1. Find an equation of the tangent line to the curve $y = \frac{x}{1+x^2}$ at $x = 3$.

$$\frac{3}{1+3^2} = \frac{3}{10}$$

$$\left(3, \frac{3}{10}\right)$$

$$y' = \frac{(1+x^2)(1) - (x)(2x)}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$$\frac{1-3^2}{(1+3^2)^2} = \frac{-8}{100} = -\frac{2}{25}$$

$$y - \frac{3}{10} = -\frac{2}{25}(x-3)$$

$$y - \frac{3}{10} = -\frac{2}{25}x + \frac{6}{25}$$

$$y - \frac{15}{50} = \frac{-2}{25}x + \frac{12}{50}$$

$$\rightarrow \boxed{y = -\frac{2}{25}x + \frac{27}{50}}$$

2. Find an equation of the tangent line to each of the following curves:

a. $y = \frac{2x}{x+1}$ at $x = 1$

$(1, 1)$

$\frac{2(1)}{1+1} = \frac{2}{2} = 1$

$y' = \frac{(x+1)2 - 2x(1)}{(x+1)^2} = \frac{2x+2-2x}{(x+1)^2} = \frac{2}{(x+1)^2}$

$\left.\frac{2}{(x+1)^2}\right|_{x=1} = \frac{2}{(1+1)^2} = \frac{1}{2}$

$y - 1 = \frac{1}{2}(x-1) \rightarrow y - 1 = \frac{1}{2}x - \frac{1}{2} \rightarrow \boxed{y = \frac{1}{2}x + \frac{1}{2}}$

b. $y = x + \sqrt{x}$ at $x = 4$

$(4, 6)$

$y = 4 + \sqrt{4} = 6$

$y' = 1 + \frac{1}{2\sqrt{x}}$

$\left.1 + \frac{1}{2\sqrt{x}}\right|_{x=4} = 1 + \frac{1}{2(2)}$

$= \frac{5}{4}$

$y - 6 = \frac{5}{4}(x-4) \rightarrow y - 6 = \frac{5}{4}x - 5$

$\boxed{y = \frac{5}{4}x + 1}$

c. $y = (1 + 2x)^2$ at $x = 2$ $(2, 25)$

$y = (1+4)^2 = 25$

$y = u^2$ $y' \; 2(1+2x)(2) = 4(1+2x)\Big|_{x=2} = 4(1+4) = 20$

$u = 1+2x$

$y - 25 = 20(x-2)$

$y - 25 = 20x - 40$

$\boxed{y = 20x - 15}$

3. Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.

$y' = 18x^2 + 5 \quad \rightarrow \quad 18x^2 + 5 = 4$

$\rightarrow 18x^2 = -1$

$x^2 = -\frac{1}{18}$

has no real solution

so TL of y can not $= 4$

4. Find equations of the tangent and normal lines to the graph of the function at the given point, P. Then, find all x values for which the tangent line is horizontal.

a. $y = (2x - 1)^{10}; P(1,1)$

$y = u^{10}$

$u = 2x - 1$

$10(2x-1)^9(2) = 20(2x-1)^9 \Big|_1 = 20$

$m = 20$ $\quad m_{NL} = -\frac{1}{20}$

TL: $\boxed{y - 1 = 20(x - 1) \to y = 20x - 19}$

$\boxed{NL: y - 1 = -\frac{1}{20}(x - 1)}$

$20(2x-1)^9 = 0$

$2x - 1 = 0$

$\boxed{x = \frac{1}{2} \to \text{horiz. TL} \ @}$

b. $y = \sqrt{2x^2+1}$; $P(-1, \sqrt{3})$

$y = u^{\frac{1}{2}}$

$u = 2x^2+1$

$$\frac{4x}{2\sqrt{2x^2+1}} = \frac{2x}{\sqrt{2x^2+1}}\Bigg|_{-1} = \frac{-2}{\sqrt{3}}$$

$m = \frac{-2}{\sqrt{3}}$ TL: $\boxed{y - \sqrt{3} = -\frac{2}{\sqrt{3}}(x+1)}$

$$\frac{2x}{\sqrt{2x^2+1}} = 0 \rightarrow 2x = 0 \rightarrow \boxed{\text{Horz. TL @ } x = 0}$$

$m_{NL} = \frac{\sqrt{3}}{2}$ $\boxed{NL: y - \sqrt{3} = \frac{\sqrt{3}}{2}(x+1)}$

5. Find equations of the tangent and normal lines to the graph of the equation $y = 2x + \cos(4x)$ at the point, (0,1). Then, find all x values for which the tangent line is horizontal on $[0,2\pi)$.

$$y' = 2 - 4\sin(4x) \Big|_0 = 2 - 4\sin(0) = 2$$

$$m = 2$$

TL $\boxed{y - 1 = 2(x - 0) \text{ or } y = 2x + 1}$

$$m_{NL} = -\frac{1}{2}$$

$\boxed{\text{NL: } y - 1 = -\frac{1}{2}(x - 0) \text{ OR } y = -\frac{1}{2}x + 1}$

$$2 - 4\sin(4x) = 0$$

$$\sin(4x) = \frac{1}{2}$$

$$4x = \frac{\pi}{6} + 2\pi n \qquad 4x = \frac{5\pi}{6} + 2\pi n$$

$$x = \frac{\pi}{24} + \frac{\pi n}{2} \qquad x = \frac{5\pi}{24} + \frac{\pi n}{2}$$

$$x = \frac{\pi}{24},\ \frac{13\pi}{24},\ \frac{25\pi}{24},\ \frac{37\pi}{24} \qquad x = \frac{5\pi}{24},\ \frac{17\pi}{24},\ \frac{29\pi}{24},\ \frac{41\pi}{24}$$

6. Find an equation to the tangent line to the graph of $y = x^2 + \ln(2x - 5)$ at $x = 3$.

$$y' = 2x + \frac{1(2)}{2x-5} \Big|_{x=3} \qquad 6 + \frac{2}{1} = 8$$

$$m = 8$$

$$y = 3^2 + \ln(6-5) = 9$$

$$(3, 9) \qquad \boxed{y - 9 = 8(x-3)}$$

7. At what point on the graph of $y = 2e^x - 1$ is the tangent line perpendicular to the line $y = -3x + 2$

$m = \frac{1}{3}$

$y' = 2e^x$

$2e^x = \frac{1}{3}$

$e^x = \frac{1}{6}$

$\ln e^x = \ln \frac{1}{6}$

$x = -\ln(6)$

$y = 2e^{(-\ln(6))} - 1$

$y = -\frac{2}{3}$

$\boxed{\left(-\ln(6), -\frac{2}{3}\right)}$

8. Find an equation of the tangent line to the graph of $y = x + \ln(x)$ that is perpendicular to the line whose equation is $2x + 6y = 5$.

$$6y = -2x + 5$$

$$y = -\frac{1}{3}x + \frac{5}{6}$$

$$m_{\perp} = 3$$

$$y' = 1 + \frac{1}{x}$$

$$1 + \frac{1}{x} = 3$$

$$\frac{1}{x} = 2$$

$$x = \frac{1}{2}$$

$$y = \frac{1}{2} + \ln\left(\frac{1}{2}\right)$$

$$y = \frac{1}{2} - \ln(2)$$

$$\left(\frac{1}{2}, \frac{1}{2} - \ln(2)\right)$$

$$y - \frac{1}{2} + \ln(2) = 3\left(x - \frac{1}{2}\right)$$

$$\boxed{y = 3x - 1 - \ln(2)}$$

9. The curve with the equation $y^2 = 5x^4 - x^2$ is called a *Kampyle of Eudoxus*. Find an equation of the tangent line to this curve at the point (1,2).

$$2y\frac{dy}{dx} = 20x^3 - 2x$$

$$\frac{dy}{dx} = \frac{20x^3 - 2x}{2y} = \frac{10x^3 - x}{y} \longrightarrow \left.\frac{10x^3 - x}{y}\right|_{(1,2)} = \frac{10-1}{2} = \frac{9}{2}$$

$$y - 2 = \frac{9}{2}(x-1) \longrightarrow y - 2 = \frac{9}{2}x - \frac{9}{2}$$

$$\boxed{y = \frac{9}{2}x - \frac{5}{2}}$$

10. At how many different x-coordinates might the "bouncing wagon" curve have a horizontal tangent line? What are those potential x-coordinates? You do not need to determine the actual coordinate pairs where the horizontal tangent lines occur.

$$2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2$$

$$6y^2 \frac{dy}{dx} + 2y\frac{dy}{dx} - 5y^4 \frac{dy}{dx} = 4x^3 - 6x^2 + 2x$$

$$\frac{dy}{dx} = \frac{4x^3 - 6x^2 + 2x}{6y^2 + 2y - 5y^4} = 0 \quad \text{when } 4x^3 - 6x^2 + 2x = 0$$

$$\rightarrow \quad 2x(2x^2 - 3x + 1) = 0$$

$$\rightarrow \quad 2x(2x - 1)(x - 1) = 0$$

$$\rightarrow \quad \boxed{x = 0 \quad x = \frac{1}{2} \quad x = 1}$$

11. Where does the normal line to the ellipse $x^2 - xy + y^2 = 3$ at the point $(-1,1)$ intersect the ellipse a second time?

$$2x - \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y - x) = y - 2x$$

$$\frac{dy}{dx} = \frac{y-2x}{2y-x}\bigg|_{(-1,1)} \quad \frac{1-2(-1)}{2(1)-(-1)} = \frac{3}{3} = 1$$

$$m_{TL} = 1 \rightarrow m_{NL} = -1$$

$$y - 1 = -1(x+1)$$

$$y - 1 = -x - 1$$

$$y = -x$$

so $x^2 - x(-x) + (-x)^2 = 3$

$$x^2 + x^2 + x^2 = 3$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

so @ $x = 1$ on the ellipse,

$$1^2 - y + y^2 = 3$$

$$y^2 - y - 2 = 0$$

$$(y-2)(y+1) = 0$$

$$y = 2 \quad y = -1$$

$$(1,2) \quad (1,-1)$$

only $\boxed{(1,-1)}$ satisfies $y = -x$