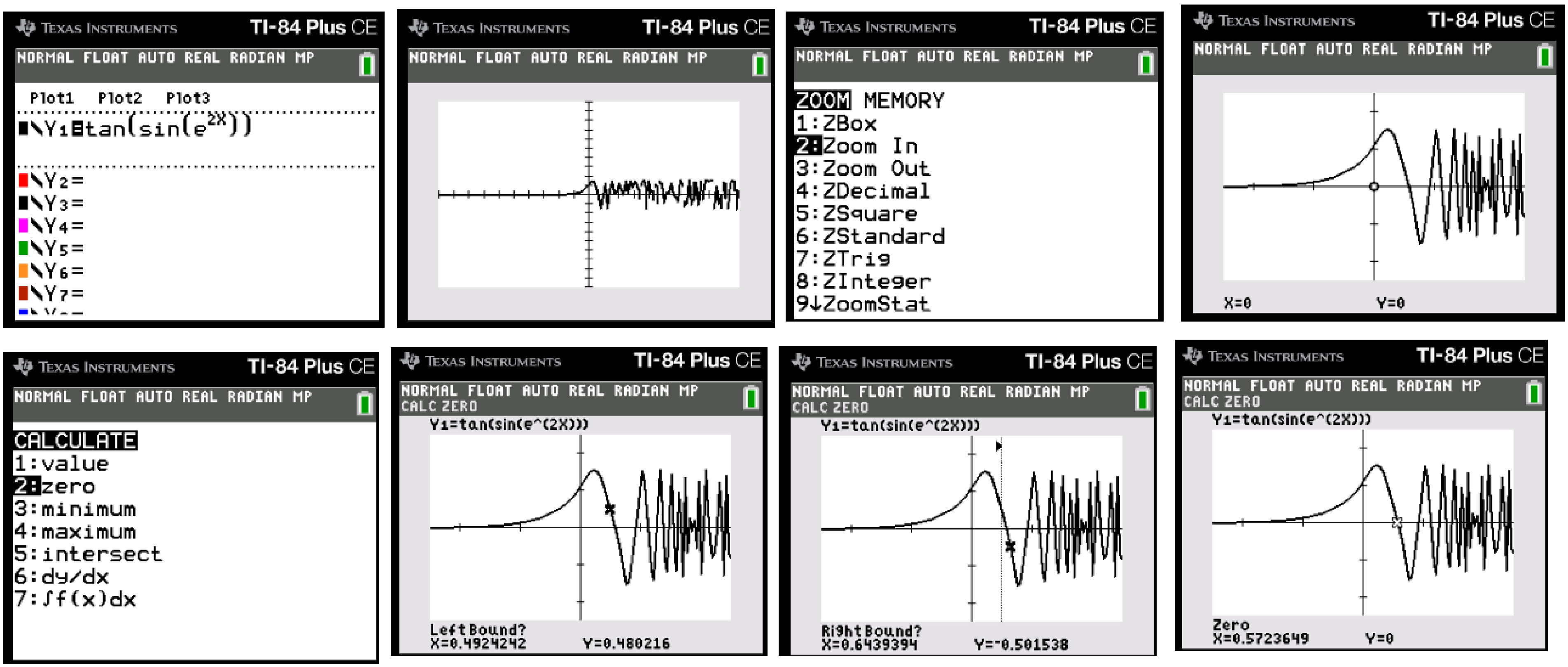
$$y^2\left(\frac{9y^3}{4} + 1\right) = 0$$

$$9y^3 = -4$$

$$\boxed{y = \sqrt[3]{\frac{-4}{9}} \approx -.763}$$

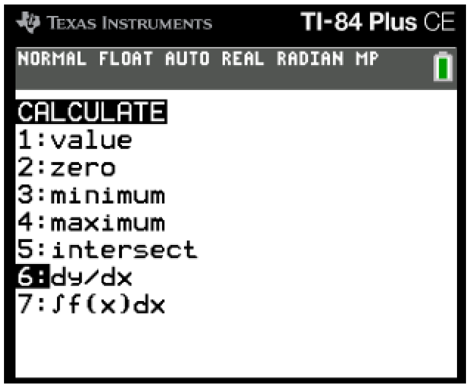
5. A particle moves according to the position equation $x(t) = \tan(\sin e^{2t})$ for $t \geq 0$. Find the velocity and the acceleration at the first time it is at the origin.

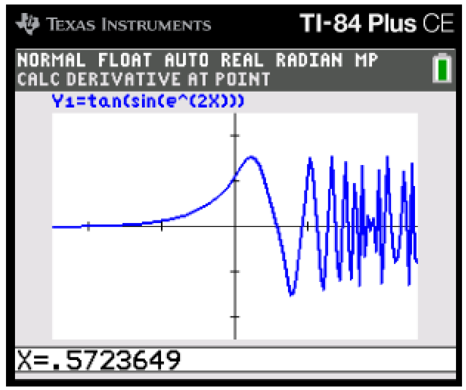
$$x(t) = \tan(\sin(e^{2t})) = 0$$

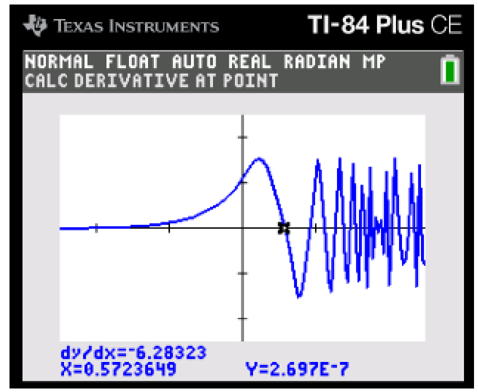


$$t \approx 0.5723649$$

$$V(.5723649) = x'(.5723649) = \boxed{-6.28323}$$

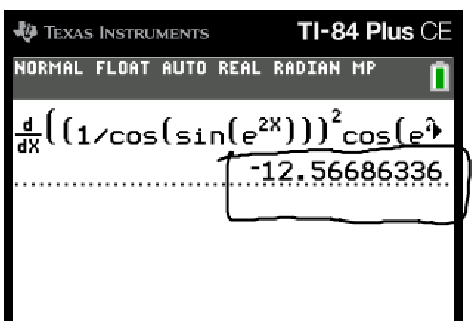


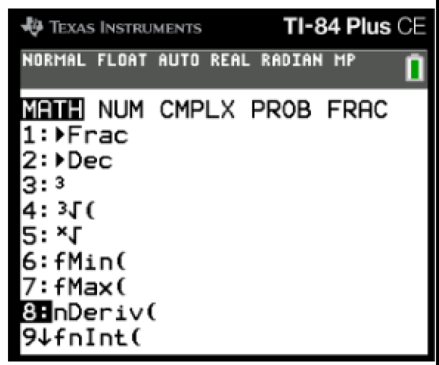


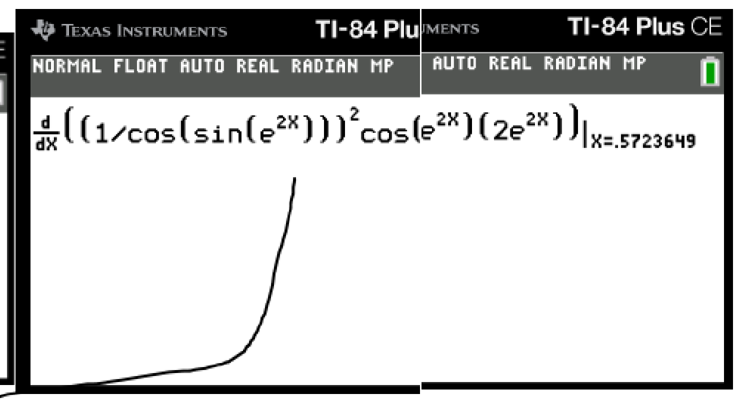


$$v(t) = x'(t) = \sec^2(\sin(e^{2t}))\cos(e^{2t})(2e^{2t})$$

$$a(.5723649) = v'(.5723649)$$

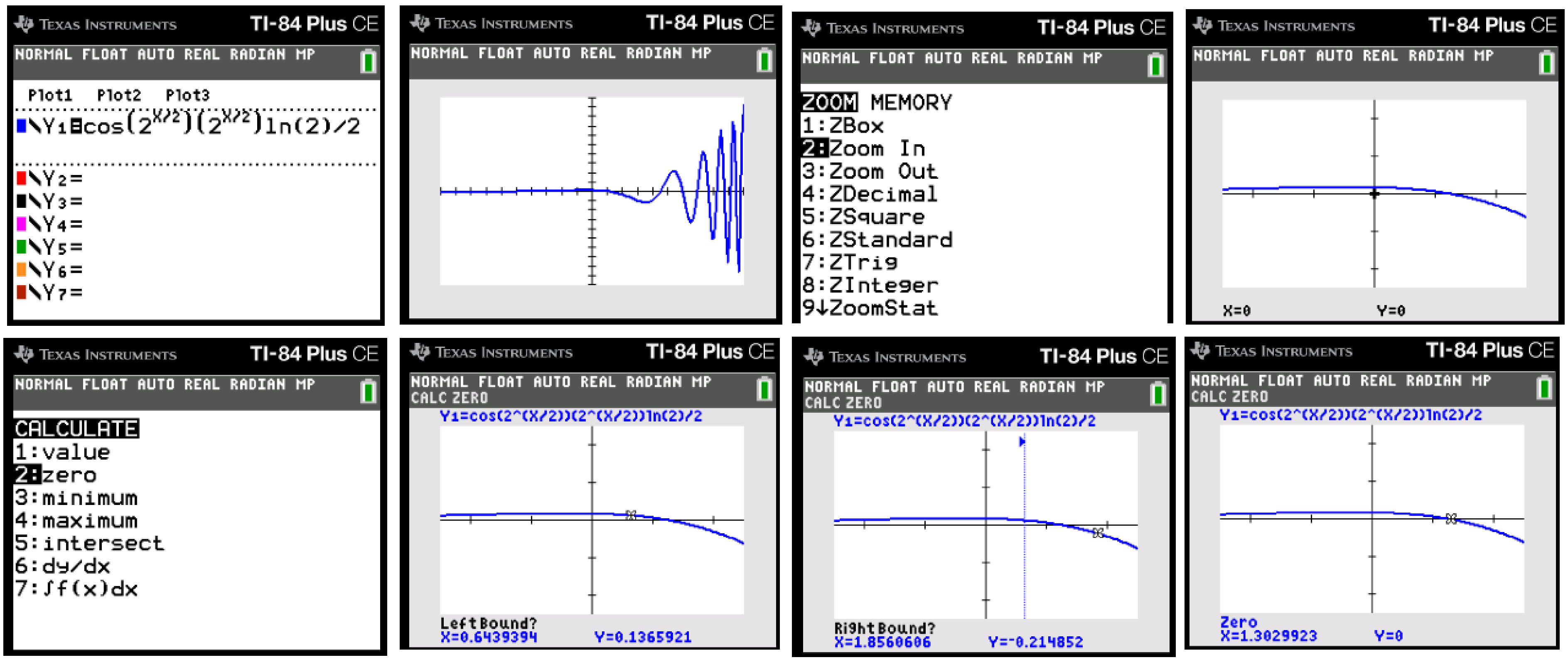






6. A particle moves according to the position equation $x(t) = \sin\left(2^{\frac{x}{2}}\right)$ for $t \geq 0$. Find the acceleration of the particle at the first time its velocity is zero.

$$x'(t) = v(t) = \cos\left(2^{\frac{x}{2}}\right)\left[2^{\frac{x}{2}}\right](\ln 2)\left(\tfrac{1}{2}\right) = 0$$



$$t \approx 1.3029923$$

$$a(1.3029923) = v'(1.3029923) \approx \boxed{-0.296368}$$