

1. Which of the following could be true if $f''(x) = x^{-\frac{2}{3}}$?

I. $f(x) = 3x^{\frac{4}{3}}$

II. $f'(x) = 10 + 3x^{\frac{1}{3}}$

III. $f'''(X) = -\frac{2}{3}x^{-\frac{5}{3}}$

- a. I only
- b. I and II
- c. II and III
- d. III only
- e. I, II, and III

2. If $y = x \sin(x)$, $\frac{dy}{dx} =$

- a. $\sin(x) + \cos(x)$
- b. $\sin(x) + x \cos(x)$
- c. $\sin(x) - x \cos(x)$
- d. $x(\sin(x) + \cos(x))$
- e. $x(\sin(x) - \cos(x))$

3. If $y = (x^3 - \cos(x))^5$, then $y' =$

- a. $5(x^3 - \cos(x))^4$
- b. $5(3x^2 + \sin(x))^4$
- c. $5(3x^2 + \sin(x))$
- d. $5(3x^2 + \sin(x))^4(6x + \cos(x))$
- e. $5(x^3 - \cos(x))^4(3x^2 + \sin(x))$

4. Let f be the function defined by $f(x) = \sqrt{|x+2|}$ for all x . Which of the following statements is true?

- a. f is continuous but not differentiable at $x = -2$
- b. f is differentiable at $x = -2$
- c. f is not continuous at $x = -2$
- d. $\lim_{x \rightarrow -2} f(x) \neq 0$
- e. $x = -2$ is a vertical asymptote of the graph of f

5. If $f(x) = \cos^3(4x)$, $f'(x) =$

- a. $3 \cos^2(4x)$
- b. $-12 \cos^2(4x) \sin(4x)$
- c. $-3 \cos^2(4x) \sin(4x)$
- d. $12 \cos^2(4x) \sin(4x)$
- e. $-4 \sin^3(4x)$

6. If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$

- a. $-\frac{7}{11}$
- b. $-\frac{7}{13}$
- c. $-\frac{1}{2}$
- d. $-\frac{3}{14}$
- e. 7

7. If $f(x) = x^3 - x^2 + x - 1$, then $f'(2) =$

- a. 10
- b. 9
- c. 7
- d. 5
- e. 3

8. Let f be the function given by $f(x) = x^3 - 6x^2 + 8x - 2$. What is the instantaneous rate of change of f at $x = 3$?

- a. -5
- b. $-\frac{15}{4}$
- c. -1
- d. 6
- e. 17

9. If $y = 5x\sqrt{x^2 + 1}$, then $\frac{dy}{dx}$ at $x = 3$ is

- a. $\frac{5}{2\sqrt{10}}$
- b. $\frac{15}{\sqrt{10}}$
- c. $\frac{15}{2\sqrt{10}} + 5\sqrt{10}$
- d. $\frac{45}{\sqrt{10}} + 5\sqrt{10}$
- e. $\frac{45}{\sqrt{10}} + 15\sqrt{10}$

10. The values of f and g and their derivatives at $x = 3$ are given as $f(3) = -1$, $g(3) = 2$, $f'(3) = 5$, and $g'(3) = -2$. Let k be the function given by $k(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$. What is the value of $k'(3)$?
- a. $-\frac{5}{2}$
 - b. -2
 - c. 2
 - d. 3
 - e. 8
11. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is
- a. $\frac{7}{\sqrt{5}}$
 - b. $\frac{14}{\sqrt{5}}$
 - c. $\frac{18}{\sqrt{5}}$
 - d. $\frac{15}{\sqrt{21}}$
 - e. $\frac{30}{\sqrt{21}}$
12. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?
- a. $(0,0)$ only
 - b. $\left(\frac{1}{2}, \frac{1}{5}\right)$ only
 - c. $(0,0)$ and $(-4,2)$
 - d. $(0,0)$ and $\left(4, \frac{2}{3}\right)$
 - e. There are no such points.

13. If $f(x) = 8x^{-2} + \frac{1}{2}x^4 + 8$, then $f'(2) =$

- a. 22
- b. 14
- c. -92
- d. -84
- e. 0

14. $\frac{d}{dx}[\cos^4(x^3)] =$

- a. $-12x \cos^3(x^3)$
- b. $-\sin^4(x^3)$
- c. $4 \cos^3(x^3)$
- d. $-12x^2 \cos^3(x^3) \sin(x^3)$
- e. $-4 \cos^3(x^3) \sin(x^3)$

15. $\frac{d}{dx}\left(\frac{2x-1}{x^2+1}\right)$

- a. $-\frac{2(x^2-x-1)}{(x^2+1)^2}$
- b. $\frac{1}{x}$
- c. $\frac{2(x+1)}{x^2+1}$
- d. $\frac{4x}{(x^2+1)^2}$
- e. $\frac{2x(2x-1)}{x^2+1}$

16. If f is the function defined below, then $f'(1)$ is

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < 1 \\ 3x^2 + 5 & \text{if } x \geq 1 \end{cases}$$

- a. 2
- b. 6
- c. -2
- d. -6
- e. nonexistent

17. If $y = x^2 + 3x$ and $u = 4x + 1$, then $\frac{dy}{du} =$

- a. $2x + 3$
- b. $\frac{x^2 + 3x}{4x + 1}$
- c. $\frac{x}{2}$
- d. $\frac{2x + 3}{4}$
- e. $\frac{1}{2x + 3}$

18. If $f(x) = 24\sqrt[3]{x} - \frac{6}{\sqrt[3]{x}}$, then $f'(8) =$

- a. 45
- b. 12
- c. -30
- d. $\frac{17}{8}$
- e. 0

19. If $f(x) = x^3 - 1$ and g is a differentiable function of x , what is the derivative of $f(g(x))$?

- a. $3x^2 g(x)$
- b. $3x^2 g'(x)$
- c. $3x^3 g'(x)$
- d. $3[g(x)]^2 g'(x)$
- e. $3x^2 g(x) - 1$

20. If $y = \sin(x) \tan(x)$, then at $x = \frac{\pi}{4}$, $\frac{dy}{dx} =$

- a. $\frac{3\sqrt{2}}{2}$
- b. $\sqrt{2}$
- c. 1
- d. $\frac{\sqrt{2}}{2}$
- e. $-\frac{\sqrt{2}}{2}$

21. $\frac{d}{dx}(x + (x + 1)^2)^3 =$

- a. $3(x + (x + 1)^2)^2$
- b. $x^2 + 3(x + 1)^2$
- c. $3x(x + (x + 1)^2)^2$
- d. $(2x + 1)(x^2 + 3x + 1)^2$
- e. $(6x + 9)(x^2 + 3x + 1)^2$

22. If $y = 2x^{\frac{5}{2}} \tan(x)$, then $\frac{dy}{dx} =$

a. $x^{\frac{3}{2}}(5 \tan(x) + 2x \sec^2(x))$

b. $5x^{\frac{3}{2}} \tan(x) - x \csc^2(x)$

c. $5x^{\frac{5}{2}} \tan(x) + 2x^{\frac{3}{2}} \sec^2(x)$

d. $x^{\frac{3}{2}}(5 \tan(x) + x \sec^2(x))$

e. $x^2 \tan(x) - 5x \csc^2(x)$

23. If $f(x) = x^2 \cos^2(x)$, then $f'(\pi) =$

a. -1

b. 2π

c. 0

d. π

e. -2π

24. Suppose $f(x) = \frac{1}{2}x^5 + x^{-2} + 3$. Then $f'''(x) =$

a. $30x^2$

b. $\frac{20}{3}x^2 - 24x^{-5}$

c. $\frac{20}{3}x^3 - 24x^{-4}$

d. $30x^2 + 24x^{-5}$

e. $30x^2 - 24x^{-5}$

25. If $g(x) = 3 \tan^2(2x)$ then $g' \left(\frac{\pi}{8} \right)$ is

- a. 6
- b. $6\sqrt{2}$
- c. 12
- d. $12\sqrt{2}$
- e. 24

26. If $y = e^x \cos x$, then $\frac{dy}{dx} =$

- a. $e^x \cos x$
- b. $-e^x \sin x$
- c. $e^x(\cos x - \sin x)$
- d. $e^x(\sin x - \cos x)$
- e. $e^x(\sin x + \cos x)$

27. What is the $\lim_{h \rightarrow 0} \frac{\csc\left(\frac{\pi}{4}+h\right) - \csc\left(\frac{\pi}{4}\right)}{h}$?

- a. $\sqrt{2}$
- b. $-\sqrt{2}$
- c. 0
- d. $-\frac{\sqrt{2}}{2}$
- e. Undefined

28. What is $\lim_{\Delta x \rightarrow 0} \frac{\sin(\frac{\pi}{3} + \Delta x) - \sin(\frac{\pi}{3})}{\Delta x}$?

- a. $-\frac{1}{2}$
- b. 0
- c. $\frac{1}{2}$
- d. $\frac{\sqrt{3}}{2}$
- e. Nonexistent

29. If $f(x) = 5 \cos^2(\pi - x)$, then $f'(\frac{\pi}{2})$ is

- a. 0
- b. -5
- c. 5
- d. -10
- e. 10

30. What is the slope of the tangent to the curve $x^3 - y^2 = 1$ at $x = 1$

- a. $-\frac{3}{2}$
- b. 0
- c. $\frac{3}{2\sqrt{2}}$
- d. $\frac{3}{2}$
- e. Undefined

31. If $f(x) = \frac{\sqrt{x+1}}{x^3+2}$, then $f'(0) =$

a. $-\frac{1}{4}$

b. $\frac{1}{2}$

c. $\frac{3}{8}$

d. $\frac{1}{4}$

e. undefined

32. If $f(x) = x^2$, $g(x) = 3x$, and $h(x) = f(x) \cdot g(x)$, then $h'(2)$ equals

a. -12

b. 12

c. 24

d. 36

e. 324

33. Find $\frac{dy}{dx}$ if $y = 3^{(4-x^2)}$

a. $(\ln 3)3^{(4-x^2)}$

b. $-2x(\ln 3)3^{(4-x^2)}$

c. $-2x(4-x^2) \ln 3$

d. $(-2x)3^{(4-x^2)}$

e. $(4-x^2)3^{(3-x^2)}$

34. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y , $\frac{dy}{dx} =$

a. $-\frac{x^2+y}{x+2y^2}$

b. $-\frac{x^2+y}{x+y^2}$

c. $-\frac{x^2+y}{x+2y}$

d. $-\frac{x^2+y}{2y^2}$

e. $\frac{-x^2}{1+2y^2}$

35. If $f(x) = 3 \tan^5(2x)$, the, $f'(x)$ is

a. $3 \sec^{10}(2x)$

b. $30 \sec^{10}(2x)$

c. $15 \tan^4(2x)$

d. $30 \tan^4(2x)$

e. $30 \tan^4(2x) \sec^2(2x)$

36. Let f and g be differentiable functions such that $g(x) = f^{-1}(x)$. If $f(2) = 4$, $f(3) = 9$, $g'(4) = \frac{1}{4}$, and $g'(9) = \frac{1}{6}$, what is the value of $f'(3)$?

a. 0

b. 3

c. 4

d. 6

e. 9

37. If $g(x) = \frac{1}{32}x^4 - 5x^2$, find $g'(4)$

- a. -72
- b. -32
- c. -24
- d. 24
- e. 32

38. If $f(x) = \frac{x^5 - x + 2}{x^3 + 7}$, find $f'(x)$

- a. $\frac{5x^4 - 1}{3x^2}$
- b. $\frac{5x^4 - 3x^2 - 1}{x^3 + 7}$
- c. $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{x^3 + 7}$
- d. $\frac{(x^5 - x + 2)(3x^2) - (x^3 + 7)(5x^4 - 1)}{(x^3 + 7)^2}$
- e. $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)^2}$

39. Evaluate $\lim_{h \rightarrow 0} \frac{5\left(\frac{1}{2} + h\right)^4 - 5\left(\frac{1}{2}\right)^4}{h}$

- a. $\frac{5}{2}$
- b. $\frac{5}{16}$
- c. 40
- d. 160
- e. The limit does not exist.

40. If $f(x) = x^2 \cos(2x)$, find $f'(x)$

- a. $2x \sin(2x)$
- b. $-2x \cos(2x) + 2x^2 \sin(2x)$
- c. $-4x \sin(2x)$
- d. $2x \cos(2x) - 2x^2 \sin(2x)$
- e. $2x - 2 \sin(2x)$

41. Find $\frac{dy}{dx}$ if $x^3y + xy^3 = -10$

- a. $3x^2 + 3xy^2$
- b. $-(3x^2 + 3xy^2)$
- c. $\frac{3x^2y+y^3}{3x^2y+x^3}$
- d. $-\frac{3x^2y+y^3}{3xy^2+x^3}$
- e. $-\frac{x^2y+y^3}{xy^2+x^3}$

42. If $f(x) = \sqrt{1 + \sqrt{x}}$, find $f'(x)$

- a. $-\frac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$
- b. $\frac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$
- c. $\frac{1}{4\sqrt{1+\sqrt{x}}}$
- d. $\frac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$
- e. $-\frac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

43. The derivative of $\arctan(x^2 + 3x)$ is

- a. $\frac{1}{1+(x^2+3x)}$
- b. $\frac{x^2+3x}{1+x^2}$
- c. $\frac{2x+3}{x^4+6x^3+9x^2+1}$
- d. $\frac{2x+3}{1+x^2}$
- e. $\frac{x^2+3x}{x^4+6x^3+9x^2+1}$

44. If $f(x) = \sin^2 x$, find $f'''(x)$

- a. $-\sin^2 x$
- b. $2 \cos 2x$
- c. $\cos 2x$
- d. $-4 \sin 2x$
- e. $-\sin 2x$

45. Find the slope of the normal line to $y = x + \cos(xy)$ at $(0,1)$

- a. 1
- b. -1
- c. 0
- d. 2
- e. Undefined

46. If $f(x) = 3^{\pi x}$, then $f'(x) =$

a. $\frac{3^{\pi x}}{\pi \ln 3}$

b. $\frac{3^{\pi x}}{\ln 3}$

c. $\frac{3^{\pi x}}{\pi}$

d. $\pi(3^{\pi x} - 1)$

e. $\pi(\ln 3)3^{\pi x}$

47. If $y = \left(\frac{x^3-2}{2x^5-1}\right)^4$, find $\frac{dy}{dx}$ at $x = 1$

a. -52

b. -28

c. -13

d. 13

e. 52